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APPLIED KINEMATICS

For Students and Mechanical Designers

BY

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Dedicated

to those who have effectively labored to forward the science and art of building machines, on whose accomplishments all must build who hope to contribute to human welfare through the service of things mechanical, and whose work helps to make every generation insolvent debtors to the thinkers of the past.

PREFACE TO SECOND EDITION

In this revision which is, in fact, a re-writing, the treatment of some topics has been considerably extended. To meet the urgent demand of present engineering practice for more comprehensive education on acceleration, that topic has been given special attention, and much illustrative problem material has been included. The increasing usefulness of the phorograph as an engineering tool seemed to justify the presentation of its theory and application in a separate chapter.

As far as possible, all material has been related to pre-requisite subjects by rigorous development; empirical presentation has been avoided. As suggested by the adjective in the title, no effort has been spared to relate the illustrative material and the problems at the end of each chapter to engineering practice.

Special attention has been given to graphical methods. Teachers of kinematics would seem to have a large measure of responsibility for this phase of engineering education. The time allotted in engineering curricula to courses on kinematics or mechanics of machinery is generally brief, so the choice of topics is as difficult as it is important. In this text, Chapters I, II, III, V, VI, and VII are basic. Beyond this, the instructor will find it possible to add the other topics, in order and amount to suit his course, except that Chapters IX and X on gearing form a sequence.

It is a pleasure to record the assistance and advice in this work given by my colleagues: Professor William J. Stevens, Mr. Kenneth W. Riddle, and Mr. C. R. Connell in manuscript reading; Mr. William R. Berry and Lieut. S. Herbert Raynes in manuscript reading and in the preparation of illustrations. Appreciation is also expressed for the cooperation of the many industrial companies mentioned in the text, for furnishing illustrations, and in many cases, special data and advice.

J. Harland Billings

Philadelphia, Pa.

CONTENTS

CHAPTER I

MOTION IN MACHINES

1–1. **The Qualities of a Machine.**—Kinetics is the science of motion. Engineering kinematics deals with the relative motions and velocities of machine parts and with their accelerations.

A *machine* may be defined as a combination of two or more resistant bodies, the motions of which are constrained and which modifies energy to do work.

A machine is a combination of resistant bodies. Resistant does not mean rigid. We have no materials that are rigid or unyielding under stress but all solid materials resist deformation. Indeed, even liquids and gases, when their volumes are confined, become resistant bodies and can serve as machine parts. The oil in the hydraulic brake transmits the force which applies the brake. The burning gas in the cylinder of the automotive engine, Fig. 1–1, performs two functions. It is the medium by which heat energy is transformed into mechanical energy. It also functions as a machine part, transmitting force from the cylinder head to the piston as the connecting rod does from the piston to the crank pin. In general, however, the deformations of machine parts under load are so small as to be neglected when considering the velocities and accelerations of the parts. In kinematics, therefore, we assume solid bodies to be rigid.

The motions of machine parts are constrained; that is, the parts can move in a certain manner only, dĕpending on the connections and on the nature of the machine. Every point in the platen or moving table of the planer, Fig. 1–2, is limited to straight-line motion. The fly-wheel of an engine can have no motion with respect to the frame but rotation. The theory of constrainment will be treated later.

The simplest example of a machine is the lever and fulcrum. Whether simple or complex, however, a machine can generate no energy. If supplied a certain amount of energy in a given time, it will use a certain portion as friction which will in turn be delivered as heat; it may store some temporarily as kinetic energy of its moving parts, and the rest will be delivered as work, useful or otherwise. This is simply a statement of the law of conservation of energy as applied to a machine. No machine, therefore, can do more than modify energy. Our so-called prime movers change such energy as heat, or the action of gravity on falling

water, into mechanical energy but are *not generators of energy* in any sense. A lathe[1] receives energy from a belt and delivers it at the point of the tool for the removal of the chip. If the work could be performed satisfactorily by attaching the tool to the belt, the lathe building industry would be ruined. Machines are useful only to the extent that they usefully perform the modifying function.

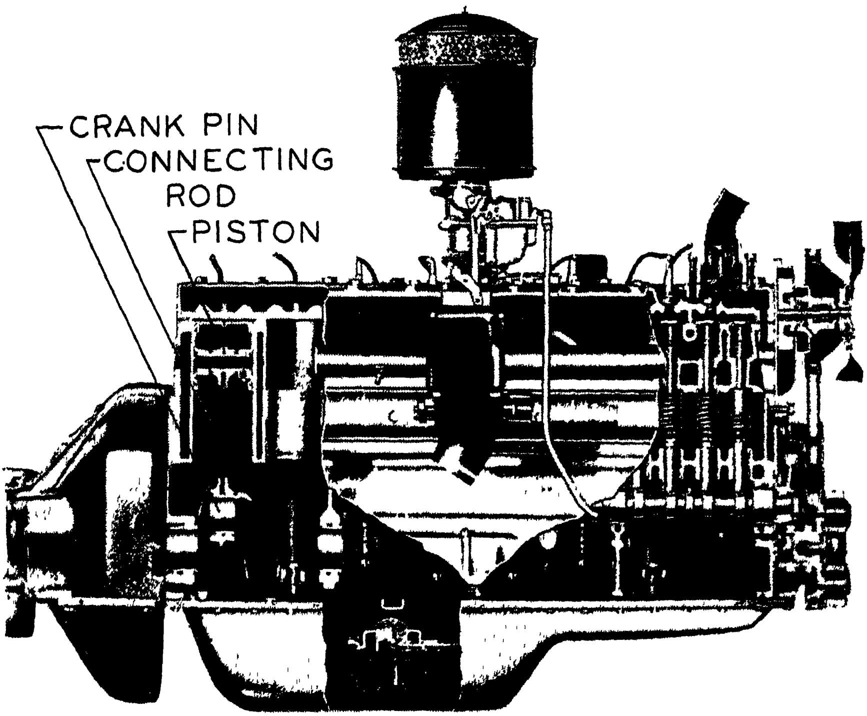


FIG. 1–1. Automotive Engine.

Pontiac Motors Division, General Motors Corporation, Pontiac, Mich

1–2. Relation of Kinematics to Machine Design.—The bearing of kinematics on machine design will be seen from an analysis of the steps necessary in the design and development of an entirely new machine for a specified purpose. These steps are:

(1) The choice of mechanisms that will give the desired motions to the various parts, either using existing mechanisms or inventing new or using new combinations.

[1] For a description of Maudsley's invention of the cross-slide tool support and all metal lathe see A History of Mechanical Inventions by Usher (McGraw-Hill), p. 328.

(2) The choice of materials to be used.

(3) The determination of primary loads.

(4) The computation of the form and dimensions of the parts necessary to resist the above loads without excessive stresses or deformations.

(5) The determination of secondary loads occasioned by the necessary accelerations of the parts themselves, with consequent adjustment of dimensions where necessary.

Mastery of the principles of kinematics is necessary in dealing with items one, three, and five, and also leads to an appreciation of the possibilities and limitations of machinery.

1–3. **Motion and Velocity are Relative; Acceleration, Absolute.**—Motion is change of position, and velocity, time rate of change of position.

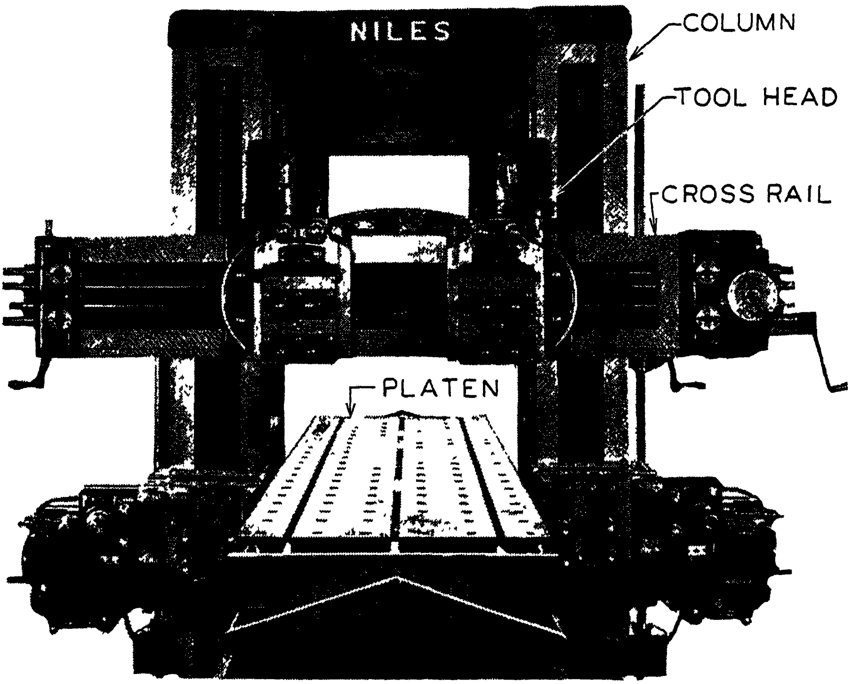


FIG. 1–2. Metal Planer.

The General Machinery Corporation, Hamilton, Ohio.

Motion and velocity can have no meaning apart from some reference body either expressed or implied. To say that a train was travelling sixty miles per hour would have no definite meaning if the hearer did not understand that the reference body implied was the earth. A man walking on the deck of a riverboat at three miles per hour may have a

motion of ten miles per hour with respect to the water and in a different direction. He will have a different motion with respect to the ground over which the water is flowing and a still different motion with respect to a train on the bank. To speak with definiteness of motion or velocity, the body of reference must be expressed or plainly implied. Principles are often made plainer by considering extreme cases. Imagine an object to be moving in a straight path with respect to the earth at forty miles per hour and further imagine that the rest of the material universe, including the earth and the atmosphere, goes out of existence. Would the object have any motion under these conditions?[1] Measurement of motion or velocity that does not involve acceleration is impossible without a reference body.

Acceleration, however, resulting as it does from the action of unbalanced force upon mass, always has a definite value independent of any body of reference and can therefore be called absolute. The term "absolute velocity" is frequently used for velocity relative to the earth. It should be remembered that such velocity is just as relative as any other.

1–4. **Classification of Motion of a Body.**—The motion of a *point* at any instant, with respect to any reference body is completely known, if the magnitude, direction, and sense of its motion be known, but in the case of a body, which contains an infinite number of points, the description of its motion is simplified by naming and defining certain classes of motion that solid bodies commonly have.

Plane motion of a body is such that all its points move in planes that are parallel to or coincident with some plane of reference. The motions, with respect to the frame, of the crosshead, connecting rod, crank, and flywheel of the air compressor, Fig. 1–4, satisfy this definition. A hockey puck has plane motion so long as its flat side maintains contact with the ice.

Rotation of a body is plane motion such that all points remain at fixed distances from a line called the axis.

Translation of a body is such motion that all plane sections of the body remain parallel to their respective first positions. Translation is subclassified into *rectilinear* and *curvilinear*. In the former, all points of the body move in straight lines; in the latter all points move in curves. In either kind of translation there is no element of rotation and every point of the body therefore has exactly the same motion. The crosshead of a steam engine has rectilinear translation. The parallel rods of a locomotive, Fig. 4–7, have curvilinear translation. Rectilinear translation is always plane motion, curvilinear not necessarily so.

[1] If the object had rotation would there be any evidence of its motion without the existence of reference bodies?

Helical motion of a body is rotation about an axis combined with a translation along the axis that is proportional to the rotation. The commonest example is the motion of a nut with respect to its bolt when the threads are properly made.

Spherical motion is such motion of a body that all its points remain at fixed distances from a point called the center. It is not necessarily rotation although rotation answers also to this definition which shows that rotation is a particular case of spherical motion. The balls of a flyball governor have spherical motion. The rollers of the conical roller bearing, Fig. 1–3, have spherical motion about the point O which is the common vertex of the rollers.

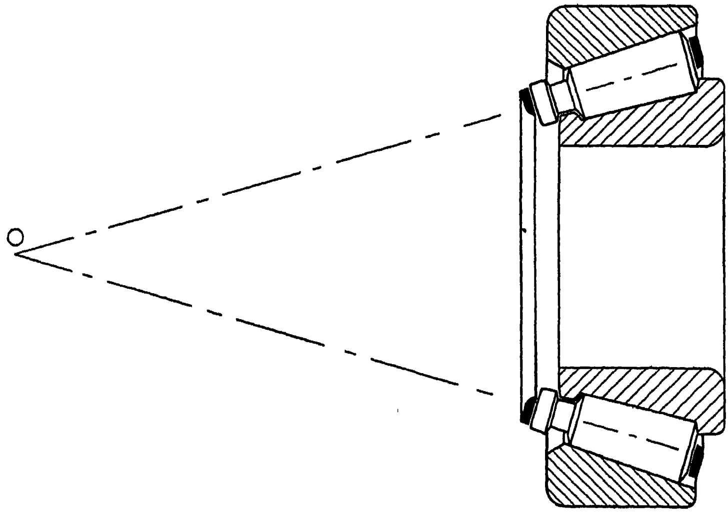


Fig. 1–3. Conical Roller Bearing.

The motion of any body is completely defined by the motion of three of its points provided the points are noncolinear (not in line). If the body is known to have plane motion, its motion is completely defined by the known motion of two of its points, provided they are not on a normal to the plane of motion. If the body has a motion of translation, its motion is defined by the motion of one of its points, since all its points have identical motion.

1–5. **Cycle, Period, Phase.**—The *cycle* of a machine is the motion from the instant when the parts have certain relative positions and velocities, to when they next assume the same relative positions and relative velocities. The four-cycle automotive engine, Fig. 1–1, accomplishes a cycle for every two revolutions of the crankshaft. The crankshaft, connecting rods, and pistons go through a cycle for every revolution of the crankshaft,

but the cam shaft and valve mechanisms have accomplished only a half cycle. "Four cycle" is an abbreviation of "four-stroke cycle" and indicates that four piston strokes are required for a cycle of the machine.

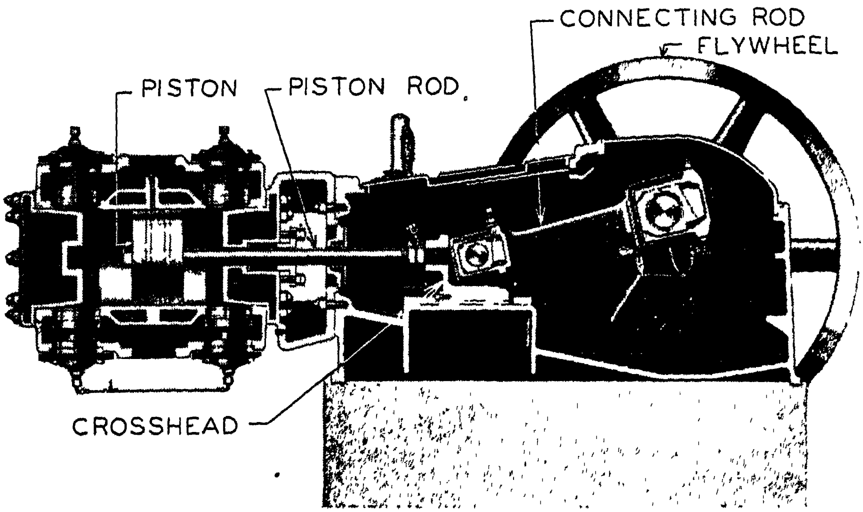


FIG. 1-4. Air Compressor.
Ingersoll-Rand Co., New York, N. Y.

The *period* of a machine or machine part is the elapsed time during a cycle.

A *phase* is the relative positions occupied by the parts of a machine at any given instant. For example, in the air compressor, Fig. 1-4, when the piston is at the left or head end of its stroke, its connecting rod and crank are in line and it is said to be on dead center, head end. When the crank has turned 180° from this position, it is on dead center, crank end. These are important phases for any reciprocating compressor or engine for they represent phases of no work.

1-6. Locus and Instantaneous Motion.—A *locus* is the path traced by a point in moving from one position to another. Sky writing by airplane is accomplished by using a stream of smoke which, if there is little wind, remains for a time as the locus of the motion of the airplane. When an engine lathe is used to cut a thread on a screw, the first cut leaves a helical mark on the bar which is the locus of the motion of the point of the tool with respect to the bar.

Let the curve AB, Fig. 1-5, be the locus of a point moving on the paper. At a certain instant when the point is at C, its motion is represented in magnitude, direction, and sense by the vector CD. This vector

represents an *instantaneous motion* of the point and must be tangent to the locus at C. An instant later the point will have motion in a slightly different direction which means that it is being accelerated as to the direction of its motion whether or not the magnitude is being changed. Note particularly that a point cannot have *instantaneous* motion in a curve. That is, at any instant, the point has motion in a single definite direction, regardless of the curvature of its continuing path. Any motion may be considered as a succession of instantaneous motions.

FIG. 1-5. Locus.

1-7. **The Result of any Motion of a Body Can be Accomplished by a Translation Plus a Rotation.**—In Fig. 1-6, the body AB has been moved from the position A_1B_1 to position A_3B_3 by any intermediate motion whatsoever. The change of position resulting from this motion can be accomplished by a rectilinear translation from A_1B_1 to A_2B_3 plus a rotation about the axis B_3 into the final position A_3B_3. Obviously the rotation could have taken place in such a way as to give a final position for AB not in the plane of the paper, so the proposition is perfectly general and will apply, as well, to any portion of the motion of AB. Any conceivable motion of a body can be nothing more than various combinations of translation and rotation.

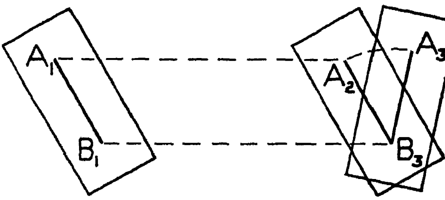


FIG. 1-6.

While it is convenient in practical problems to distinguish between translation and rotation, identifying each according to its definition, it is of interest here to note that translation of a body is rotation about an axis infinitely distant. Thus translation is a particular case of rotation.

1-8. **Velocity and Acceleration Relations.**—The following symbols will be used:

s = distance travelled or length of locus, ft,
t = time in sec to travel distance s,
V = linear velocity, ft per sec,
a = linear acceleration, ft per sec per sec,
N = rpm = revolutions per minute,
r = radial distance of point from axis of rotation, ft,
θ = total angle of rotation of line or body, radians,
ω = angular velocity, radians per sec,
α = angular acceleration, radians per sec per sec.

Linear	Angular	
$V = \dfrac{ds}{dt}$	$\omega = \dfrac{d\theta}{dt}$	(1)
$a = \dfrac{dV}{dt} = \dfrac{d^2s}{dt^2}$	$\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$	(2)

If the velocity is uniform,

$$s = Vt \qquad \theta = \omega t \tag{3}$$

$$V = \frac{2\pi r N}{60} = r\omega \qquad \omega = \frac{2\pi N}{60} = \frac{V}{r} \tag{4}$$

If the acceleration is uniform,

$$V = V_1 + at \qquad \omega = \omega_1 + \alpha t \tag{5}$$

subscript one denoting initial velocity.

$$s = V_1 t + \tfrac{1}{2}at^2 \qquad \theta = \omega_1 t + \tfrac{1}{2}\alpha t^2 \tag{6}$$

The student is probably familiar with these relations but they are assembled here for review and reference because they are basic to much of the theory that will be developed in later chapters.

Consider the wheel B, of Fig. 1–7, mounted in bearings in the frame A, in which it is free to turn. Suppose its speed of rotation is 10 radians per second clockwise relative to the frame which is fixed to the earth. Then from equation (4)

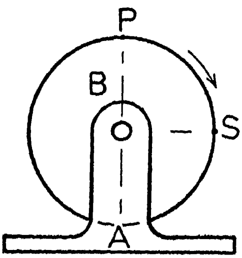


Fig. 1–7.

$$N = \frac{60\omega}{2\pi} = \frac{60 \times 10}{2\pi} = 95.5 \text{ rpm}$$

If the wheel is 4 ft in diameter, the velocity of the top point P, relative to the frame, is $r\omega = 2 \times 10 = 20$ ft per sec and in direction toward the right horizontal. The velocity of S relative to $P = (SP)\omega = 2\sqrt{2} \times 10$ ft per sec and is normal to the line SP.

1–9. Vectors.—Quantities that have magnitude, direction, and sense, such as force, velocity, or acceleration, are called vector quantities because they can be completely described by vectors but not by numbers alone. Quantities such as mass, time, money, are nondirectional and are called *scalar* quantities. The solution of problems involving vector quantities is generally much simpler by graphical than by analytical methods. For some problems a combination of the two methods is most effective.

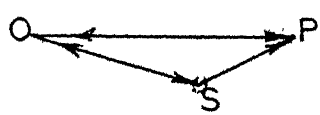


Fig. 1–8.

In Fig. 1–8, the vector OP is shown resolved into two components OS and SP. Another way to express this relation is to say that the vector addition of OS and SP yields OP as resultant, or $OS \mathrel{+\!\!\!\!\rightarrow} SP = OP$.

In Fig. 1–9, with a right angle at R, the vectors OR and RP are *rectangular components* of OP. They are sometimes called *normal components*. Fig. 1–9 also shows the vector addition,

$$OT \mathrel{+\!\!\!\!\rightarrow} TU \mathrel{+\!\!\!\!\rightarrow} UV \mathrel{+\!\!\!\!\rightarrow} VP = OP$$

Vector subtraction of SP from OP, Fig. 1–8, yields OS. Vector subtraction of OP from SP yields SO with the arrow at the O end, since $SO \mathrel{+\!\!\!\!\rightarrow} OP = SP$.

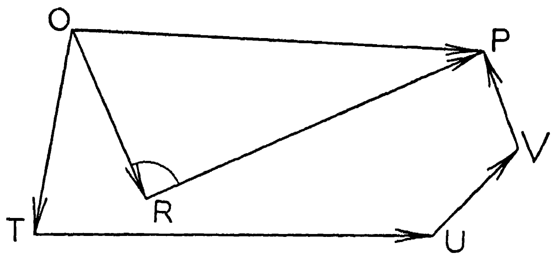


Fig. 1–9.

If any number of vectors placed in addition yields a closed polygon, the vector sum is zero. *The vectors are not in addition however unless the arrows are all in the same sense around the polygon.* Manifestly, the vector sum of any number of vectors, plus their resultant with reversed sense, equals zero.

Principle of Rectangular Vectors. *If one rectangular component and the direction only of the resultant vector are given, the magnitude and sense of the resultant can be found.* For example, suppose that rectangular component OR, Fig. 1–9, is given as well as direction OP of the resultant. Normal to OR at R draw the other component of such length as to intersect the given direction OP at P. This determines the resultant OP with arrow at P. This principle has important applications in problems of velocity in mechanisms.

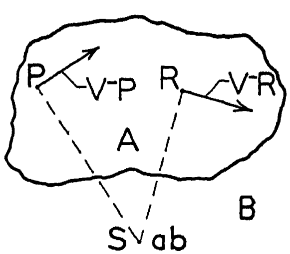


Fig. 1–10. Centro.

1–10. **Instantaneous Axis, Centro.**—The body A of Fig. 1-10 has plane motion such that all its points move in the plane of the paper B, or in parallel planes. The instantaneous motion of body A with respect to B is completely known and defined if the instantaneous motion of two of its points is known, provided these two points are not in one straight line normal to the paper. Suppose that when A is going through the position shown, the velocity of the point P in A is as represented by the vector V^-P, and the instantaneous velocity of R is V^-R. From the

previous article we know that the motion of body A relative to B at any instant must be either translation or rotation. At this particular instant, it cannot be translation, for if so P and R would have the same velocity in magnitude, direction, and sense. The motion is therefore rotation about an axis at a finite distance.

Now since any point in a rotating body can only move in a direction normal to the radius from the point to the axis, it is plain that the axis of rotation in this case must lie somewhere on a line through P normal to $V^{-}P$. Similarly it must lie on a line through R normal to $V^{-}R$. This locates the instantaneous axis at the point S. Since A has plane motion in the plane of the paper, the axis about which A is turning with respect to B, runs through S normal to the paper and is projected on the paper as the point S.

It happens that in machines plane motion is the rule; non-planar motion, the exception. When all parts have relative plane motion, all the instantaneous axes will be normal to the plane of motion and will project on it as points. It is convenient to call these points *centers of rotation*, and if instantaneous motion is being considered, *instantaneous centers* or *centros*. S is the centro of the instantaneous motion of A relative to B. All centros will in future be regularly designated by placing together the lower-case letters of the two bodies concerned. In this case, the centro is ab.

If A is rotating, for the instant, about ab with respect to B, it is equally true that B is rotating about ab with respect to A.

The centro ab fell outside the body A. It can still be considered a point of body A. In kinematics it is necessary to think of any body as infinitely extensible in space, so that points can be located on it at any position in space. Suppose, for example, a point be chosen on a flywheel 2 ft outside the rim. As the flywheel revolves, this point will travel with it as if supported by an extended arm. This point can travel through the floor and through adjacent parts of the machine, but kinematically it is a point on the flywheel. Hence whether centros fall inside or outside of the bodies to which they belong, they must be regarded as points of the bodies for the instant considered. Since every centro is an instantaneous axis of two bodies which have relative motion, it follows that the centro is a point common to those two bodies. Further, if there is a third body, C, having motion with respect to both A and B, the centro ab will have the same motion with respect to C whether considered as a point in A or in B.

These facts will now be brought together in a definition. The *centro* of two bodies having relative plane motion represents the axis about

which one body is rotating with respect to the other for the instant, and it is a point common to the two bodies having the same velocity in each with respect to any third body.

1–11. **Centros on a Machine.**—A common street railway truck is represented in Fig. 1–11. It is moving towards the right, the wheels rolling without sliding on the straight level track A. The rear wheel C turns relative to the frame B about the center of its axle O. Therefore the centro bc is at O. A point on this wheel, such as P, can have motion relative to the frame only in direction $P2$ normal to its radius PO.

Point R of the wheel is coincident with S of the track. Since there is no sliding, this is a common point of wheel and track for the instant and must be the centro ac. It follows that all points of the wheel must

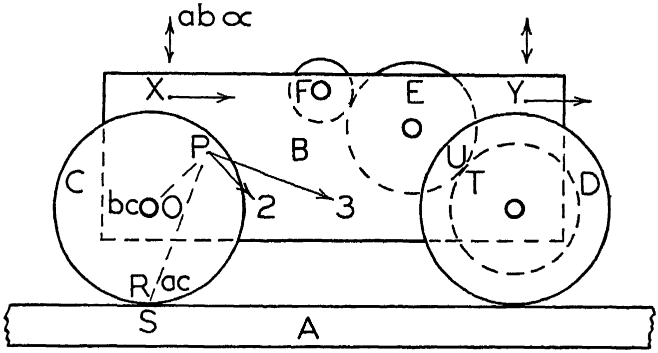


FIG. 1–11. Street Railway Truck.

move, relative to the track, about ac for the instant. Hence the instantaneous motion of P relative to A must be in direction $P3$ normal to $P - ac$.

Wheel D at the other end carries a gear meshing with gear E, and all are driven by the motor pinion F. Where is the centro of the meshing gears D and E? According to the nature of gear action there are two imaginary circles, one on each gear, that roll together without sliding. These are called pitch circles, and their point of tangency is the pitch point. In this case the point of tangency is U on E and T on D. Since this is the common point having the same velocity on both E and D relative to third bodies such as the frame, it is the centro ed.

Next we shall find the centro of the frame B and the track A. No common point is apparent, so consider the motion of two points of the frame such as X and Y, relative to the track. Their motion is obviously as shown by the vectors, in direction parallel to the track. To make this

motion possible the centro ab must lie somewhere on a vertical line through X, also somewhere on a vertical through Y. These lines intersect at infinity, and that is the location of ab. Note that this is a particular point at infinity, namely the one at the end of all vertical lines that are a finite distance apart.

The motion of the frame relative to the track is rectilinear translation, and it will be found that when the relative motion is translation, either rectilinear or curvilinear, the centro is at infinity.

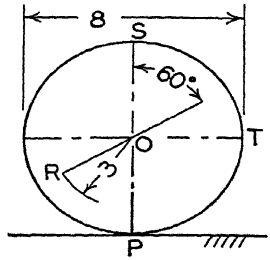
QUESTIONS AND PROBLEMS

1. What are the principal steps necessary in the design of an entirely new machine and in which steps is kinematics applied?
2. Name and define eight kinds of motion that a body can have and cite two examples of each. Choose your examples, if possible, from your observations of machinery.
3. A locomotive is running along a straight level track. What is the locus with respect to the track of a point on a driving wheel tread? (The tread is the face of the rim which makes contact with the track.) What is the locus of this point with respect to the locomotive frame? (See Fig. 4–7.)
4. An automobile having a tire of 32 in. outside diameter is travelling on a level road at 40 miles per hour.
    (*a*) At what speed are its wheels turning in rpm and in radians per sec?
    (*b*) What is the locus of a point P on the periphery of a wheel with respect to the road?
    (*c*) Find the instantaneous motion of P, with respect to the road, in miles per hour, when it is going through its highest position.
    (*d*) Find the motion of P, with respect to the automobile frame, when it is in contact with the road. Give direction and sense.
5. The flywheel of a steam engine is 4 feet in diameter and is turning at a constant speed of 40 radians per second.
    (*a*) What is its rpm?
    (*b*) What is the linear velocity of a point on the rim relative to the frame?
    (*c*) What is the linear velocity of a point on one end of a horizontal diameter relative to the opposite end?

    Ans. (*a*) 382.2, (*b*) 80 ft per sec, (*c*) 160 ft per sec.

6. Consider the flywheel of the air compressor, Fig. 1–4, to run clockwise and suppose a cycle to begin when the piston is on head-end dead center. When the flywheel has turned through one quarter cycle, that is through 90°, what per cent of its travel for a cycle has the piston covered? The crank is 6 in. long and the connecting rod 20 in. Solve graphically. Ans. 28.8%.
7. A wheel 3 ft in diameter is held stationary and a second wheel 2 ft in diameter is directly under it and in contact with it. The lowest point of the 2-ft wheel is P. If the 2-ft wheel is *translated* completely around the stationary wheel maintaining contact with it, what is the locus of P?
8. A resultant vector has one rectangular component 4 in. long which makes with it an angle of 37°. Find the length of the resultant graphically and check your result by trigonometry.
9. Find the resultant in magnitude and inclination by adding the following vectors:
   A is 3 in. vertically upward,
   B is 4 in. 30° above right horizontal,
   C is $2\frac{1}{2}$ in. 45° below right horizontal,
   D is 2 in. 30° left of vertically downward.
10. An airplane lands at 60 miles per hour while its propeller which is 8 ft in diameter is turning 600 rpm. Assuming the propeller shaft to be horizontal, find graphically the ground speed of the propeller tip when it is in its lowest position.
11. The water in a river is flowing due south between parallel banks at 6 miles per hour. A boat pointing 30° north of east makes a water speed of 18 mph, and a man is walking directly across the boat from right to left at 3 mph. Find graphically the ground speed of the man.
12. How far up or down river from its starting point on the west side will the boat of problem 11 reach the opposite bank if the river is 2 miles wide? Ans. 0.385 miles up.
13. A resultant vector is 6 in. long and due north. The magnitude of its two components are unknown but one is known to run 30° north of east, the other 45° west of north. Find the magnitude of each component.
14. Sketch the mechanism of the locomotive (Fig. 4–7) and, for the particular phase shown, locate the following centros:
   (*a*) of the driver (driving wheel) and the track,
   (*b*) of the driver and the frame,
   (*c*) of the parallel rod and one driver,
   (*d*) of the parallel rod and the frame.

15. The 8-in. wheel shown is rolling toward the right along the horizontal track without sliding, at 20 radians per second. Draw a vector diagram showing the relative velocities of the five points indicated. Choose and state your scale for velocity.

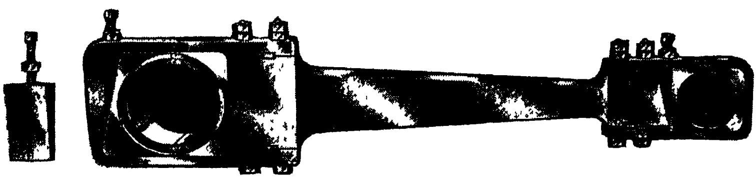


16. A four-link mechanism, Fig. 3–18, has A horizontal, fixed, and 5 in. long. B is $1\frac{1}{2}$ in., D 2 in. and C, opposite A, is $4\frac{1}{2}$ in. long. B is the driver and D carries a flywheel. Find the locus of the midpoint of C for a complete cycle. Choose your own scales.

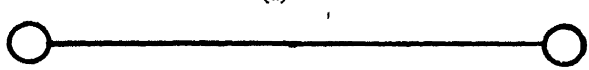
CHAPTER II

MECHANISMS AND CENTROS

2–1. Mechanism, Link, Chain.—The difference in technical meaning of the terms machine and mechanism is a difference in viewpoint. In kinematics, we use the term mechanism when dealing with the relative positions and connections of machine parts, their relative velocities and accelerations, without regard to the shape of the parts or the forces transmitted. In non-technical parlance, a mechanism is a machine in which the forces transmitted are small, as in a clock, weighing balance, measuring instrument, or model, but strictly, these are all machines since they must transmit and modify energy if their parts have any relative motion. A mechanism might be called the kinematic representation of a machine. Of course, a machine may contain a number of integrated mechanisms, as in the automotive engine, Fig. 1–1. Similarly



(*a*)



(*b*)

FIG. 2–1. Machine Part and Link.

when a machine part is dealt with as part of a mechanism it is called a link. The link shown at (*b*) Fig. 2–1, is a convenient representation of the connecting rod shown at (*a*) when the motions and connections of the part only, are under consideration.

A *mechanism* is a combination of links so connected that relative motion of any two compels definite relative motion of every part of the mechanism.

This definition allows us to distinguish between combinations of machine parts that conform to the laws of mechanism, and combinations that do not.

FIG. 2–2. Lever Shear.
A. Garrison Foundry Co., Pittsburgh, Pa.

Link may be defined as the kinematic representation of a machine part. While the term, in its specialized meaning, is distinct from the link of a chain (for example the common oval link of a haulage chain) the two are related. The haulage-chain link connects two adjacent links, although such a chain, by itself, is not a mechanism. The kinematic link connects other kinematic links in the manner required to constitute a mechanism.

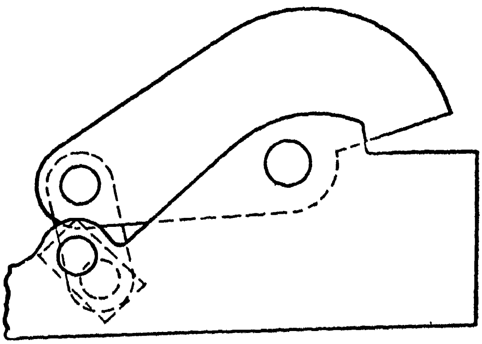


FIG. 2–3. Outline Drawing of Lever Shear.

The analogy is carried farther in the term kinematic chain. A *kinematic chain* is a number of connected kinematic links. The meaning of the term link is made broad enough to include all machine parts that can serve as parts of a mechanism. Some, such as belts, ropes, chains, wires, can transmit tension only. Gases or liquids, such as the water in the cylinder of a hydraulic press, can transmit only compression. Links like the connecting rod and the frame of the air compressor, Fig. 1–4, can transmit tension or compression.

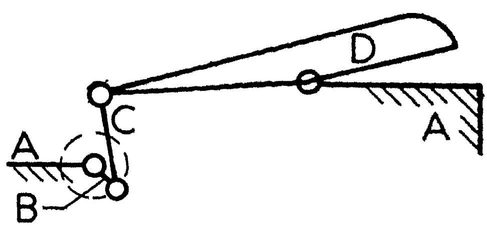


FIG. 2–4. Kinematic Drawing of Lever Shear.

2–2. Kinematic Drawings.—Mechanisms can be represented by simpler drawings than are required for the machine. The mechanism is the skeleton of the machine and a kinematic drawing is required to

show only the joints, connections, or pairing, as well as the essential dimensions of the links between joints. For example, Fig. 2–2 pictures a lever-type shear, sometimes called an alligator shear, which is used to shear small bars of iron or steel, cut up scrap, etc. Fig. 2–3 is a line drawing of the essential mechanism omitting the reduction gearing, and Fig. 2–4 is the kinematic drawing. This latter drawing, if accurately scaled, will serve as the basis for solution of all problems of displacement, velocity, and acceleration. B represents the crankshaft and crank, C the connecting rod or pitman, D the moving shear link, and A the frame. Note that the frame or fixed link is only shown where it becomes part of a joint or where otherwise kinematically necessary, as for example, to show the position of the fixed shear jaw. The fixing of a link is indicated by hatching called *ground lines.*

2–3. **Significance of the Fixed Link.**—Moving vehicles such as automobiles and airplanes have no link fixed to the earth, but they have a frame which for convenience is considered the kinematic link of reference. The frame is generally the largest link though not necessarily so. The revolving wing of an autogyro may have larger dimension than the fuselage. It would be more accurate to define the frame of a vehicle as the link that carries the operator and passengers, and the frame of an automatic moving machine, such as a torpedo, as the link carrying the cargo. However, since all motion is relative, the designation of any fixed link is a convenience and is not essential in the solution of any kinematic problem.

2–4. **Constrainment.**—It was shown in §1–7 that the result of any motion of a body in space could be accomplished by a single translation plus a single rotation. A corollary statement is that any instantaneous motion of a free body in space can be only a combination of translation and rotation. Since the translation can be in any direction, it can consist of components along any of three normal axes, the x, y, and z axes. Similarly the rotation can have components about any of these three axes. The free body in space is therefore considered to have six **degrees of freedom.**

If a body or link is prevented from having one or more of these six motions, it is to that extent constrained. Moreover, if one motion becomes dependent on another, more constrainment results; in fact, one degree of freedom is lost. Consider the three bodies, Fig. 2–5. The plate represents a toolmaker's surface plate having a smooth accurate plane upper surface. On this surface is a flat gage and a sphere. Assuming that the only limitation on the motion of the gage is that it maintains flat contact with the plate, it can rotate about a vertical axis

and translate in x and y directions. As these three motions are independent, the gage has three degrees of freedom relative to the plate.

Suppose that the only limitation on the motion of the ball is that it maintains contact with the surface plate. The ball can rotate about any axis, and it can be translated in any except the z direction. These motions are independent, therefore five degrees of freedom exist. Now add the further restriction that the ball cannot slide on the plate. The previous five motions are possible but are no longer independent. Translation in the y direction is now related to rotation about the x axis, a relation which can be expressed by the equation, $V_y = r\omega_x$, where r is the radius of the sphere and ω_x is the component of rotation about the x axis. Similarly, $V_x = r\omega_y$. The degrees of freedom have been reduced to three.

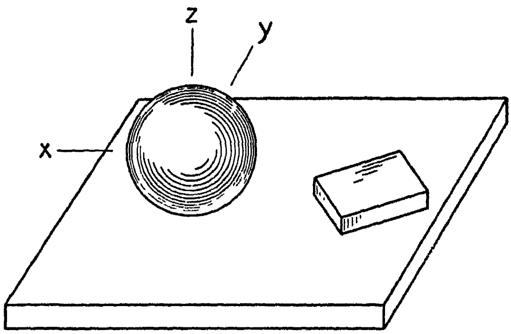


FIG. 2–5.

In Fig. 2–6 (*a*), a bar link B connects a gear C to the fixed link A. Link B can rotate about O relative to A, and C about P relative to B. So C can have, relative to A, at any instant, rotation about one axis only, and independent translation in one direction only, the translation being in direction normal to OP for the phase shown. C has two degrees of freedom relative to A.

In Fig. 2–6 (*b*), A has been made a gear, its stationary teeth meshing with those on C. C can have the same two motions as before, but they are no longer independent and only one degree of freedom exists.

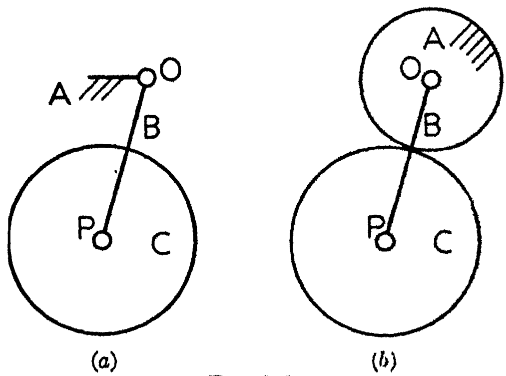


FIG. 2–6.

The relation between this theory and the definition of a mechanism should now be apparent. In a true mechanism, only one degree of freedom can exist between any two links regardless of how complex the

relative motions may be. In Fig. 2–6, (*b*) is a mechanism, (*a*) is not, because of lack of constrainment. The theory of constrainment will be covered further in the next two articles.

2–5. **Pairing of Links.**—Pairing is the term applied to the various connections between the links of a mechanism. The part of each link involved is called an *element* of the pair. **Turning pairs** are those that constrain the connected links to relative rotation. In Fig. 2–7 (*a*), links *A*, *B*, and *C* are connected by turning pairs on a common axis so that each can have rotation about it. At (*b*) there are only two links joined by one turning pair. This is indicated kinematically by the line crossing the circle.

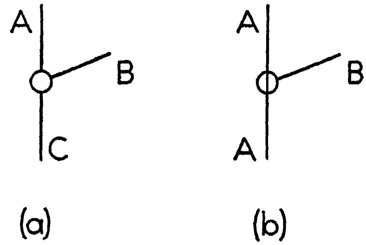


FIG. 2–7.

Sliding pairs are those that constrain two links to relative translation which, as shown in Fig. 2–8, can be either rectilinear (*b*), or curvilinear (*a*). It is usual in kinematic drawings to show the sliding pair with one guide line as at (*c*), it being understood that there is only one degree of freedom. Good examples of sliding pairs are to be found on the planer, Fig. 1–2. The platen which carries the work to be machined slides on the bed, being guided by accurately finished V's. The cross rail slides on the vertical columns, being positioned by elevating screws. The tool heads slide across the rail, driven by the feed screws.

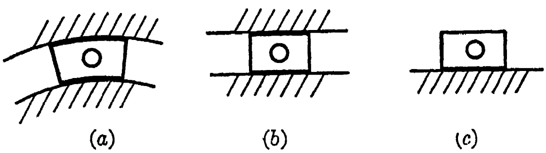


FIG. 2–8.

Spherical pairing constrains the connected links to relative spherical motion. The ball and socket joint, Fig. 2–9, is an example. In the universal joint, spherical pairing is accomplished by the use of two turning pairs with normal, intersecting axes, Fig. 13–4. Spherical pairing, of itself, allows two degrees of freedom. However it will presently be shown that there are other means of constrainment in addition to pairing.

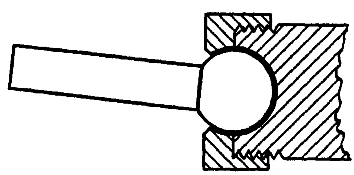


FIG. 2–9. Spherical Pair.

Screw pairing constrains two links to relative helical motion. The pipe vice, Fig. 2–10, illustrates the use of the screw pair to obtain large

mechanical advantage in holding pipe for cutting and threading. Bolts, screws, and fastenings in great variety use the screw pair.

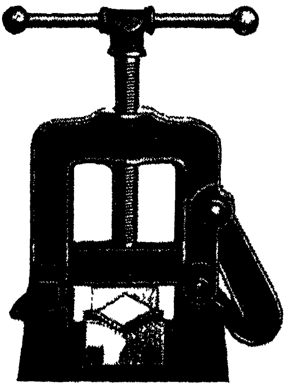


FIG. 2–10. Pipe Vise.
Yost Manufacturing Co., Meadville, Pa.

Pairing of links has been classified into **higher pairing,** where the contact between mating links is theoretically a point or a line, and **lower pairing,** where contact is on surfaces. Higher pairing is that between balls and races of ball bearings, Fig. 2–11, between rollers and races of roller bearings (Fig. 2–12), between the driver and follower of a cam, and between gear teeth. Lower pairing is exemplified by the plain journal bearing such as the crank-pin and main bearings of the automobile engine, Fig. 1–1, and sliding bearings such as those of the planer, Fig. 1–2.

A higher pair can generally be replaced by two lower pairs. For example, the higher-pair contact at P, Fig. 2–22, could be replaced by a crosshead sliding on the surface (plane or cylindrical) of B or C, and having a turning-pair connection with the other. Lubrication could be much improved thereby.

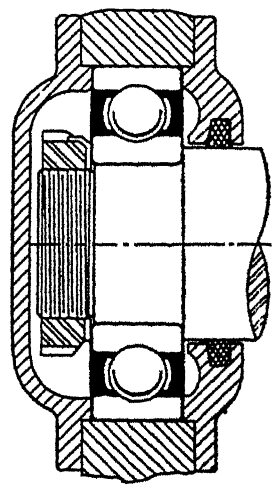


FIG. 2–11. Ball Bearing with Mounting.

In practical application, however, the difference between higher and lower pairing is less than appears on first view. In the case of the ball or roller bearing it has been found that there is a measurable area of contact at normal loading. Also in the case of the cylindrical bearing, the bore of the shell must be slightly larger than the shaft, and if the load is not carried completely by the oil film, the metal to metal contact is a narrow area similar in shape to the contact between a roller and its race in a roller bearing.

However a principle of pairing is involved. With higher pairing only line or point contact would result as long as no force was transmitted, while with lower pairing there would be surface contact.

2–6. Constrainment from the Nature of Mechanisms.—The *slider-crank* mechanism of Fig. 2–13 is the basic mechanism for all reciprocating engines whether operated by steam, gas, gasoline, or oil, and for other

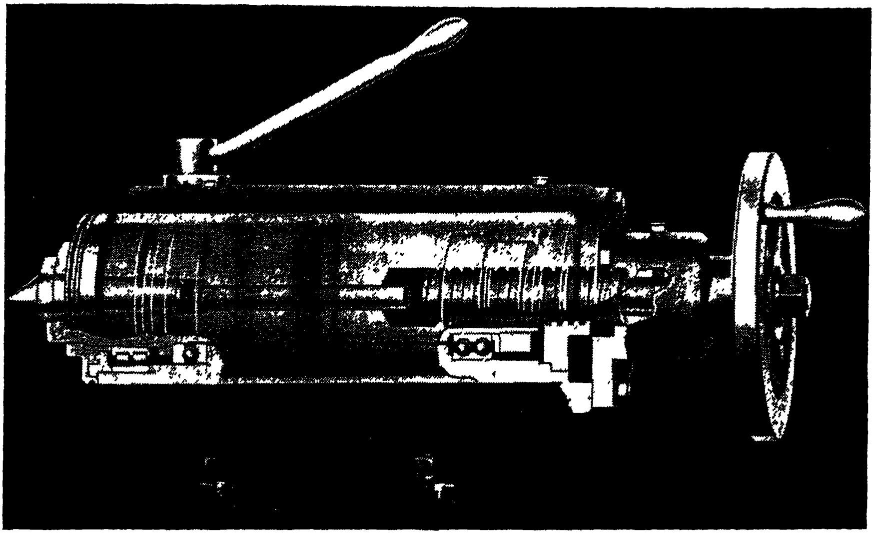


FIG. 2-12. Lathe Tailstock with Ball and Roller Bearings—Higher Pairing.

Jones and Lamson Machine Co., Springfield, Vt.

The load on the tailstock center is both radial (downward) and axial. The radial load is taken by the double roller bearing, the axial load by the thrust ball bearing, while the right end is supported by the double-row ball bearing. This is a *live* tailstock center, as it turns with the work. Axial adjustment is obtained by turning the screw which moves the barrel (light) in the housing (dark).

applications of great variety. Since the development of the automobile it is undoubtedly the most used mechanism we have, with the single exception of the simple wheel and bearing. Note that link *D* of the mechanism, Fig. 2-13, represents the crosshead, piston rod, piston, piston rings, and all minor connected parts. All these parts have identical instantaneous motions and accelerations and since in a mechanism the form of the parts is not necessarily represented, the block *D* can be conveniently used to represent them all.

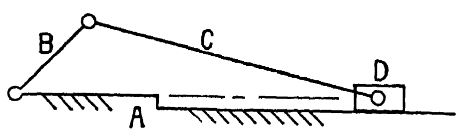


FIG. 2-13. Slider Crank Mechanism.

While the origin of the connecting rod and crank as a device to change reciprocation to rotation should probably be credited to James Watt, it is a fact that Pickard first obtained the patent for it about 1775 and prevented Watt, who believed he had been plagiarized, from using it during the life of the patent.

The constrainment in this mechanism due to pairing is apparent. Each turning pair limits the two links it connects to relative rotation, and the sliding pair limits D and A to relative rectilinear translation. From the nature of the mechanism, however, the extent of travel of D is constrained to twice the length of the crank B. For the same reason the extent of rotation of C relative to D is limited to the angle $2\tan^{-1}\left(\frac{B}{C}\right)$.

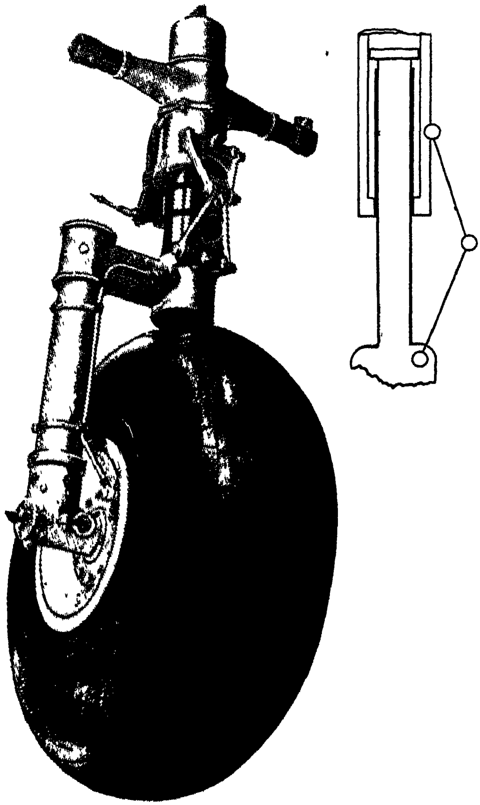


FIG. 2–14. Landing Gear Assembly.
The Glen L. Martin Co., Baltimore, Md.

An application of the slider-crank mechanism where its main purpose is to effect constrainment is shown in the airplane landing-gear assembly of Fig. 2–14. The links, shown kinematically at the right, keep the piston from turning in the cylinder and therefore keep the plane of the wheel in the forward direction. This is torsional constrainment and the links are called torque links. The landing shock is cushioned by the compression of fluid in the cylinder followed by the forcing of part of the fluid through restricted openings. The trunnion bearings on either side of the cylinder link are used to retract the wheel into the nacelle.

Fig. 2–15 (*a*) represents the quadric or four-bar mechanism, one having a wide field of application. At (*b*) is shown the same links, but C and D are not connected. C can now have motion with respect to B without compelling relative motion of B, A, and D, so it does not fulfil the requirements of the definition and is not a mechanism. A mechanism can be called, with good logic, a *closed kinematic chain.* If the chain is opened at any point the combination ceases to be a mechanism.

Fig. 2–16 illustrates a five link combination which, even though it is a closed kinematic chain, is not a mechanism, for B, C, D, and A can

all have relative motion without compelling F to move with respect to A. Thus, while closure is a requisite it is not a guarantee of a mechanism. This chain lacks the kind of constrainment that depends on the nature

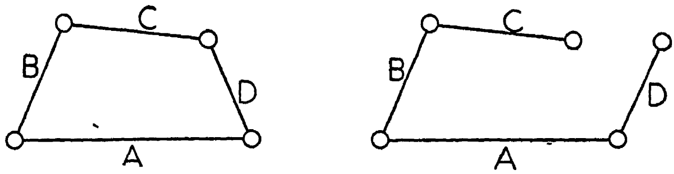


FIG. 2–15. Quadric Mechanism.

of the mechanism. On the contrary, Fig. 2–17 shows what happens when too much of this kind of constrainment occurs.

In a true mechanism all links are completely constrained; they have one degree of freedom only. There are many practical mechanisms, however, which have *incomplete constrainment* and therefore depend on outside forces to supply it. The rear-axle housing of an automobile can rotate, in some degree, about both transverse and longitudinal axes. Constrainment results from the forces between road and tires below, and spring forces above.

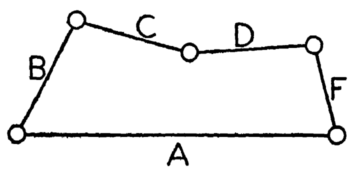


FIG. 2–16.

The autogyro wing has two degrees of freedom. Constrainment results from the balanced action of air pressures and centrifugal forces. The flyball governor allows two degrees of freedom to the ball links, but the balanced action of centrifugal force and the pull of gravity constrains each ball to a definite circle of rotation for each speed.

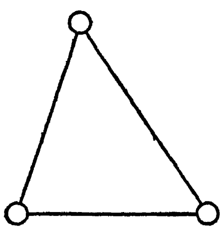


FIG. 2–17.

2–7. **Driver and Follower.**—The term *driver* is applied to that link of the mechanism initially moved with respect to the frame. The term *follower* may be applied to any link which is driven by another. Most mechanisms are reversible, not only as to sense of motion but as to sense of drive, so that the same link can be driver and follower at different times. For example, the belted flywheel of the air compressor, Fig. 1–4, is the driver, but the same slider-crank mechanism is used on the steam engine in which case the piston becomes the driver.

2–8. **Centros in Mechanisms.**—The theory of the instantaneous center was introduced in the previous chapter. It was shown that whenever

two bodies have relative motion of any kind, at a particular instant that motion will be rotation about a certain axis, and further, if the relative motion is plane motion, the instantaneous axis, being always normal to the plane of motion, is projected on that plane in a point which for brevity is called the centro of the two bodies.

From the definition of a mechanism, each link must be capable of relative motion with respect to every other link. A mechanism will therefore have as many centros as the number of combinations of two that can be formed of its links. The number of combinations of n objects taken r in a group is

$$\frac{n(n-1)(n-2)\ \ldots(n-r+1)}{\underline{|r}} \tag{1}$$

Applied to the case of centros, r is two and the number of centros S is given by

$$S = \frac{n(n-1)}{2}. \tag{2}$$

The centros for the quadric mechanism, Fig. 2–18, will now be found using the direct method. From equation (2) there will be six centros. The four centros *ab*, *bc*, *cd*, and *ad* are seen at once to be at the centers of the respective turning pairs. These are called *permanent centros* because for all positions of the mechanism they remain at the same points on the links of which they are the centros.

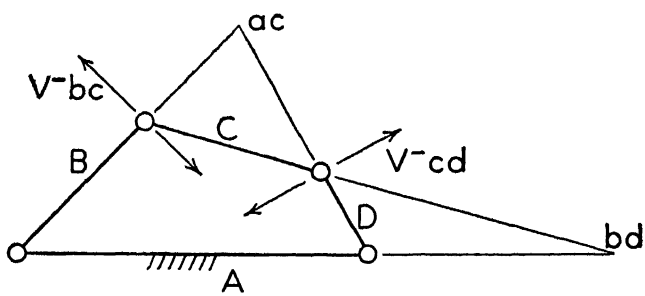


FIG. 2–18. Centros of Quadric Mechanism.

To find the centro *ac*, use is made of the known direction of motion of two points in C relative to A. The centro *bc* is a point on C and also a point on B. For the instant when the mechanism is going through the position shown, *bc*, as a point in B, is moving in direction $V^{-}bc$, with respect to A. It must have this same direction of motion relative to A, as a point in C. This direction of motion, $V^{-}bc$, is possible only if the centro *ac* lies somewhere on the line running through *bc* and *ab*.

Similar reasoning leads to the conclusion that $V^{-}cd$ with respect to A is only possible if the centro *ac* is located somewhere on the line through *ad* and *cd*. The only point that satisfies both these requirements is *ac* at the intersection. This means that, for the instant, link C is turning about *ac* with respect to A. Furthermore, *ac* is a point common to A and C and a pin driven through an extension of C and an extension of A at *ac* would not interfere with the action of the mechanism for the instant. Of course *ac* is not a permanent centro. An instant later, *ac* would have a slightly different location, both on C and on A as can be readily demonstrated by drawing the mechanism in a slightly different phase.

The remaining centro *bd* is found by a procedure exactly similar to that used for *ac*. The two points in D of known velocity with respect to B are *ad* and *cd*. Now all points in A move relative to B about the centro *ab*. (The fact that A is fixed to the earth does not affect the above statement for if B moves with respect to A it follows that A must move with respect to B.) Therefore *ad* must move with respect to B in a direction normal to *ab–ab*, and to allow such motion *bd* must lie somewhere on the line *ab–ad*. Similar reasoning regarding *cd* will limit *bd* to a location somewhere on the line *bc–cd*. Hence *bd* must lie at the intersection as shown.

The finding of centros, particularly in complex mechanisms, is simplified by the use of a theorem first formulated by Professor Kennedy.

2–9. Kennedy's Theorem and Applications.—A, B, and C, Fig. 2-19, are three bodies which have relative plane motion. They may be links in a mechanism or they may not be connected in any way. The only limitation imposed is that their motions must be plane motion, and the figure is drawn so that the motion is parallel to the plane of the paper.

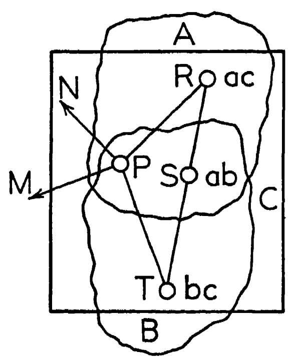


FIG. 2–19. Kennedy's Theorem.

For the instant when the three bodies are going through the positions shown, it is assumed that the location of centros *ac* and *bc* are known to be at R and T respectively. It is required to prove that the third centro *ab* must be on the line of the other two.

Using the Euclidian method, assume that *ab* is not in line with *ac* and *bc* but at any other point, as P. Now *ab*, as a point in A, must move relative to C in a direction PN, normal to its radius to the centro *ac*. Also *ab*, as a point in B, must move relative to C, in a direction

PM, normal to its radius to the centro *bc*. Thus, if *ab* is at *P*, it will have motion in two different directions, *with respect to the same body C, at the same time,* which is impossible. Therefore *ab* cannot have a location off the line *ac–bc*. It will further be seen that any location for *ab* on this line will make the directions *PN* and *PM* coincident.

Any three bodies having relative plane motion will have three centros and those three centros must be on one straight line.

The above proof of Kennedy's theorem is sufficiently general to make it apply to three stars in space provided they had relative plane motion. Application will now be made to mechanisms, reverting first to the quadric chain of Fig. 2–18. Having found the permanent centros by inspection we proceed to find *ac*. The following form will prove helpful:

$$\begin{array}{ll} & \underline{ac} \\ B & (ab,\ bc) \\ D & (ad,\ cd) \end{array}$$

The centro sought is underlined and below it is written the name of some link *B*, to be considered as a third body with *A* and *C*. Now *A*, *C*, and *B*, having relative plane motion, must have their three centros on one straight line. Centros *ab* and *bc* are known, so the third centro *ac* must lie somewhere on the line *ab–bc*. Then bring in another third body *D*, giving three centros *ac*, *ad*, and *cd* on one straight line. Since *ad* and *cd* are known, the third centro *ac* must lie on the line *ad–cd*. These two facts locate *ac* at the intersection.

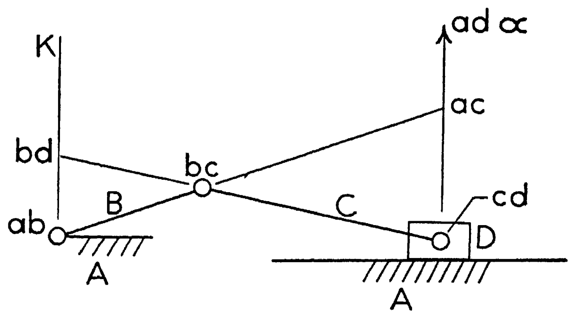


FIG. 2–20. Slider-Crank Centros.

Note that two applications of Kennedy's theorem are necessary to locate one centro. In locating *bd* by this method the following form may be used:

$$\begin{array}{ll} & \underline{bd} \\ A & (ab,\ ad) \\ C & (bc,\ cd) \end{array}$$

The centros of the important slider-crank mechanism, Fig. 2–20, will now be found. After writing in the permanent centros, *ab*, *bc*, *cd*, and *ad*, the remaining two, *bd* and *ac*, will be found using Kennedy's theorem.

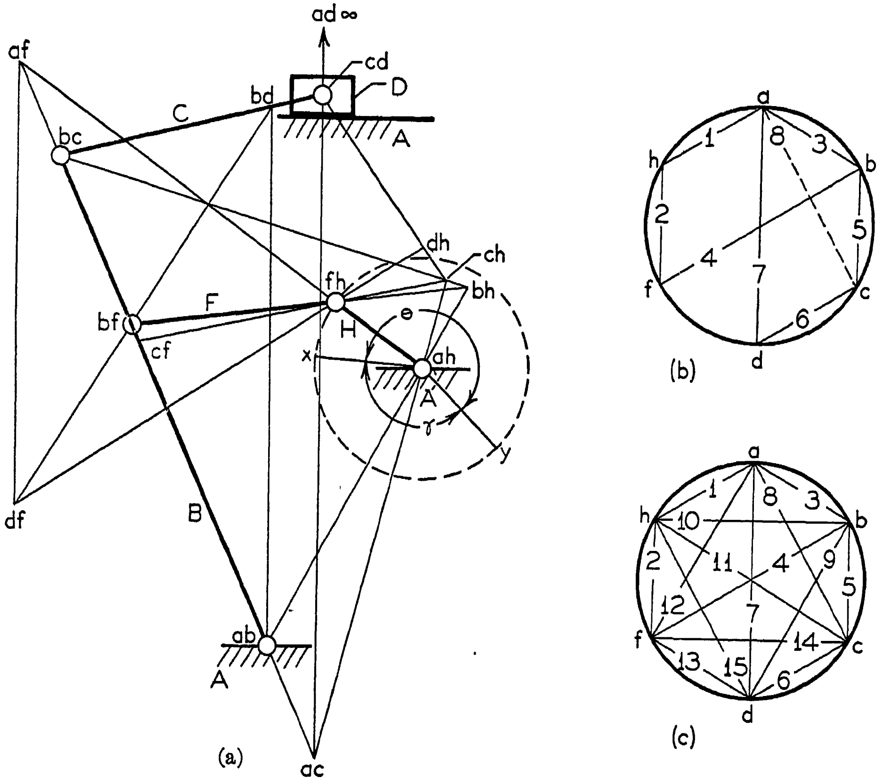


FIG. 2–21. MacCord's Quick-Return Mechanism. Circle Diagram for Centros.

The centro *ad* is at infinity at the end of a line normal to the travel of *D* relative to *A*, since *D* has a motion of translation. Further, since all parallel lines, a finite distance apart, meet at infinity, *ad* will also be found at infinity at the ends of all lines on the figure, such as *ab–K*, that are parallel to *cd–L*. Formal solution then is:

$\underline{bd}$	$\underline{ac}$
$A\ (\overline{ab},\ ad)$	$B\ (\overline{ab},\ bc)$
$C\ (bc,\ cd)$	$D\ (cd,\ ad)$

Another illustration of locating centros will be given. Fig. 2–21 (*a*) shows a quick-return mechanism. Link *D* might be the ram of a shaper

carrying the cutting tool. The driving link H, revolving at constant speed, would cause link D to advance on the cutting stroke while H moved through the angle θ from x to y clockwise, and the return would occupy less time as γ is less than θ. The object of all quick-return mechanisms, a number of which will be discussed later, is to make more efficient use of the time of the operator and the machine.

From equation (2) there are 15 centros. The circle diagram is a helpful guide to the necessary order in which the centros must be found, especially when the mechanism has more than four links. A circle is drawn and the letters representing the links are equally spaced on the circumference. The permanent centros are first located on the mechanism and, as they are found, chords are drawn on the circle diagram joining the letters involved. The chords should be numbered in the order in which the centros are found. The circle diagram as at (*b*) shows that the seven permanent centros have been found.

The only centros that can next be found are those for which two applications of Kennedy's theorem can be made, and that requires four particular known centros. The diagram shows that *ac* can be found because the line *ac* completes two triangles *abc* and *adc*. Also *af* and *bh* could be found as number 8, but *cf* could not be. When all 15 centros have been found the diagram has the balanced appearance of Fig. 2–21 (*c*). Having the chords numbered is helpful in checking.

2–10. **Centros of Direct Contact Bodies.**—There is one class of mechanism where Kennedy's theorem proves inadequate for finding the nonpermanent centros. In cases where motion is transmitted by direct contact as in Fig. 2–22, it is possible to make one application of the theorem bringing in the third body A to prove that centro *bc* is on the line *ab*–*ac*, but there is no other third body to yield centros for a second application necessary to locate *bc* on this line.

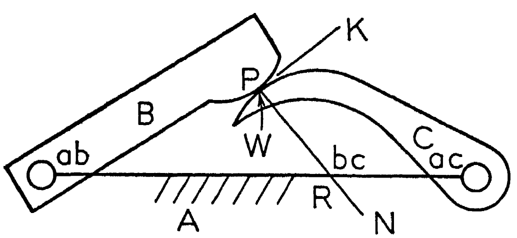


Fig. 2–22. Bodies Driving by Direct Contact.

The surfaces at the contact point will always have a common tangent and common normal. Draw this common normal PN intersecting the line of centers at R. PN is the line of action as B drives C. There are two points of contact, P on B, and W on C. They have the same position but different velocities with respect to A.

The only motion that P can have *relative* to W is in the direction of the common tangent PK, since, if B and C are not to separate from

each other nor to crush each other, there can be no *relative* motion in the direction of the common normal PN. Such relative motion of P with respect to W is only possible if the centro bc is on the line of PN. This is the additional fact necessary to locate bc at the intersection R.

When one link drives another by direct contact, their centro is at the intersection of their line of centers with the connecting link, and the common normal to their surfaces at the contact point.

This statement is true whether the mechanism has three links or more. In complicated linkages it may at times be easier to use some other application of Kennedy's theorem than the one involving the fixed link.

2–11. **Application of Centros to Steering Problems.**—Fig. 2–23 represents the four wheels of an automobile or truck in proper relative position to make a sharp turn. The center lines of the axes of all four wheels intersect in the common point ac which is therefore the centro of the car and the ground. For the instant, all points of the frame of the car are turning about ac relative to the ground. If the axes of the front wheels D and E intersected the line of the rear axle in different points, there would be side scrubbing of the front tires.

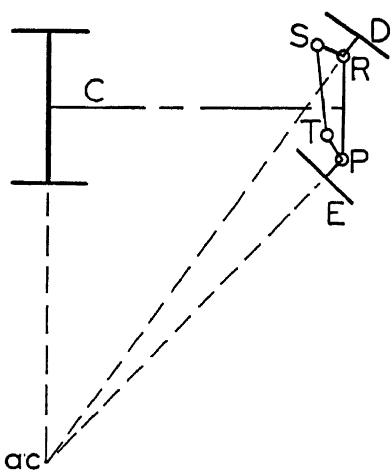


FIG. 2–23. Automobile Steering.

Perfect steering action would require that the two front wheels be in agreement as to the turning centro for all possible turning radii. No simple mechanism has been devised that will connect the front wheels in this manner. The mechanism shown in Fig. 2–23, which is the one in general use, gives an approximation good enough for practical purposes. The length of the tie rod ST must be shorter than the distance RP between knuckle pins. With independent front-wheel suspension, a slightly different linkage is generally used, but the same principle is employed.

The caster, Fig. 2–24, commonly used on furniture, illustrates a useful principle. Assume that motion occurs normal to the plane of the paper due to a force at P. Resistance on the line ST causes rotation about axis PR, the centro of the caster frame and the leg, and the caster is drawn into position for pure rolling. This is the simplest form of **caster action.**

The steering mechanism of the bicycle, Fig. 2–25, has been developed by utilizing the caster principle in an entirely different manner. The front wheel turns in the *fork link*. The centro axis of the fork link and the frame is WT. Since WT passes above O, the fork link and its wheel are in unstable equilibrium when the bicycle is vertical and stationary. In forward motion, however, a large measure of automatic balancing results.

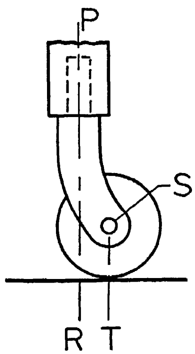


Fig. 2–24. Simple Caster.

Suppose the machine tends to tip toward the right (toward the reader), the handle bars being free. A component of the force of gravity acts normal to the frame in the plane of WT, and is resisted at O, causing the fork link to turn right. If the bicycle has fôrward motion, the resulting curved path causes centrifugal force toward the left which tends to restore balance. In addition, a part of this force acts in the plane of WT about O as fulcrum, turning the fork link counterclockwise and bringing the machine out of its curve as balance is restored. Manifestly, "hands off" riding depends on the offset s.

The offset r serves a different purpose. With ordinary steering, a tendency to overbalance toward the right is met by turning the fork link clockwise. This moves the axis WT to the left of the line of support PO, which is a factor in restoring balance.

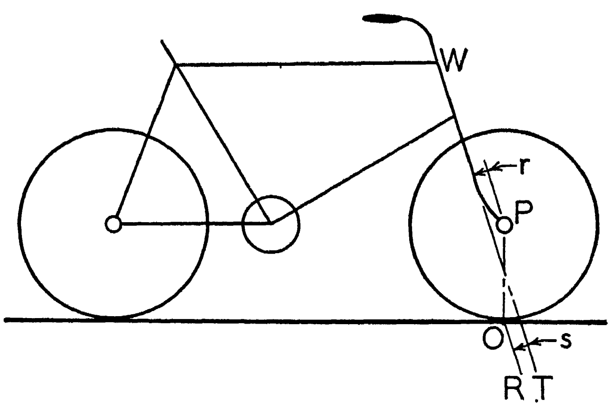


Fig. 2–25. Steering Mechanism of the Bicycle.

The caster principle is an important factor in automobile steering. The front wheels turn on short axles called *knuckles*. Steering is effected by turning the front wheel assemblies about knuckle-pin (also called king-pin) bearings. The center line of one of these bearings is ST and

PR in the two views of Fig. 2–26. Caster is shown in the left view where the knuckle-pin axis intersects the road in a point *ahead* of the theoretical wheel-contact point *O*. When going forward on a curve, centrifugal force acting in the plane of *PR* about *O* as fulcrum tends to restore the front wheels to straight-running direction.

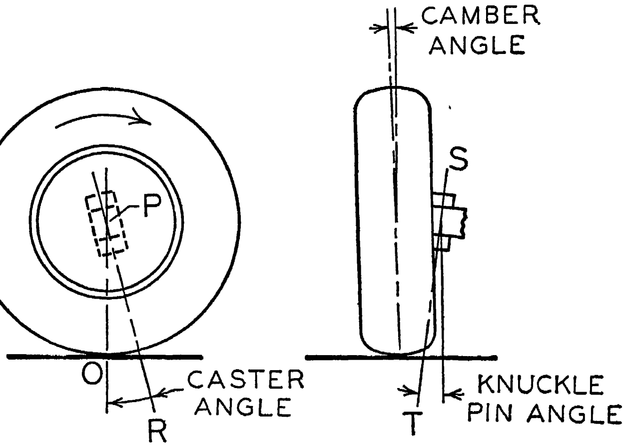


FIG. 2–26. Caster and Camber.

In straight-ahead position, caster places the wheels in unstable equilibrium, leaving the automobile with a tendency to wander at low speeds when the centrifugal force is small. But, unlike the similar situation on the bicycle, a remedy is here possible. In the right-hand view, the knuckle axis is seen to be inclined to the vertical, intersecting the road closer to the plane of the wheel. Hence, when the front wheel assemblies are turned, the front end of the car is raised a small amount. Gravity therefore tends to restore the wheels to straight-ahead position. This offsets the caster wander at low speeds, and contributes to finger-tip steering stability at all speeds.

Camber is the inclination of the planes of the front wheels so that the distance between rims is greater at the top than at the bottom. It reduces bending at the knuckle-pin bearings, and places more load on the large inner wheel bearing than on the smaller outer bearing. It has little effect on steering. Camber should be distinguished from *toe-in,* the latter causing the front-wheel rims to be slightly closer together in front of the axle than behind it. With camber, there is a tendency for the two front wheels to roll away from each other. A slight amount of toe-in is required to offset this.

2–12. **Centrodes.**—Permanent centros have fixed locations on the links to which they belong. Nonpermanent centros move on their links

in paths that are continuous as the mechanism goes through its cycle and the locus or point-path is called a centrode.

A *centrode* is therefore the locus on a link of the centro it shares with some other link. Manifestly, one travelling centro will make two centrodes, one on each of its links. These mating centrodes have some interesting properties.

Consider the quadric chain, Fig. 2–27. For the instant that the mechanism is going through the solid-line position, the centro ac is at T.

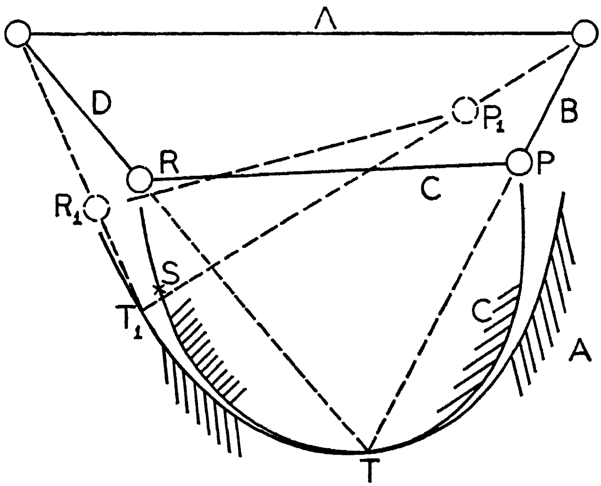


FIG. 2–27. Centrodes on the Quadric Chain.

For the instant, T is a common point of A and C. It is also a point on the two centrodes which are described, one on A being TT, and one on C being TS, as link C moves relative to A. Moreover, as C moves with respect to A, the two centrodes will roll on each other and *they will roll without sliding* since every successive point of contact is the centro ac for that instant.

The unique property of centrodes then is that a conjugate pair provides surfaces on which the two links concerned can roll without sliding. Such cam surfaces, eliminating as they do the friction of sliding, have valuable though limited application.

If we assume the A link fixed, the centrode TT_1 on A can be plotted directly from successive positions of centro ac. As C is moving relative to the paper, a construction is necessary to locate all these centros on the full line position of C. When ac is at T_1, C is at R_1P_1. It is only necessary to construct triangle SRP equal to $T_1R_1P_1$ to locate S as the corresponding point on the centrode of C.

Another method which is best for some cases is to fix C and move A, B, and D, locating ac directly on C.

Unless some means of preventing sliding, such as placing gear teeth on the centrodes, were used, it would not be practicable to substitute the centrode surfaces for the links D and B. The next example will put the centrode in a more practical setting.

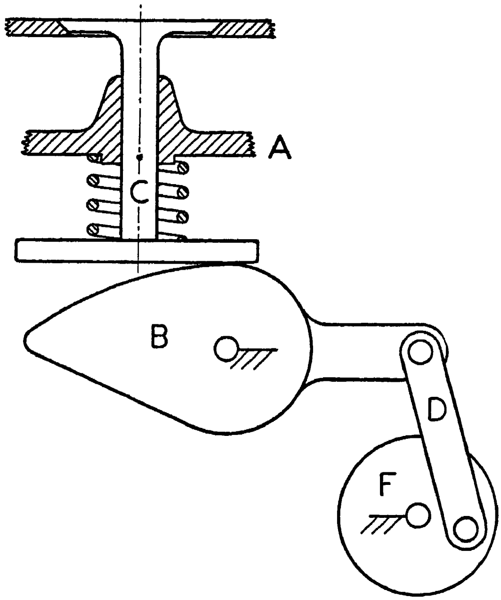


FIG. 2–28. Gas-Engine Valve Mechanism.

Fig. 2–28 shows a mechanism designed to operate an engine poppet valve. F is the driving link, and the cam B oscillates, opening the valve for about one quarter of the cycle. There is sliding at the cam surfaces causing friction and possibly wear. Is it possible to use the centrodes of the cam B and follower C to achieve pure rolling action?

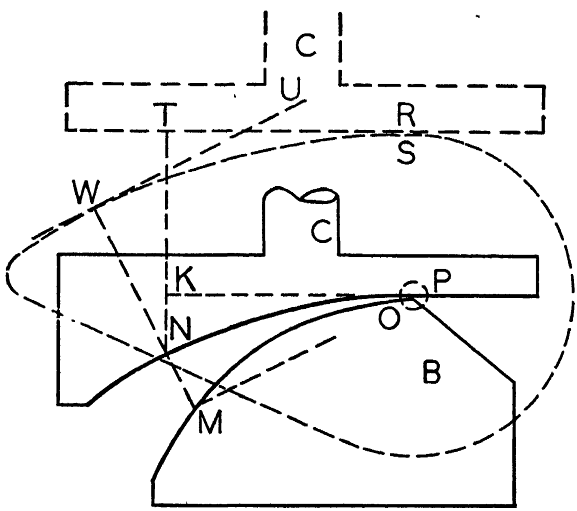


FIG. 2–29. Centrode Mechanism for Valve.

For clarity, the construction is shown in a separate figure (Fig. 2–29), drawn to double size. The cam links are represented by broken lines and the centrode links by solid lines. It is convenient to hold B fixed and give to C the entire relative motion. With contact at RS, centro bc is at O which is the axis of the shaft carrying B. (Centro ac is at infinity in horizontal direction.) O on B, and P on C are coincidental points on the two centrodes.

Next let contact on the original links move to W so that the contact surface of C will have the tangent position WU. Centro bc is now at M, a point of centrode MO on B. M must now be relocated on the original position of C. The total motion of C relative to B, as contact changed from R to W, was rotation about O through the angle KPM plus a translation of amount $(WM) - (SP)$. Hence we rotate M about O to K, and then move it vertically to N so that TN equals WM. Sufficient points located in this manner give the centrode PN on C.

The new cam and follower with centrode surfaces will drive the valve, giving it precisely the same motion as the original mechanism. As B rotates with its shaft about O, there will be pure rolling contact. Even in that portion of the cycle when C is stationary, there will be knife-edge contact at O without sliding.

As previously intimated, the centrode cams have limitations and disadvantages. The irregular curves are more costly to machine. Compare for example the centrode curves, with the flat surface and circular arc of the original of the foregoing case. If the valve were to be open for a larger part of the cycle so that a part of the centrode on B became vertical, the drive would no longer be definite, sliding might occur, and the combination would no longer fulfil the definition of a mechanism. However, the designer should be alert to recognize cases where the advantages of centrode surfaces outweigh the disadvantages.

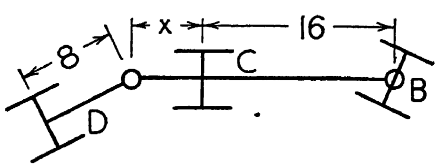
2–13. **Axodes.**—The term centro was defined not as a point but as an axis of instantaneous motion of one body relative to another. An *axode* is the surface described on a body by its moving centro axis. For a cycle, the axodes are closed cylinders or cones. They are cylinders if the bodies concerned have relative plane motion. The terms cylinder and cone are used here in their geometric sense as bodies having bases of any shape whatever.

For all cases of plane motion the axodes are fully represented by their traces on the plane of motion which are centrodes. In the previous article the curves found as centrodes satisfactorily represented the axode surfaces needed for the cam and follower of Fig. 2–29. For non-plane motion, the axodes, being cones, cannot be described by a single projection unless the cones happen to be circular. The axodes of a pair of mating

bevel gears, Fig. 10–2, are right circular cones with axes coincident with those of the shafts connected. As most practical mechanisms have relative plane motion only, their axodes are cylinders. Whether cylinders or cones, each conjugate pair of axodes makes pure rolling contact.

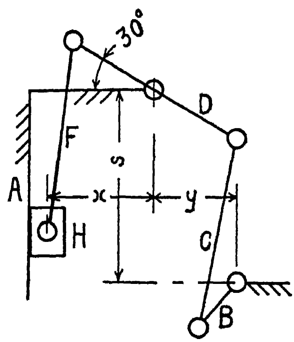
QUESTIONS AND PROBLEMS

1. Define mechanism and machine. State clearly the difference in technical meaning of the two terms.
2. Draw a closed kinematic chain that has one sliding pair and is not a mechanism. How many degrees of freedom has each link?
3. Devise a mechanism having
   (*a*) eight links,
   (*b*) five links,
   (*c*) six links with two of the connections sliding pairs.
4. Draw the Scott-Russell mechanism, Fig. 4–14, and find all the centros by direct reasoning, that is without using Kennedy's theorem.
5. Find the centro of two driving wheels of the mechanism of the locomotive, Fig. 4–7.
6. Two cylinders, B of 4 in. diameter and C of 2 in. diameter, turn on fixed and parallel axes and are in contact. Draw an end view and locate the centro bc for the following cases:
   (*a*) when the cylinders roll on each other without sliding,
   (*b*) when both cylinders rotate at the same angular velocity (with sliding at the surfaces in contact),
   (*c*) when B rotates at 12, and C at 6 radians per second in opposite sense.
7. A truck and trailer are traveling on a curve such that the center of the front axle B of the truck moves on an arc of 36 ft radius. With given dimensions in feet, find the value of x that will cause the trailer
   (*a*) to follow the track of the rear wheels,
   (*b*) to follow the track of the front wheels of the truck.

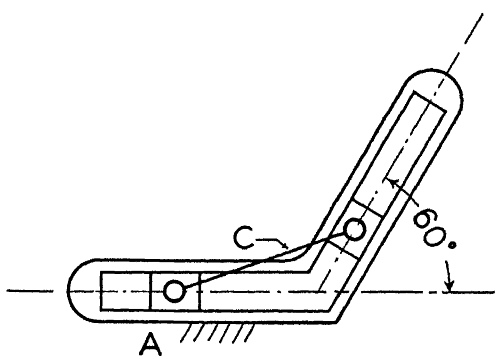


Based on your answers to (*a*) and (*b*), make the kinematic design for a train of several 4-wheel trucks for shop transportation, to be drawn by a single battery car, so that the train will travel around corners where the space is just sufficient for one of the 4-wheel trucks to clear.

8. "Kennedy's theorem is proved by using the fact that a point cannot have two different motions at the same time." Is this statement correct?
9. Convert the kinematic chain, Fig. 2–16, to a mechanism
   (*a*) by the addition of a single link,
   (*b*) without changing the number of links.
10. What is the difference between a permanent and a non-permanent centro?
11. Find all the centros of the crossed-link quadric mechanism, Fig. 3–19, when link D is vertical. Make $A = 3\frac{1}{2}$ in., $B = 1$ in., $C = 3\frac{1}{2}$ in. and $D = 1\frac{1}{2}$ in.
12. Draw the Tchebicheff mechanism, Fig. 4–19, making $A = 2$ in., $C = 1\frac{1}{2}$ in., and B and D each $2\frac{1}{2}$ in., with D 30° to the left of vertical. Find all the centros.
13. Locate all the centros for this rocking-beam mechanism. $B = 1\frac{1}{4}$ in., $C = 4$ in., $D = 4$ in. with ad at its center, $F = 4$ in., $x = 2\frac{1}{4}$ in., $y = 1\frac{3}{4}$ in., $s = 4$ in. Construct a circle diagram as you locate the centros.



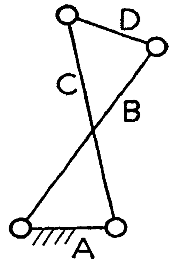
14. Making connecting rod C of length 4 in. develop the centrodes that will give equivalent motion of C relative to A.



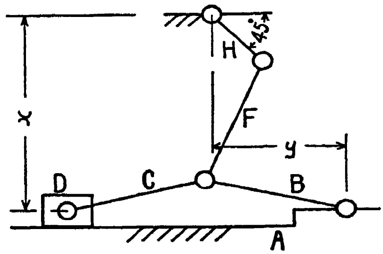
15. A four-bar linkage has A the fixed link 4 in. long, B the left crank 1 in., C $2\frac{1}{2}$ in., and D the right crank 2 in. Construct the centrodes of ac for a motion of B of 45° clockwise from the upper vertical.

16. Draw this crossed-quadric mechanism making $A = D = 1$ in., and $B = C = 2\frac{1}{2}$ in. Plot the centrodes of *ad*. The best method in this case is to plot the centrode on A first, then consider D fixed to plot its centrode.

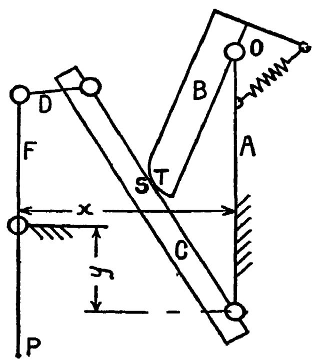
If gear teeth were placed on these centrodes what kind of gears would result?



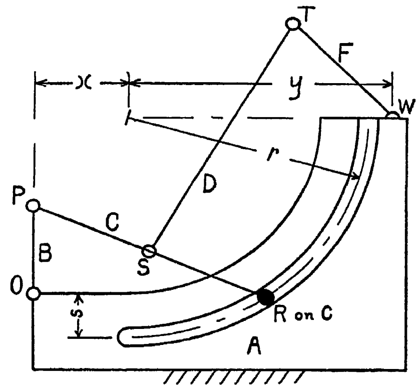
17. This toggle mechanism has B, C, and F each $2\frac{1}{4}$ in., $H = 1\frac{1}{8}$ in., $x = 3\frac{3}{8}$ in. and $y = 2\frac{1}{8}$ in. Find all the centros.



18. Find all centros for this direct contact drive in which B is held in contact with C by a spring. A and F are each $4\frac{1}{2}$ in. long and are parallel. Centro *af* is at the midpoint of F. $D = 1\frac{1}{8}$ in., $C = 4\frac{1}{2}$ in., $x = 3\frac{3}{8}$ in., $y = 1\frac{1}{2}$ in. and $OT = 2\frac{5}{8}$ in.



19. The link C ends in a pin which slides in the groove of the fixed link A. With all dimensions inches, draw $B = 1$, $C = 2\frac{5}{8}$ with cd at its center, $D = 3, F = 1\frac{1}{2}, x = 1, y = 2\frac{3}{4}$, $s = \frac{1}{2}, r = 2\frac{1}{2}$. Find all centros.



20. Devise a mechanism to connect a rotating shaft with a parallel shaft which is to reciprocate about its axis through 90° every revolution of the driving shaft. Give dimensions.
21. Design and dimension a mechanism to drive a reciprocating part in a straight path such that the backward stroke will occupy one quarter the time of the forward stroke. The drive is at constant speed.
22. How many degrees of freedom has:
    (*a*) A single ball in a ball bearing, Fig. 2–12,
    (*b*) The front wheel of an automobile relative to the car body, Fig. Prob. 6–10 (*a*)?
23. Find all the centros for an oscillating-beam, quick-return mechanism such as shown in Fig. 6–11, for the phase in which the driving crank B is 60° clockwise from the upper vertical. Make the length ac to ab 3 in., B $1\frac{1}{2}$ in., C 6 in., F 2 in., and make the path of the centro fh $2\frac{3}{8}$ in. above ab.

    Print on the drawing the definition of centro and the statement of Kennedy's theorem.

CHAPTER III

INVERSION IN MECHANISMS

3–1. Inversion of the Kinematic Chain.—Inversion may be defined as the choice of a different link in a mechanism to be the fixed link or frame. This simple operation produces such remarkable results that it will be considered in some detail.

In the slider-crank chain, there are four variations, as indicated in the four views of Fig. 3–1, obtained by fixing each of the four links in turn. In this figure it is easy to identify the links by their form and dimensions; however, if the links are allowed to vary in form and dimension, there is only one certain means of identification and that is the pairing. Links A and D each have one element of a turning pair and one element of a sliding pair, while B and C have turning connections only.

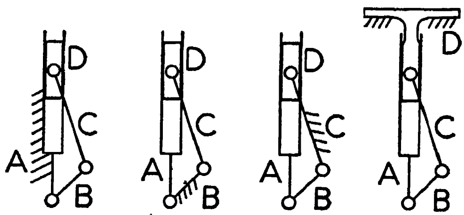


FIG. 3–1. Effect of Changing the Link to be Fixed on the Slider-Crank Mechanism.

Suppose now that a mechanism is presented for identification having links of any form and dimensions whatsoever. If it has four links, three turning pairs and one sliding pair, it is the slider-crank chain, since there is only one order in which three turning pairs and one sliding pair can be looped. Suppose that a certain link is indicated as the fixed link or frame, which inversion is it?

It will be quite evident whether the fixed link has both a turning and sliding element or only turning elements but further classification is impossible. Fig. 3–2 has been devised to make this clear. Links A and D are identical, as are B and C. Fixing B or C yields a mechanism quite distinguishable in properties from that resulting from the fixing of A or D. Each is a distinct inversion of the other. But fixing B yields the same mechanism as fixing C, and no essential difference results in changing the fixing from A to D.

It might be contended that C could be distinguished from B as being that link clockwise from those having the sliding pair. This is a fallible criterion as will be appreciated by imagining the mechanism viewed from the opposite side.

Returning to Fig. 3–1 where links with the same pairing have different form and dimensions, it is seen that four distinct variations occur as each link in turn is fixed. It becomes evident that inversion falls logically into two classes:

class (1) inversion based on pairing,

class (2) inversion based on the form and dimensions of the links.

In the first class, inversion is independent of the form of the links. In the second class the links must retain their form during inversion. The number of inversions of the first class will depend on the variety of the pairing but if inversion is extended to the second class there will be as many as the number of links in the mechanism.

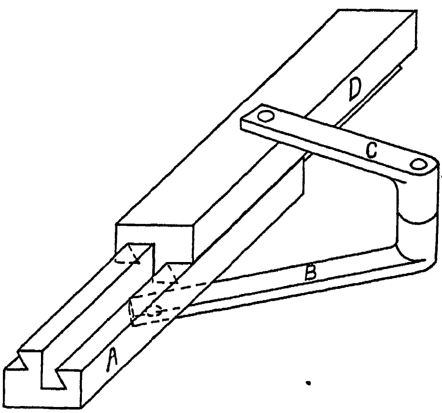


FIG 3–2. Slider Crank Chain with Uniform Links.

The quadric chain has no inversions of class one but four of class two. Unless the links differed in form (including dimension), inversion of this chain would not yield significant results.

Observe that inversion does not change the relative motion of the links in any manner. The change of performance arises from new motions with respect to the frame, that is with repects to the new fixed link.

3–2. **Applications of the Slider-Crank Chain where Links having Both Turning and Sliding Elements are Fixed.**—This is the inversion shown in Fig. 3–1 (*a*), also in Fig. 1–4. It is the original slider-crank mechanism and is by all measures the most useful four-link combination. The relative motion of the links at the dead-center phases, when power is applied to the piston, is made definite by the carry-through effect of a flywheel attached to the crank link.

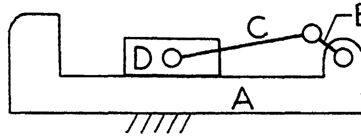


FIG. 3–3. Bulldozer Mechanism.

An application from the machine-tool field, the bulldozer, is shown pictorially in Fig. 3–4 and kinematically in Fig. 3–3. In the machine pictured, there are two connecting rods and two large side gears which act as cranks and also as flywheels. Dies are attached to the face of the block *D*, Fig. 3–3, and to the opposing face of *A*, and metal in hot billets

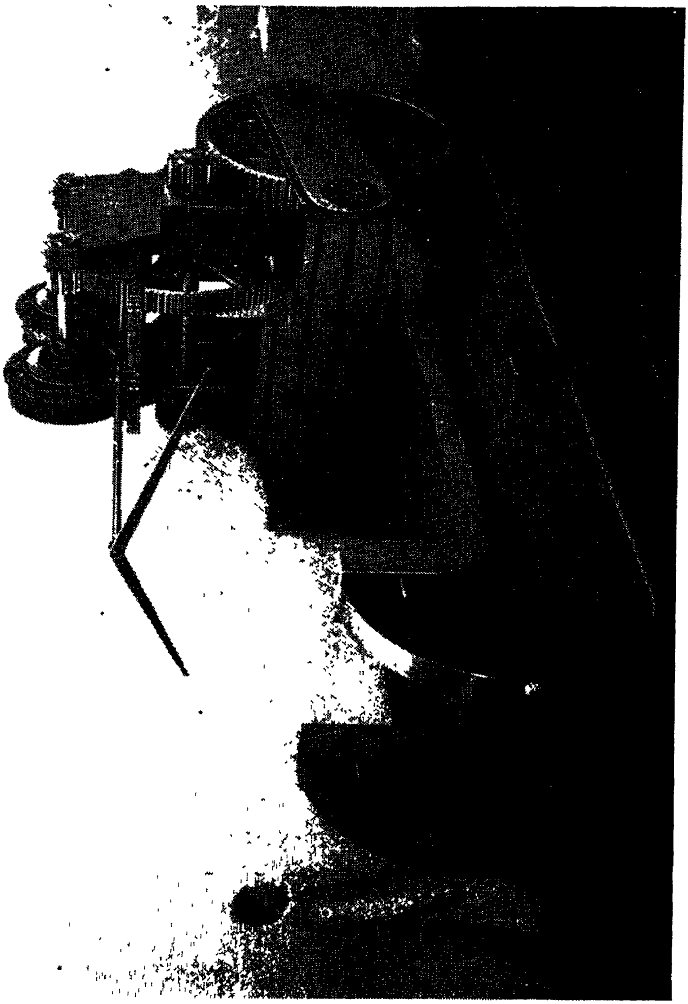


FIG. 3-4. Bulldozer.
The Ajax Manufacturing Co., Cleveland, Ohio.

or in bars, either hot or cold, can be formed or pressed to desired shapes in one working cycle of the machine. When B and C are nearly in line, the mechanical advantage of the driving crank B is very large and the dies are adjusted to complete the operation at that phase.

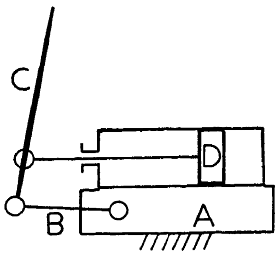


FIG. 3–5. Pump Mechanism.

Fig. 3–5 illustrates one form of the mechanism of the common hand pump which may be horizontal or vertical. Air pumps and hand and power presses are often designed using this mechanism. It will be noted that here, as in the case of the bulldozer, the fixed link has one element of a sliding pair and one element of a turning pair.

3–3. Applications of the Slider-Crank Chain where Links Having only Turning-Pair Connections are Fixed.—In applications of the slider-crank chain the forms of links A and D, Fig. 3–1 (a), are generally quite different and if, in the inversions, these links are allowed to retain this difference in form they can be identified thereby. This allows us to

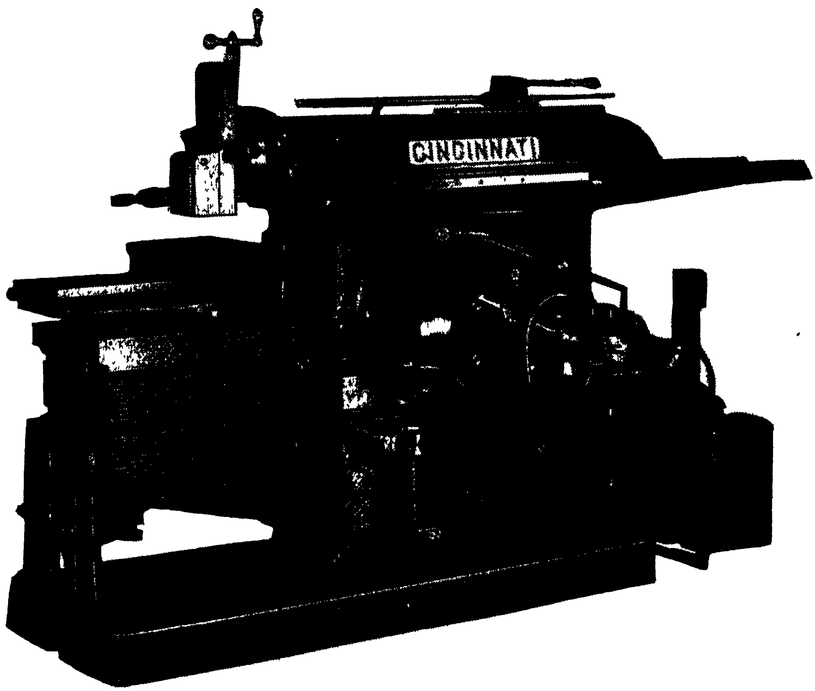


FIG. 3–6. Oscillating-Beam Shaper.

The Cincinnati Shaper Co., Cincinnati, Ohio.

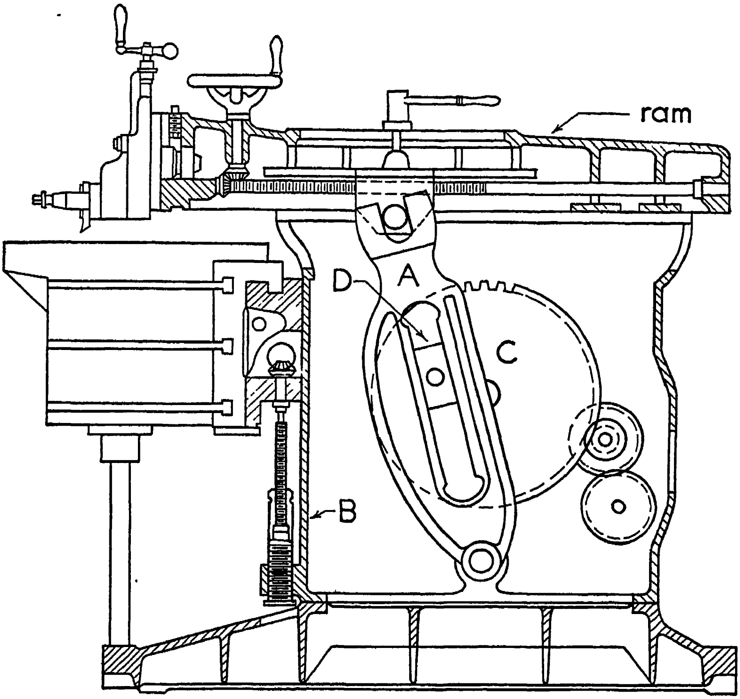


FIG. 3-7. Sectional View of an Oscillating-Beam Shaper.

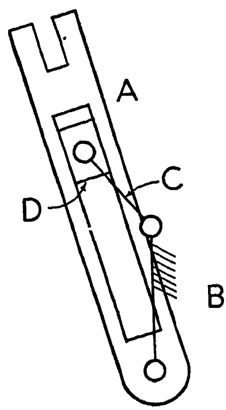


FIG. 3-8. The Basic Mechanism of the Beam Shaper—B Fixed.

extend the inversions to the second class and the results have important practical application.

First fix B as in Fig. 3-1 (b). Two distinct cases arise. If C is shorter than B, A will oscillate as C revolves, and, if C is rotating counter clockwise, A will move more slowly toward the left than toward the right. A valuable application is to be found in the oscillating-beam shaper pictured in Fig. 3-6, shown sectioned in Fig. 3-7, while the basic mechanism appears in Fig. 3-8. The block is D and is driven by the bull gear C. The cutting tool is carried on the front of the ram. The position of the pin which drives D is adjustable on the bull gear and when the pin is moved closer to the center of the gear, the stroke of the ram is shortened.

The tool, carried on the front of the ram, can only cut on the forward stroke, toward the left. Suppose the proportions of the mechanism are

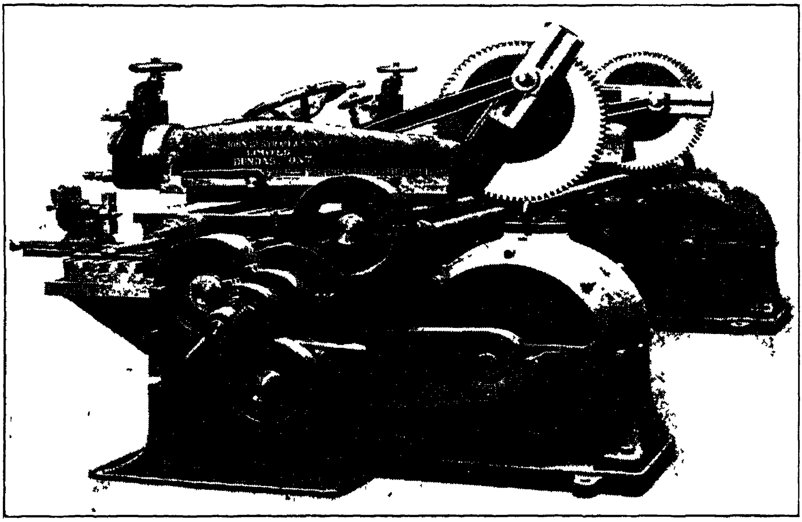


FIG. 3-9. Whitworth Quick-Return Shaper.

The John Bertram and Sons Co., Dundas, Ont.

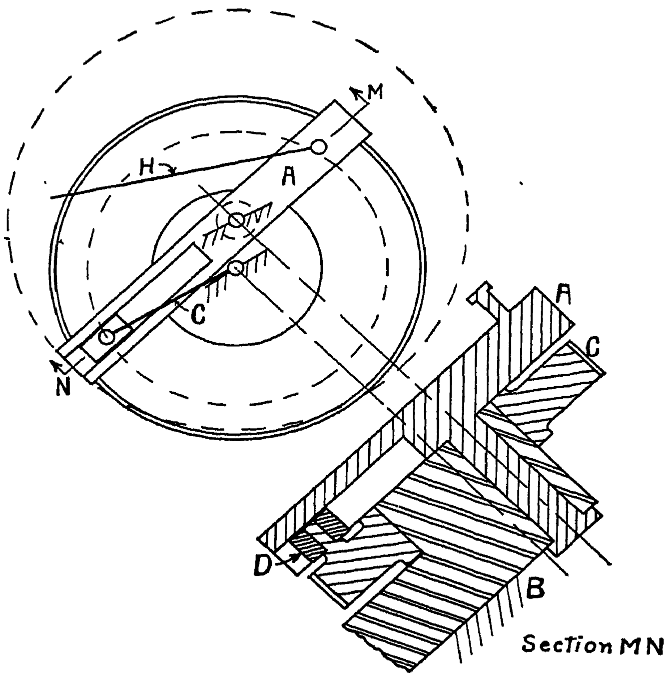


FIG. 3-10. Whitworth Quick-Return Mechanism.

such that the back idle stroke occupies one second while the cutting stroke requires three. If the mechanism were not "quick return," a cycle would occupy six seconds for the same cutting speed. Hence a saving of $33\frac{1}{3}$ per cent in the time of the operator and of the machine results in this case from the quick-return feature.

Returning now to Fig. 3–1 (b), if the crank C is longer than link B, the beam A will revolve. The speed of A during the revolution will be variable however, so this mechanism is also used for shaper and slotter drives. It is called the Whitworth Quick-Return mechanism and the twin shaper (two on one frame) shown in Fig. 3–9 is an example of its application. Fig. 3–10 shows two views of the essential parts in the phase corresponding to that of the front twin. The sectional view shows C, the large driven gear, revolving on the large hub of B and driving A through the sliding block D. With C driven at constant speed, A will revolve slowly in the phase shown, while 180° later, D will be closer to the axis of A, and A must therefore turn faster, speeding the ram on its idle return stroke.

The *rotary* cylinder engine, the mechanism of which is shown in Fig. 3–11, was used extensively for airplanes in the early days of the industry.

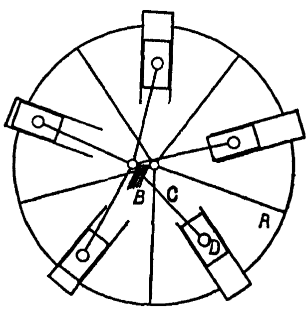


FIG. 3–11. The Rotary Cylinder Engine.

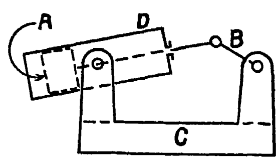


FIG. 3–12. Rocking-Cylinder Engine.

The cylinders acted as flywheel. The means of supplying fuel and ignition to the moving cylinders was complicated, however, and the design was superseded by fixed cylinder engines.

Modern airplanes and tanks are powered by either radial or in-line engines, both of which are examples of the first inversion, Fig. 3–1 (a), of the slider-crank mechanism. The student must be careful not to confuse the terms *rotary* engine (revolving cylinder) with *radial* engine (stationary cylinders arranged like arms in a wheel).

Applications where C is used as fixed link are not so frequently met. The toy steam engine, Fig. 3–12, called a rocking-cylinder engine, makes

use of the motion of the cylinder past ports in the frame to admit and exhaust steam, so valves and valve gear are unnecessary. The General Electric Co. has used this rocking-cylinder inversion as a compressor in one design of domestic refrigerator.

3–4. **Inversions of the Double-Slider Four-Link Chain.**—From Fig. 3–13 it is apparent that of the four links, only B and D have like pairing. It follows that there will be three inversions of this chain on the basis of pairing, and since B and D have here like form, there will be no inversion of the second class unless these links assume distinctive form.

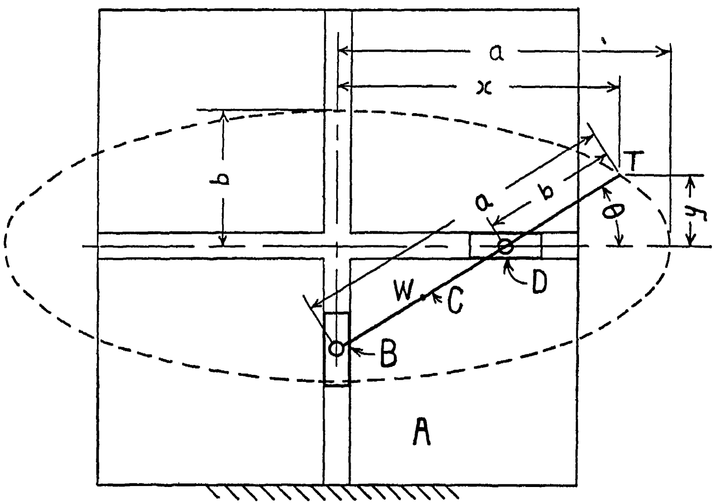


FIG. 3–13. Ellipsograph Mechanism.

An application of this chain with link A fixed is to be found in the *ellipsograph* or elliptic trammel, an instrument for drawing true ellipses. Suppose the pen or pencil is attached to the bar C at the point T, the bar being inclined at angle θ from the line of travel of D. Taking x and y as the variable coordinates of T, these relations obtain:

$$x = a \cos \theta$$
$$y = b \sin \theta$$

from which

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2 \theta + \cos^2 \theta = 1 \qquad (1)$$

the characteristic equation of the ellipse. In order to make the instrument adaptable for drawing ellipses of varying size and of varying ratio of major to minor axes, the bar C is made so that its turning-pair elements

can be fastened at any point along its length, thus making a and b adjustable.

If T is carried down to cd, the ellipse becomes flattened to a straight line. If T is brought to W, the mid-point of C,

$$x^2 + y^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta \tag{2}$$

and since $a = b$,

$$x^2 + y^2 = a^2 \tag{3}$$

the equation of a circle of radius a. Then this value of a is the radius at which W travels in a circle about the intersection of the slides on A. It appears that the mechanism could be driven from a shaft at the intersection of the cross-slides and normal thereto, having a crank with crank-pin connection to C at W.

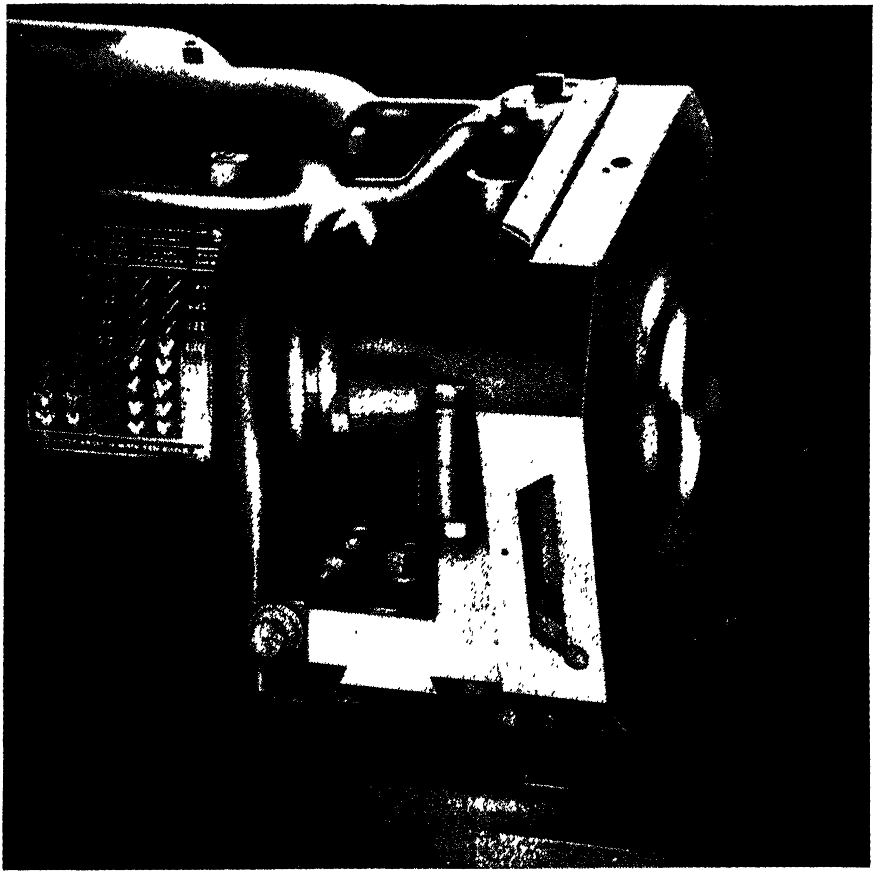


FIG. 3-14. Elliptical Chuck Mounted on Lathe.

The Monarch Machine Tool Co., Sidney, Ohio.

The **elliptical chuck** is a device that can be attached to the headstock and bed of an ordinary lathe, as in Fig. 3–14, for the machining of objects of elliptical outline such as the blanks for elliptical gears. If we invert the elliptic trammel of Fig. 3–13, fixing C and driving D, the plate A will be given elliptical motion; that is each point of A will describe a particular ellipse relative to the new fixed link C.

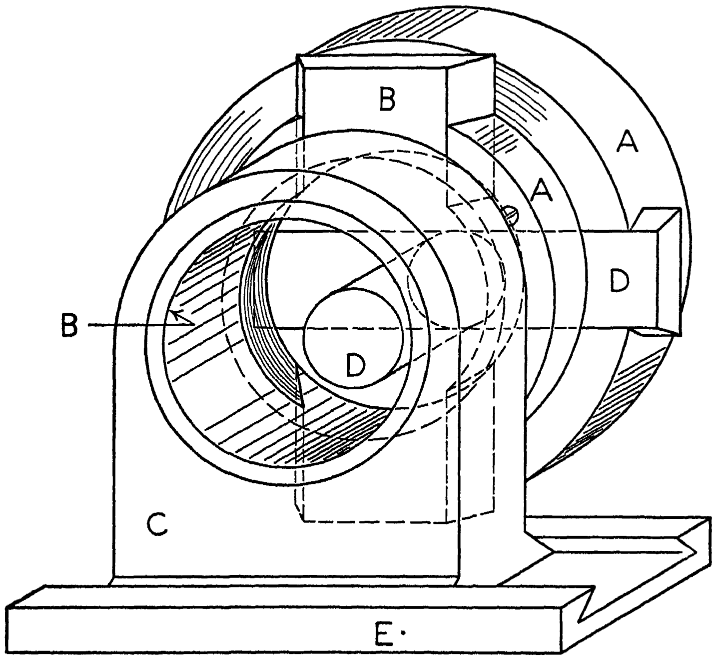


FIG. 3–15. Elliptical Chuck. Oblique view looking toward tailstock. D is attached to the lathe spindle, A is the face plate. C is fixed but adjustable on E.

One method of adapting this inversion to the lathe is shown in Fig. 3–15.[1] The drive is through link D which is fastened to the nose of the lathe spindle. D gives rotation to the face plate A, but A must slide on D as required by the action of the other slide B. Link B rotates in the *fixed* housing C. In operation, C is clamped in E, and E is bolted to the lathe bed. Link B must be hollow, including its slide, to allow D to operate through it. Face plate A with work attached will have elliptical motion relative to a tool supported from the bed.

Returning now to Fig. 3–13, if the point of the cutting tool is placed in line with the centers of D and B, it will have the relative position of T.

[1] Prepared with the aid of descriptive material furnished by the Monarch Machine Tool Co., Sidney, Ohio.

The distance of the tool point from the center line of D equals the minor semiaxis of the ellipse to be turned on the work. The distance of the tool point from the center line of B equals the major semiaxis. This latter dimension is adjusted, Fig. 3–15, by clamping C in different positions along the slide E. See also Fig. 3–14 which shows a screw traverse for positioning C.

This inversion has also found a limited application in the **Oldham coupling,** Fig. 3–16, which can be used to connect two parallel shafts that are not quite in line. In the figure the shafts are shown with a large center-line displacement s, as it better serves the purpose of illustration. The center link A is a disk having machined on it two projecting bars normal to each other. These engage the grooves on B and D and transmit the torque. The amount of sliding action, and consequently of frictional lost work, increases as the displacement increases. However this coupling transmits a theoretically constant angular velocity ratio.

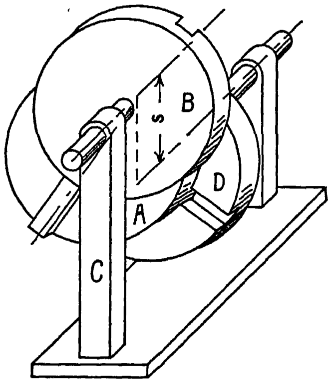


FIG. 3–16. The Oldham Coupling.[1]

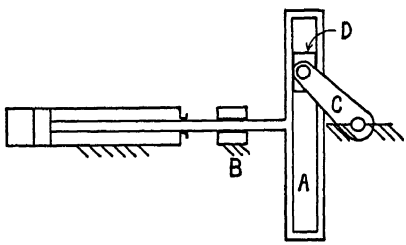


FIG. 3–17. The Scotch Yoke.

Fixing B or D gives the mechanism known as the **Scotch yoke,** Fig. 3–17, which can be used instead of the slider-crank mechanism where true harmonic motion for the reciprocating parts is desired. Harmonic motion with a slider-crank connection would require an infinitely long connecting rod. The Scotch yoke has been used to drive pumps and compressors, C being the crank of the driving shaft. Its application has, however, been limited due to its poor mechanical efficiency caused by the added friction of the slider D.

3–5. **The Quadric Chain or Four-Bar Linkage.**—All links having uniform pairing, the only inversions of the quadric chain will be those of class two. Considerable variation in performance, however, results from

[1] After an illustration in "Kinematics" by R. J. Durley.

the possible variation in the lengths of the three moving links compared to that of the fixed link.

The **parallelogram linkage,** Fig. 3–18, allows crank B to drive crank D at uniform relative angular velocity except for the two dead-center

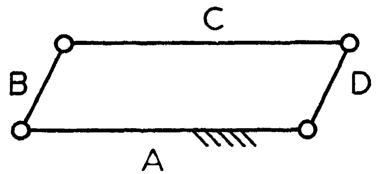


FIG. 3–18. $A = C, B = D.$

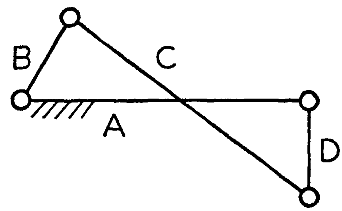


FIG. 3–19. $A = C, B = D.$

phases where all links are in line. A flywheel on the driven link would supply kinetic energy to carry it past dead center at speed, but would not insure a satisfactory start from this position. Indeed, the mechanism might take up the crossed link position of Fig. 3–19. A satisfactory way to avoid the dead-center starting difficulty is to duplicate the mechanism with phase difference. This is the method used on the steam locomotive as shown diagrammatically in Fig. 3–20, the phase difference being 90°. The same arrangement is used on hoisting engines to give continuous torque.

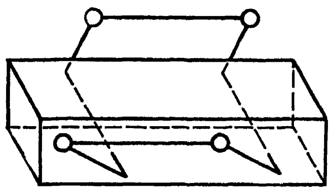


FIG. 3–20. Crank Arrangement on Steam Locomotive.

With links A and C parallel, Fig. 3–18, B can drive D, the latter carrying a flywheel, at nearly constant relative angular velocity. In the crossed-link position, Fig. 3–19, B can still drive D continuously but the speed ratio is quite variable.

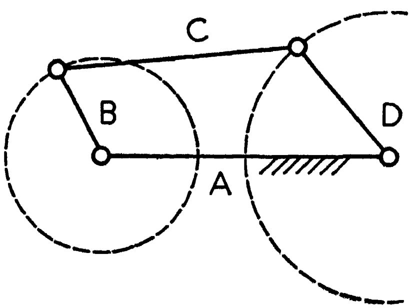


FIG. 3–21. $A = C - (D - B)$ or $A = C + (D - B)$. B rotates, D oscillates through dead center.

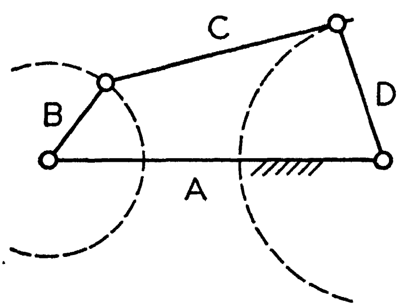


FIG. 3–22. $A > C + D - B$ and $A < C + D + B$. Both cranks oscillate. There are four dead-center positions.

We shall now examine the possibilities of the quadric mechanism when the cranks B and D are of unequal length and for convenience let B represent the shorter crank. Fig. 3–21 shows the special case where the length relations allow D to oscillate through dead center. Two revolutions of B are required for one complete oscillation of D.

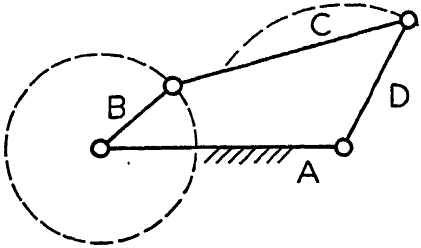


FIG. 3–23. $A < C + (D - B)$ and $A > C - (D - B)$. B rotates, D oscillates.

The drag-link mechanism, Fig. 3–24, allows one crank to drive the other with full rotation for both and no dead center phase, but with variable relative angular velocity. To avoid interference the bars must be arranged somewhat as in the two views of Fig. 3–25.

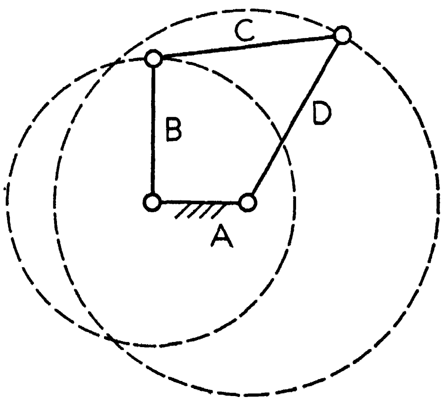


FIG. 3–24. Drag-Link Mechanism. $A < C - (D - B)$ and $A > C - (D + B)$, also $A < B$ and $A < (D + B) - C$, [$B < D$ being assumed].

It may be noted that the above cases cover all possible lengths of A in terms of the lengths of B, C, and D, from $A = C + D + B$ to $A = C - (D + B)$. Only inside these limits can a four-bar mechanism be constructed. Each case has applications for which its performance is suited. There are few complicated machines on which this mechanism in some of its inversions will not be found.

It is not recommended that the student memorize the dimensional relations that define performance, but rather that he verify them in the following manner. Cut three strips of cardboard about 6 in. by 1/2 in. and lay off scales on the center line of each showing half inches. Label these B, C, and D. Use thumb tacks, head down, for centros bc and cd,

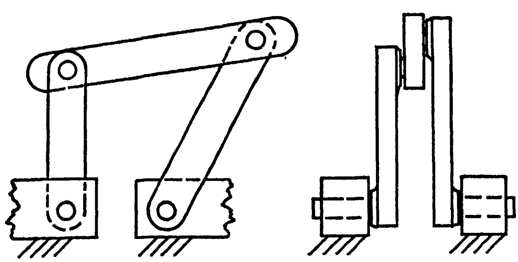


FIG. 3–25. Construction of Drag-Link Mechanism for Full Rotation.

and for *ab* and *ad* tack into the drawing board. Now all dimensions can be quickly varied and the performance noted. This technique will be found useful also for more complicated bar linkages.

3–6. **Other Four-Link Mechanisms.**—If *t* represents a turning-pair connection and *s* a sliding-pair, the four-link mechanisms so far analyzed can be represented thus:

$$\text{Slider crank,} \quad A\,t\,B\,t\,C\,t\,D\,s\,A \tag{4}$$

$$\text{Double slider,} \quad A\,s\,B\,t\,C\,t\,D\,s\,A \tag{5}$$

$$\text{Quadric,} \quad A\,t\,B\,t\,C\,t\,D\,t\,A \tag{6}$$

Evidently there are two other sequences that will give distinctive chains.

$$\text{Crossed-slider chain,} \quad A\,s\,B\,t\,C\,s\,D\,t\,A \tag{7}$$

$$\text{Four-slider chain,} \quad A\,s\,B\,s\,C\,s\,D\,s\,A \tag{8}$$

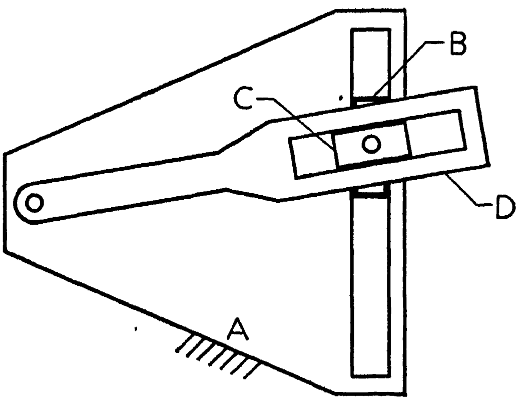


FIG. 3–26. Rapson's Slide.

There should be corresponding mechanisms.

An application of (7) is found in Rapson's slide shown in Fig. 3–26. This is a tiller mechanism, the rudder being fastened to the shaft which is keyed to the left end of *D*. A cable, chain, or sometimes a screw actuates block *B* which moves *C* by means of the turning-pair connection. The reaction of the water on the rudder is greater as it is moved off center to turn the ship, and this mechanism provides greater leverage for such positions automatically.

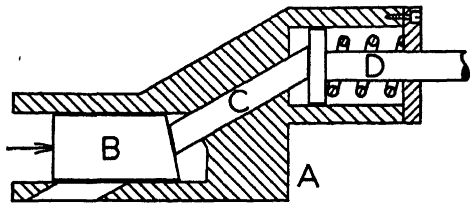


FIG. 3–27. The Four-Slider Mechanism.

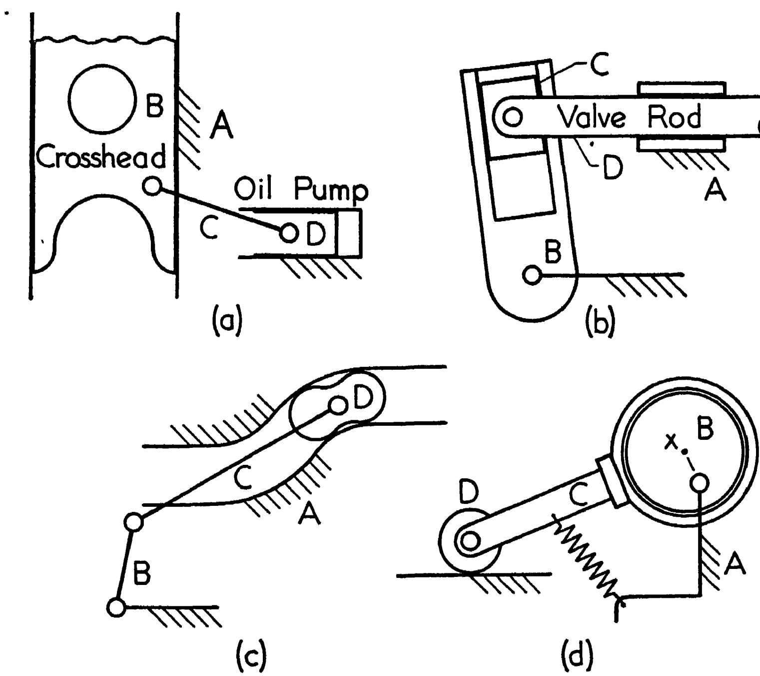
Fig. 3–27 shows a four-slider mechanism used to transmit a uniformly-reduced offset translation. This chain can be arranged to give the driven link *D* motion in any direction. The action is one-way, however, and the effect can often be achieved by simpler devices.

3–7. **Closure.**—The purpose of this somewhat extended study of inversion is to give practice in recognizing the basic mechanisms no matter how disguised, and to open the door to the possibilities of adapting these simple mechanisms to the purposes of design by inversion. It has doubtless been noticed that the complicated mechanisms of many links are nearly always the result of combining the basic chains herein presented. For example, the rocking-beam mechanism, shown in problem 13, Chapter II, is a simple addition of the slider-crank and quadric chains.

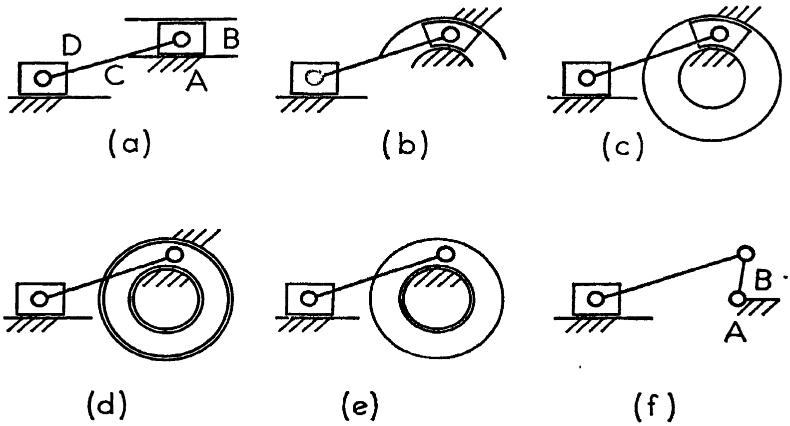
It should be remembered that inversion does not change the relative motions of the links provided the links retain their form. It is the new motions relative to the changed fixed link or frame that change the functioning of the mechanism.

QUESTIONS AND PROBLEMS

1. Define inversion of a mechanism and explain the two classes. How many inversions of each class does the slider-crank chain have?
2. Identify the following mechanisms as to basic classification and particular inversion:



3. Make a logical classification of the pairing in the mechanisms of problem 2.
4. Construct a four-bar linkage using cardboard and lettering the links as in Fig. 3–20. Make
$$A < C - (D - B)$$
and
$$A > D + B - C$$
Describe the possible action of the mechanism.
5. In this related series of mechanisms what is the basic chain at (*a*), at (*f*)? Where does the pairing between A and B change from sliding to turning?



6. An elliptical gear blank is to be turned on the chuck of Fig. 3–15, to have a major-axis diameter of 10 in. and a minor-axis diameter of 6 in.
   (*a*) How far should the axis of B be set from the axis of D?
   (*b*) How far must the point of the cutting tool be from the axis of link D when the finish cut is being taken?

CHAPTER IV

PARALLEL AND STRAIGHT-LINE MECHANISMS

4–1. The Pantograph.—The essential portion of this type of mechanism is the quadric chain with opposite links equal and therefore parallel. It can be used to perform two functions depending on the method of fixing. If connection to the fixed link is by a turning pair at a single point on the quadric chain, the mechanism can be used to give proportional motion and is regularly called a pantograph. If one link of the parallelogram is fixed, a motion of circular translation is assured for the opposite link.

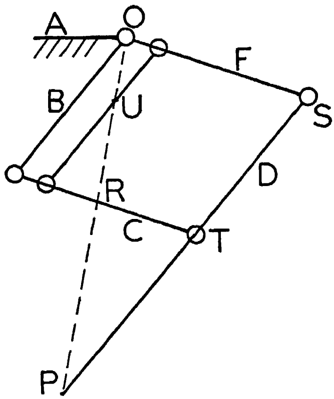


FIG. 4–1. The Pantograph.

The proportionality resulting in the first case will now be proved. In Fig. 4–1, the side D of the quadric chain is extended to P, letters at the end of the alphabet being used to represent points. R is the point on C in line with O and P. Because link C is always parallel to F, $\triangle\ PTR$ and $\triangle\ PSO$ are always similar and

$$RT = OS \times \frac{PT}{PS}$$

which equals a constant for all possible positions of the mechanism. Thus the same point on link C is always in line with O and P. Further,

$$\frac{PO}{RO} = \frac{PS}{TS} = \text{a constant.}$$

It follows from these two facts that for all possible positions of the mechanism the motion of R is proportional to the motion of P, and in the constant ratio RO/PO. So if P is given a straight-line motion in any direction, R will have a reduced straight-line motion in parallel direction. If P is made to trace any irregular figure, R will trace a reduced copy of the same figure.

Suppose a greater reduction in scale is required, such as UO/PO. The same mechanism could be used by adding a link parallel with one of the sides of the quadric chain and connecting it by turning pairs in such position as to locate U in line with P and O.

Numerous adaptations of the pantograph mechanism connected to give proportional motion have been made. It is used to reproduce maps to smaller or larger scale, also on engraving machines where a

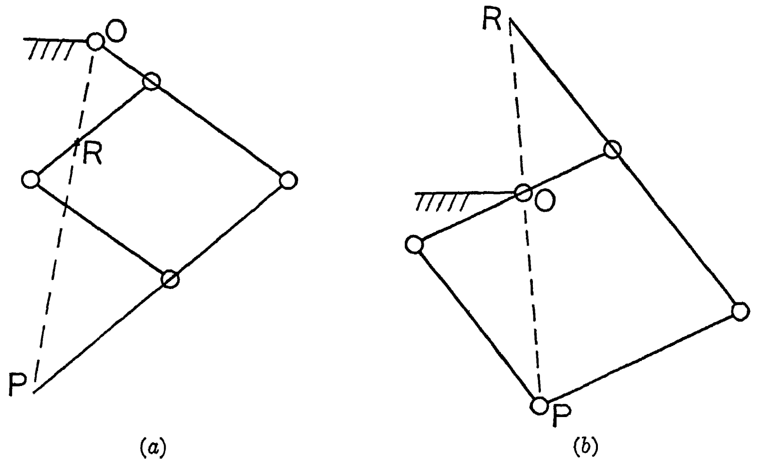


FIG. 4-2. Pantographs.

small revolving end-milling cutter is carried at U, and P is a tracing point which the operator guides around a large drawing of the object to be engraved. In Fig. 4-2 (a), the connection to the fixed link is at a point outside the parallelogram, though still on one of the quadric links. In (b), if a figure were traced with point R, a pencil point attached to P would produce an inverted copy. The compound pantograph, Fig. 4-3, often called "lazy tongs," is a multiplication of the same basic mechanism. It is used where a very large proportional reduction of motion is required, such as the indicating of long-stroke engines.

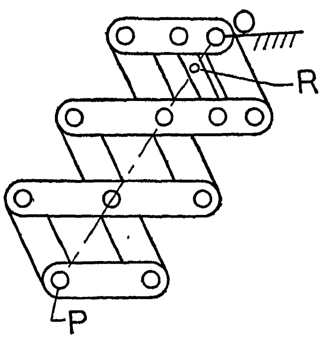


FIG. 4-3. Lazy Tongs.

The general scheme of connections required to indicate a steam engine is shown in Fig. 4-4. Two engine indicators I_1 and I_2 are shown connected to the two ends of the cylinder of a double-acting engine. The purpose of the indicator, originally developed by James Watt for use on his locomotive, is to show how the pressure in the cylinder varies, as the piston moves back and forth (as volume behind piston changes). To operate the indicator, the cock (valve) in the short pipe between the cylinder and indicator is opened for a cycle, allowing the steam or gas

pressure to compress the indicator spring, Figs. 4–20 and 4–21, and cause the pencil point to rise or fall in proportion to the pressure. The pencil point is pressed lightly against the indicator card which is fastened around the indicator drum. This is the part of the assembly necessary to record pressure.

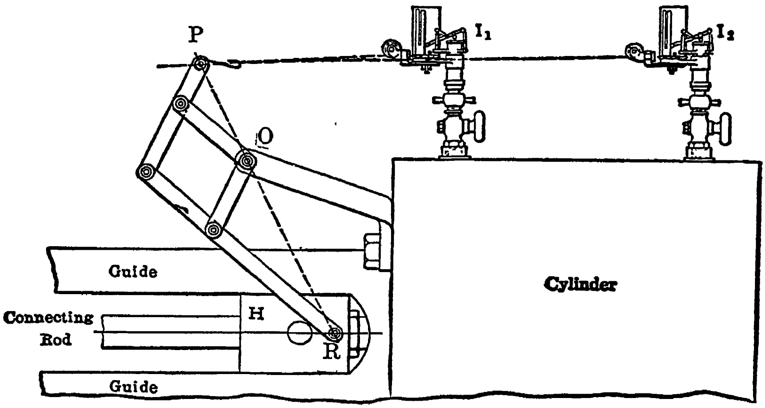


FIG. 4–4. General Arrangement of Indicators and Reducing Mechanism to Take Indicator Diagrams of an Engine.

To get the volume coordinate, the drum surface must be given motion *proportional* to the travel of the piston. This is accomplished by having a cord wound around the pivoted drum and fastened to the hook at P, Fig. 4–4. The drum spring keeps the cord tight for the return stroke. The engine crosshead H moves with the piston and we know that the pantograph (connected so that R, O, and P are in line) will give to P a reduced copy of the motion of R with proportionality. For accuracy it is essential that the cord be parallel to the travel of P.

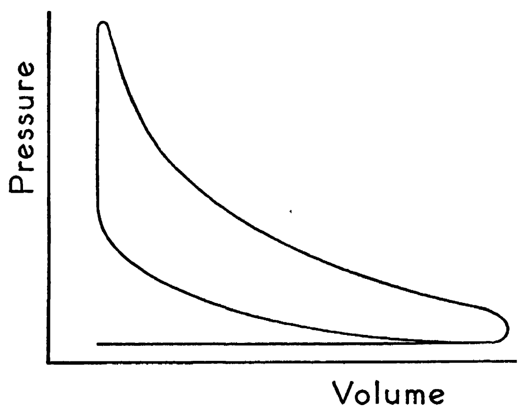


FIG. 4–5. Indicator Diagram.

An indicator diagram as taken from a gas engine appears in Fig. 4–5. The mechanisms used to move the pencil will be treated in § 4–6.

4–2. **The Parallelogram.**—The parallelogram mechanism with one side fixed gives a motion of circular translation to the opposite side. A

common application is to the Roberval balance, Fig. 4–6. To reduce friction, knife-edge bearings are used at the turning pairs. The parallel rod, driving wheels and frame of the locomotive furnish another application, Fig. 4–7.

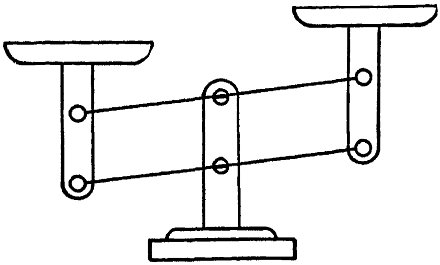


FIG. 4–6. Roberval Balance.

Some very useful devices result from compounding the parallelogram mechanism, with one side of the first parallelogram fixed. In Fig. 4–8 is a multisigner, a machine by which it is possible to sign eighteen or more checks at once by inserting a pen through the hole at P. Additional pens are held at the points marked ○, in the frame C which moves over a sheet of checks. The disc B is constrained to circular translation. Frame C has likewise circular translation with respect to B, but, by virtue of the compounding, it has free translation with respect to A, within the range of the linkage. Hence all pens will have identical motion with respect to the paper. The device is a splendid time saver where thousands of checks are required at once for a payroll.

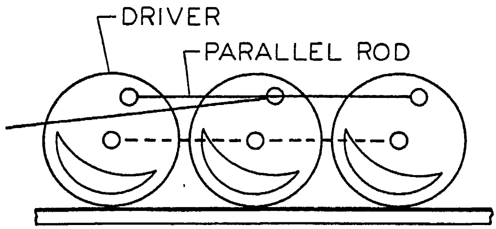


FIG. 4–7. Locomotive Parallel Rod.

A very similar application is seen in the drafting device, Fig. 4–9, an additional feature being the adjustable dial, calibrated in degrees, by which the square can be set at any desired angle.

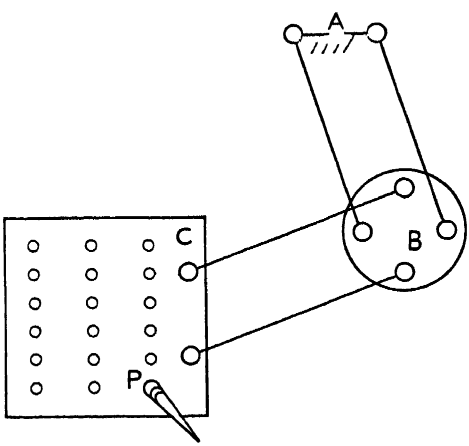


FIG. 4–8. The Multisigner.

Flame cutting of metal plates using the oxy-acetylene torch has been given accuracy and speed by the use of the parallelogram mechanism. In the pictorial illustration, Fig. 4–10, four torches are being traversed over identical paths to cut four steel plates at once. The gas tubing

obscures the mechanism somewhat. The front bar of the front parallelogram is of square steel. On it the four torches are adjustably supported, and at the end near the operator it carries a tracer which bears on the

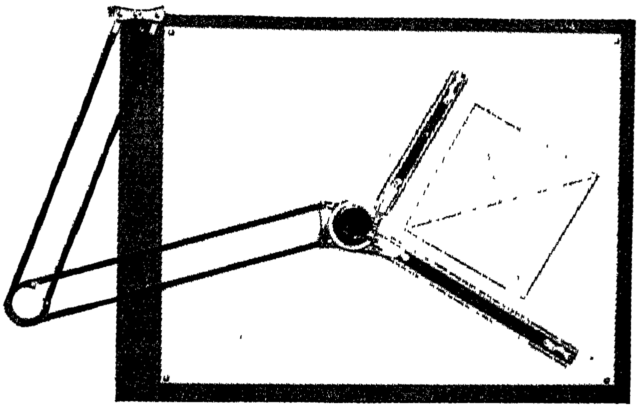


FIG. 4–9. Universal Drafting Machine.

edge of a templet. A small motor drives the tracer around the templet at constant speed and in this manner constant cutting speed is given to

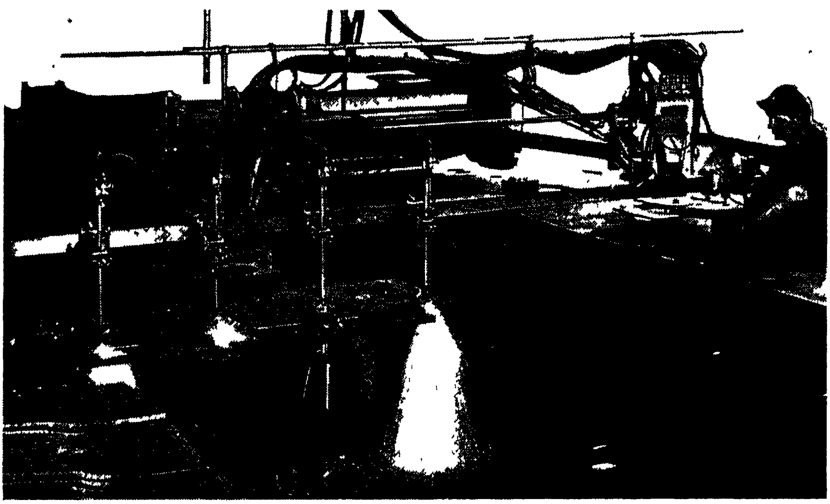


FIG. 4–10. The Travograph. Developed by Air Reduction Sales Co.

Photograph supplied by Lukens Steel Co., Coatesville, Pa.

the torches. A double parallelogram is used so the mechanism is identical in principle to the multisigner, Fig. 4–8. If the number of pieces to be

cut is too small to justify the making of a templet, the tracer can be traversed by hand over a drawing of the object.

4–3. **Generators of Rectilinear Translation.**—Rectilinear translation is easily obtained from the sliding pair but there are mechanisms having only turning pairs that will generate the motion. For kinematic simplicity, Sarrut's device, Fig. 4–11, is unique. The hinges must be on

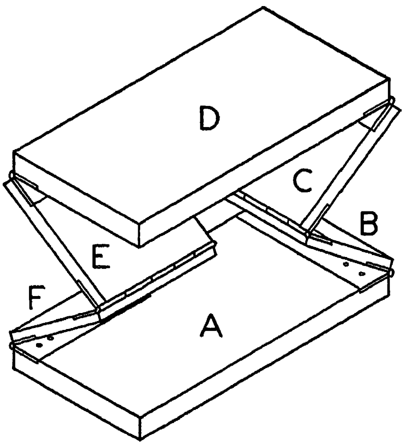


FIG. 4–11. Sarrut's Mechanism.

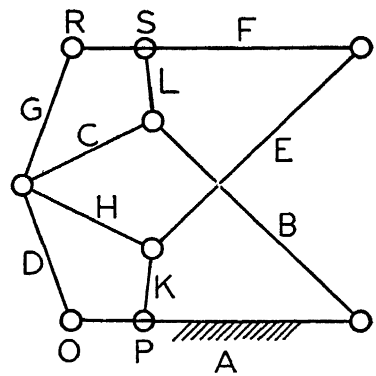


FIG. 4–12. Kempe's Kite.

adjacent sides. The warping stress in the plates is an objection however.

A glance at the ten-link mechanism of Fig. 4–12 reveals the appropriateness of the name. There are four kite-like figures arranged in a symmetry that persists throughout the possible motion. The following length relationship is essential:

$$A = B = E = F = 2D = 2H = 2C = 2G = 4L$$
$$= 4K = 4(OP) = 4(RS).$$

The centro af will always be found at infinity in the direction of these two links, proving that F and A have relative rectilinear translation.

4–4. **Exact Straight-Line Mechanisms.**—Peaucellier's mechanism, shown in Fig. 4–13, generates an exact straight-line motion by the action of links connected by turning pairs only.

The necessary proportions are that the sides of the parallelogram $WMTN$ should be equal, that PM equals PN, and the fixed link PO equals OW. Then, for any possible phase, P, W, and T will be in line. Drop a normal from M on the line PT.

In the right-angled triangles PVM and TVM,

$$(PM)^2 - (PV)^2 = (MV)^2 = (TM)^2 - (TV)^2.$$

Transposing,

$$(PV)^2 - (TV)^2 = (PM)^2 - (TM)^2 = \text{a constant.}$$

Consequently

$$(PV + TV)(PV - TV) = PT \times PW$$

is a constant for any position of the mechanism.

Therefore

$$PT \times PW = PS \times PR,$$

and

$$\frac{PT}{PS} = \frac{PR}{PW}.$$

In the triangles PWR and PST, the angle at P is common and the sides adjacent are proportional, so the triangles are similar and the angles PWR and PST are equal.

But PWR, the angle in a semi-circle subtended by a diameter, is a right angle. Hence PST is a right angle.

Now T represents the outer corner of the parallelogram in a general position. It follows that for any possible phase of the mechanism, T will be found on a straight line through S normal to PS.

The maximum travel of T above and below S is established by the closing of the parallelogram as M and N come together. Unless the joints at M and N are designed to pass each other, the travel of T will be somewhat less.

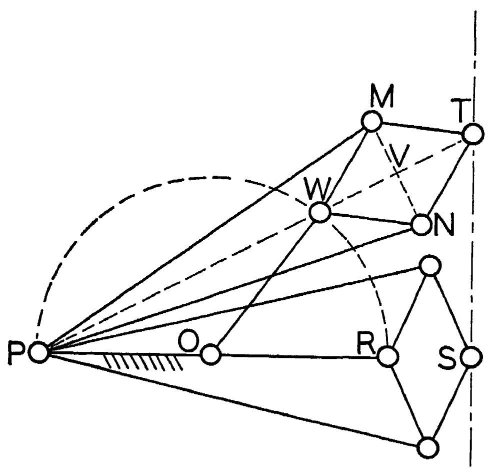


Fig. 4–13. Peaucellier's Mechanism.

The Scott-Russell mechanism, Fig. 4–14, is also exact but it can hardly be said to generate straight-line motion as Peaucellier's does. It affords a handy means of converting a straight-line motion already obtained, into another in a direction normal to the first.

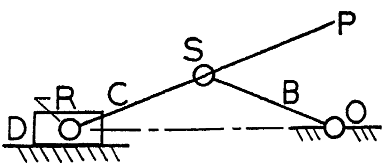


Fig. 4–14. Scott-Russell Mechanism.

The link B must be half as long as C and connect to it at its mid-point. For any position of the mechanism, a circle with center at S

and radius SO, must pass through R and P. Thus the angle ROP is always a right angle and P must travel on a straight line normal to RO.

The luffing crane, Fig. 4–15, can be used to accomplish the work ordinarily done by a jib crane where it is desirable to be able to fold the crane back against its column out of the way. The cable is connected at T to the hoisting drum, and, by driving gear V, the load can be luffed in and out. If the sheave wheels were infinitely small the load W would travel horizontally when the hoisting cable was stationary.[1]

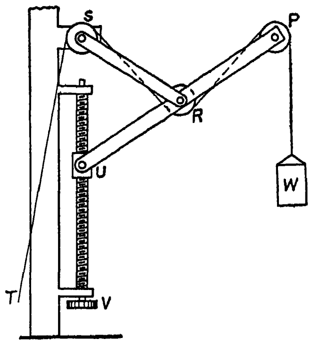


FIG. 4–15. Luffing Crane.

4–5. Approximate Straight-Line Mechanisms.—When the translated link D of the Scott-Russell mechanism, Fig. 4–14, is replaced by an oscillating link D, Fig. 4–16, the result is what is commonly called the grasshopper mechanism. The longer D is made, the more closely does the locus of P approach a straight line. Hatching the circles is another way to indicate fixing.

When Watt was doing his epoch-making development work on the steam engine there was no way to produce finished plane surfaces such as crosshead guides, except by the long laborious manual processes of chipping and filing. The lathe, though extremely crude compared with modern turning machines, was at that time developed to a state of practical usefulness, but the planer and shaper had not been devised. Indeed, had it not been for the timely invention by Wilkinson in 1775, of the boring bar supported in bearings, Watt's later engines would not have been possible. Under these conditions we find Watt, and Newcomen who previously had developed the first piston engine, the "atmospheric engine," both making use of the turning pair rather than the sliding pair wherever possible. The mechanism that bears Watt's name was devised by him in 1785 to perform the function that has, since the perfection of planing equipment, been performed by crossheads and guides. As in most approximate straight-line mechanisms the compensation principle

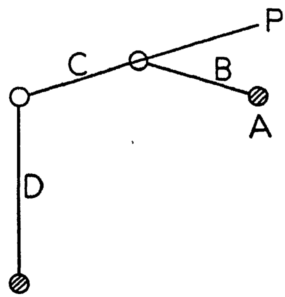


FIG. 4–16. Grasshopper Mechanism.

[1] What is the effect on the height of the load, of sheave pulleys of practical size, when the load is luffed out?

is used. If D and B are equal as in (*a*), Fig. 4–17, the point P, having the best approximation to straight line travel, is the midpoint of C. At the midtravel of P, the links B and D should be parallel. If D and B

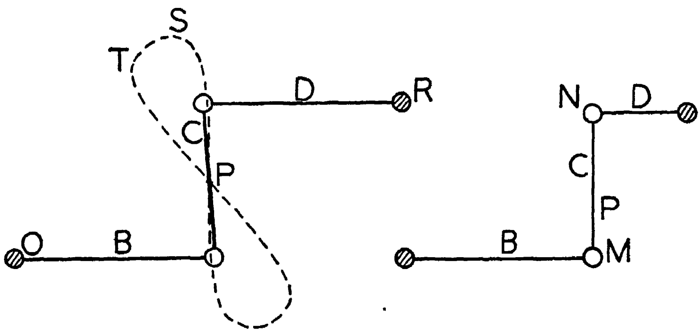


FIG. 4–17. Watt Straight Line Mechanism.

are unequal, Fig. 4–17 (*b*), as in Watt's application, P is located so that $MP/NP = D/B$. The complete locus of P is the crossed loop PST, as shown in Fig. 4–17 (*a*).

Watt's beam-engine mechanism is shown in Fig. 4–18. Links B, C, and D limit the motion of P to an approximately straight line, and B,

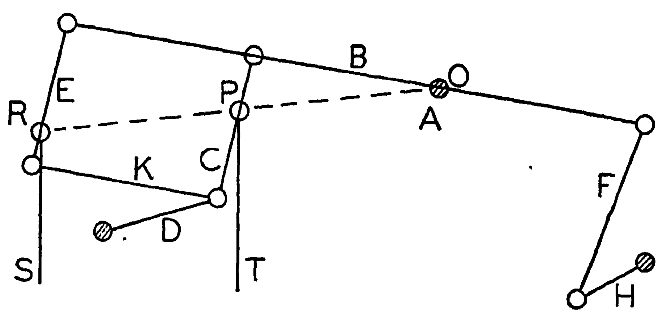


FIG. 4–18. Watt's Beam Engine.

E, K, and C constitute a parallelogram mechanism so that R, in line with P and O, must travel parallel to P, and therefore nearly in a straight line. Watt attached the piston of his driving steam cylinder at S, and the piston of his condenser pump at T. H was the crank and belt wheel.

It is only when we realize the slender foundations on which Watt and other founders of mechanical engineering had to build, and the crude tools they had to use, that we can know what splendid engineers they were.

The Tchebicheff mechanism, Fig. 4–19, constrains the point P, the midpoint of C, to an approximately straight line parallel to A. Best results are obtained from the following ratios. If A is 4, C should be 3, and B and D about 5. It is a close relative of the Watt mechanism as can be seen by turning the link B over to a position above C.

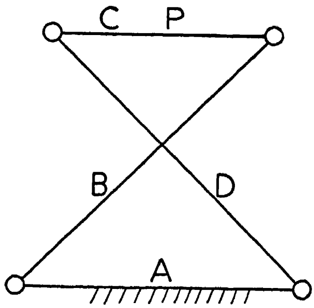


FIG. 4–19. Tchebicheff Mechanism.

4–6. **Indicator Pencil Mechanisms.**—As explained in § 4–1, the pencil mechanism must cause a pencil point to move in a straight line, parallel to the axis of the drum, and the motion must be proportional to the travel of the indicator piston. Strange as it may seem, the devices that

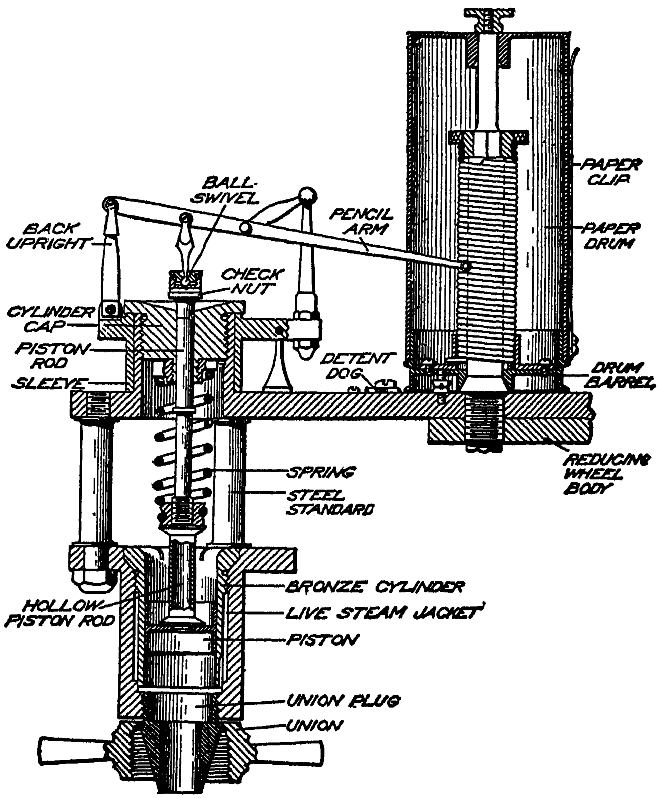


FIG. 4–20. Steam Engine Indicator.

The Trill Indicator Co., Corry, Pa.

the makers of indicators have found most satisfactory are, in general, not exact straight-line mechanisms, but operate on the principle of approximate compensation to convert motion in circular arcs to straight-line travel. They give very accurate results, however, over the limited range for which designed. The sectional view of the Trill indicator, Fig. 4–20, shows a pencil mechanism which is also used on the Thompson indicator.

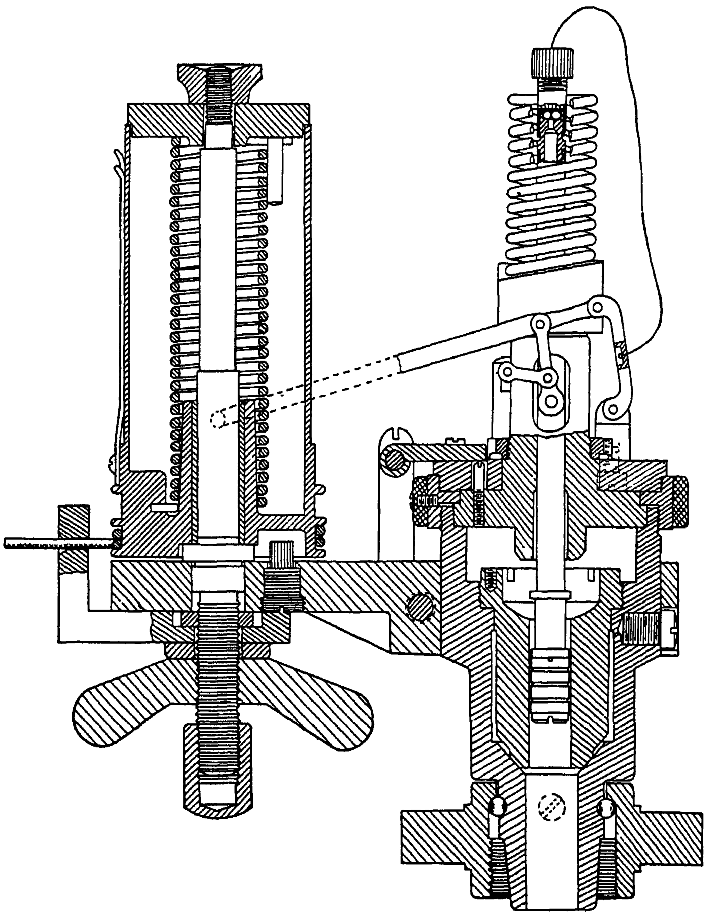


FIG. 4–21. Indicator for Gas and Oil Engines.

The Bacharach Industrial Instrument Co., Pittsburgh, Pa.

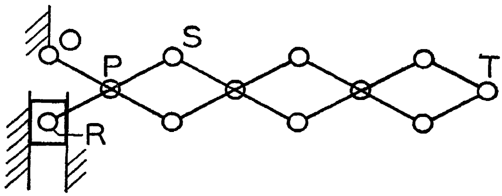
The Bacharach indicator, Fig. 4–21, with its small piston and heavy construction is a design used for internal-combustion engine testing. This type of pencil mechanism is also used on Crosby instruments. The

wire leading from the top of the piston stem is part of the ignition indicator circuit. When the electric current flows to the spark plug, it is also passed, by an auxiliary circuit, through the pencil, and burns a mark on the indicator card. Both of these pencil mechanisms classify as approximate. The pantograph would be ideal for the purpose were it not for the difficulty of keeping the weight and inertia low.

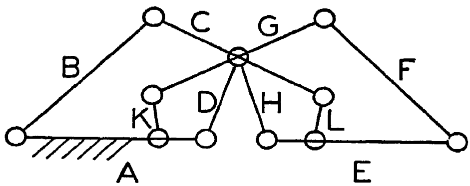
PROBLEMS

1. Into what two classes can the applications of the parallelogram mechanism be divided and on what is the classification based?
2. Prove that the pantograph mechanism can be used to reproduce any plane figure to reduced or enlarged scale.
3. Draw and dimension a mechanism that could be used to reproduce a plane figure to quarter size and inverted.
4. Does the Roberval balance, Fig. 4–6, indicate less weight if the article is placed at the side of the pan nearest the fixed link, than if placed in the center? Prove your conclusion.
5. The multisigner, Fig. 4–8, is composed of two parallelogram mechanisms in series. What would be the effect if three were used in series?
6. Prove the exactness of Peaucellier's mechanism, being careful to make a correct drawing.
7. Prove that the Scott-Russell mechanism can be used to convert straight-line motion in one direction to straight-line motion in a normal direction. Are the motions proportional?
8. Draw the pencil mechanism of the Bacharach indicator in the phase shown in Fig. 4–21 and approximately double that size. Locate the centro of the pencil and the frame, and determine whether the pencil moves parallel to the indicator piston.
9. A Tchebicheff mechanism of the proportions recommended has its fixed link 2 in. long. What length of approximately straight travel will it give if the maximum allowable deviation from straightness is 1/16 in? Solve by plotting the locus of P.
10. A circular disk of 10-in. diameter rolls without sliding on the inside of a fixed ring, the hole in the ring being of 20-in. diameter. Find a point on the disk that has straight-line motion. Design a mechanism to produce straight-line motion in this way.

11. Classify this mechanism which is frequently used for safety gates, inside elevator gates, etc. Design the attachment of a vertical link at T which will remain vertical as the gate folds back.



12. Classify this mechanism and prove its action.



13. Design a pantograph indicator reducing rig of practical proportions to reduce a horizontal piston travel of 24 in. to a proportional travel of 3 in. at the indicator-drum surface. The normal distance from the line of travel of the crosshead connecting pin to the fixed point is 20 in. Show your mechanism in three positions, one full line and two in broken line.

14. In a Watt mechanism, the two links connected to the fixed link are each 3 in. long, and the middle link, which is normal to the first two at mid-travel, is 2 in. long. What length of guided motion, straight within $\pm$ 0.03 in. will it give?

CHAPTER V

THE PHOROGRAPH—A DEVICE FOR FINDING VELOCITIES IN MECHANISMS

5–1. **Definition.**—The body C, represented in Fig. 5–1 as a disk, has the *instantaneous* angular velocity ω_C radians per second relative to A which is considered fixed. A point P on another body B has velocity $V^{-}P$ relative to A as indicated by its vector. We shall use the symbol $V^{-}X$ to represent the velocity of a point X. All velocities are to be considered instantaneous, also relative to the fixed link unless otherwise defined.

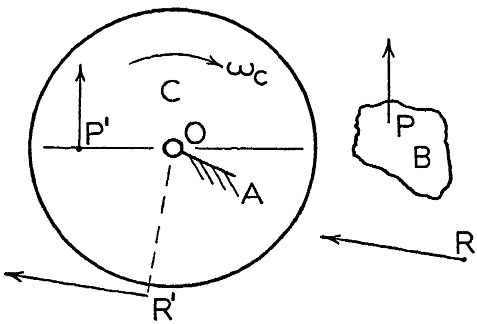


Fig. 5–1.

It is possible to find on C a point that has the identical velocity in magnitude, direction and sense with respect to A, that P has. When found, that point called P' will be the phorograph[1] of P on C. Since $V^{-}P$ is vertical, P' must be on a horizontal line through O. Only on this line do points of C have vertical motion. Manifestly, P' must be to the left of O, and the distance OP' is defined by the relation

$$V^{-}P = V^{-}P' = (OP')\omega_C$$

Similarly the phorograph of any other point such as R, of known velocity, can be located. R' falls outside the body C but it must be recalled that points can be located on C anywhere out to infinity. In this manner velocities of any magnitude from zero to infinity, and of any direction and sense can be represented. A further important fact is that because P' has the same velocity as P relative to A, it has the same velocity relative to all third points or bodies wherever located. The link on which the phorographs are located is called the phorograph link. Any link or body can be used as a phorograph link provided it has rotation however small. A link having translation will not serve, as all its points have the same velocity.

[1] The phorograph was devised and named by Professor T. R. Rosebrugh of the University of Toronto.

A point on one body is the phorograph of a point on a second body when it has the same velocity relative to all third bodies as that of the point on the second body.

5–2. **Phorographing a Mechanism.**—Knowing the velocities of points, it is possible to find their phorographs on a rotating body whether the points are on bodies entirely unconnected or joined in a mechanism, using the method of the previous article. This, of course, would be of no value in finding velocity relations in mechanisms. Fortunately there is a way to find phorographs *in a mechanism* without knowing the magnitudes of any velocities. From the phorographs, velocity relations are easily obtained, and these hold for any speed of the mechanism.

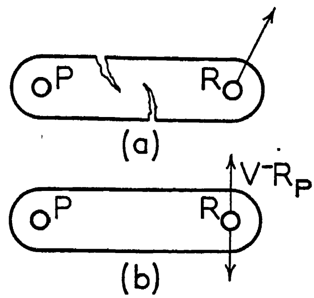


FIG. 5–2

The technique of finding phorographs from the nature of the mechanism rests on one basic concept. In Fig. 5–2 (*a*), a velocity of R relative to P as represented by the vector is impossible without rupturing the link. The only possible *direction* of motion of R relative to P is that shown at (*b*), if the link is not to be pulled apart or crushed. This applies only to relative motion. The two points may have, in addition, any common velocity whatever. Further, this principle applies only when the two points are in the same body.

Two points in the same body can have relative motion only in a direction normal to the line joining the points.

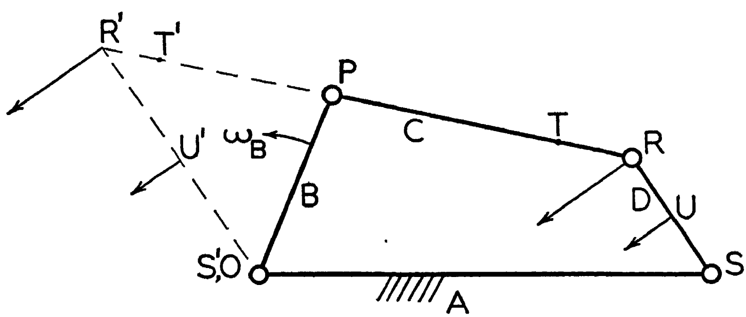


FIG. 5–3. Phorograph of the Quadric Mechanism.

The quadric mechanism, Fig. 5–3, will now be phorographed using B as the phorograph link. The phorograph of R on B will be found first. The symbol $V\text{-}R_S$ will be used to represent the instantaneous velocity of R relative to S. As the mechanism is passing through the position

shown, the direction of $V^{-}R_S$ is normal to its radius RS. Points on B that have such direction of motion are found only on the line OR' parallel to D, projected infinitely either way. Further, points that have the same sense of motion as R are found only above O. It now remains to find how far out on this line R' must be.

The only motion R can have with respect to P, when the mechanism is going through the position shown, is in a direction normal to RP. R' must have the same direction of motion with respect to P. All points on B, having motion in such direction with respect to P, are found only on a line through P parallel to PR produced both ways to infinity. Hence R' is located at the intersection of a line through O parallel to D and a line through P parallel to C.

If ω_B is the instantaneous angular velocity of B in radians per second, and linear dimensions are in inches, $V^{-}R$ (equals $V^{-}R'$ on B) $= (OR')\omega_B$ inches per second.

Further, $R'P$ must be the phorograph of the whole link C, and if T is one-quarter of the distance from R to P, then T' is one-quarter of the distance from R' to P. S' is of course at O, so the whole mechanism has been phorographed on B. The velocity of any point such as U on D is $(OU')\omega_B$ relative to the fixed link A. Vectors representing velocities of turning-pair centers are measured from the centers, though not so drawn.

Since a phorograph has the same velocity as the point it represents, it follows that two phorographs will have the same velocity relative to each other as the points they represent have relative to each other. What is the velocity of T relative to U in Fig. 5–3? $V^{-}T_U = (T'U')\omega_B$, where ω_B is the angular velocity of B. What is the direction of motion of T relative to U? The direction is normal to the line $T'U'$. If ω_B has the sense shown, the sense of $V^{-}T_U$ is toward the left, and the sense of $V^{-}U_T$ is toward the right. An additional solution is required for each phase for which velocity relations are desired.

The **choice of the phorograph link** is generally a simple matter. Velocities relative to the fixed link are usually required, and a phorograph link having a permanent centro with the fixed link gives such velocities directly. While phorographs can be found on any link that has rotation, they will generally be most useful on links that have a constant angular velocity during the cycle.

5–3. Solution for Contact Points of Bodies in Direct Contact.—Fig. 5–4 shows body B driving C by direct contact. At the contact point are the two points, P on B, and R on C, having the same location for the instant but different motions relative to A. The phorograph of R on B is required.

First locate S' at O, that being the only stationary point on B. Points R and S are both on C, so $V^{-}R_S$ has direction normal to RS. Points on B that have such direction of motion relative to S' (and therefore relative to S) are found only on a line through S' parallel to RS, namely the line OU produced both ways to infinity.

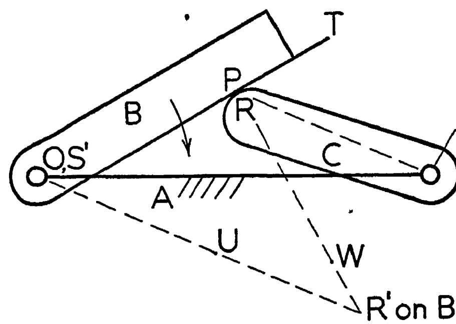


FIG. 5-4.

The other line necessary to locate R' comes from the relative motion of R and P. The only possible motion of R *relative to* P must be in direction PT which is the common tangent to the surfaces at the contact point, if bodies B and C are not to crush or separate. Points on B having the same direction of motion *relative to* P are found only on the line PW normal to PT. This locates R' at the intersection as shown. Now $V^{-}R_A = V^{-}R'_A = (OR')\omega_B$. Also, $V^{-}R/V^{-}P = OR'/OP$.

It will now be evident that to find a phorograph on a given mechanism, two intersecting lines must be located on the phorograph link, just as two applications of Kennedy's theorem are necessary to locate a centro. Each line results from the *direction* of the relative motion of points, which depends on the nature of the mechanism. An exception is the phorographing of points on the fixed link, which is direct.

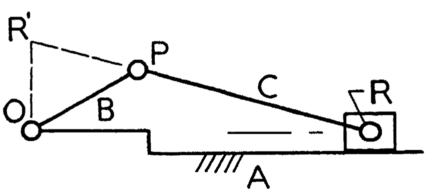


FIG. 5-5.

5-4. Angular Velocities by Phorograph—Slider-Crank Mechanism.—Link B, Fig. 5-5, has all the advantages as phorograph link. It revolves about a permanent centro and probably at constant speed. $V^{-}R_A = V^{-}R_O$ is horizontal, so R' on B must lie on a vertical line through O. $V^{-}R_P$ is normal to C. Points on B having this direction of motion relative to P are found only on the line PR', which locates R' at the intersection. This phorographs the mechanism completely.

The crosshead velocity $V^{-}R_A = V^{-}R'_A = (OR')\omega_B$. Now $R'P$ is the phorograph of C, which establishes this relation,

$$V^{-}R_P = (RP)\omega_C = (R'P)\omega_B$$

or

$$\omega_C = \frac{R'P}{RP}\omega_B$$

The angular velocity of any link is to the angular velocity of the phorograph link inversely as its length is to its phorograph length.

5–5. **Cam and Lever Mechanism.**—This mechanism with dimensional variations has been used as a trip control on punches and shears, B being the hand or foot lever.

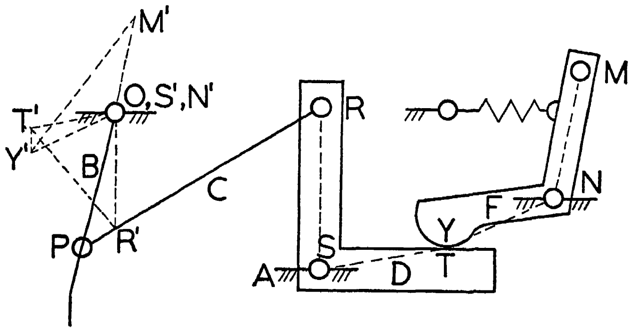


Fig. 5–6.

The contact point between the bellcranks D and F is the location of the points Y on F and T on D. Choosing B as the phorograph link we observe that S' and N' are at O. Then $V^{-}R_S$ is normal to RS, and points on B having this direction of motion relative to S', which is O, are found only on a line through O parallel to RS. The other factor controlling the motion of R relative to B is its connection through C. $V^{-}R_P$ is normal to C, and points on B having this direction of motion relative to P lie on the line of PR, locating R' at the intersection.

After R and S, the next key point in the mechanism is T. We should now be able to shorten the formula for location of the phorograph to this:

$V^{-}T_S$	gives the line	$S'T'$		
$V^{-}T_R$	gives	$R'T'$,	hence	T'.

To locate Y',

$V^{-}Y_N$	gives	$N'Y'$		
$V^{-}Y_T$	gives	$T'Y'$,	hence	Y'.

The last step rests on the fact that, if F and D are not to separate or crush, the direction of $V^{-}Y_T$ can only be tangent to the surfaces at the contact point (§ 5–3).

$V^{-}M_N$	gives	$N'M'$		
$V^{-}M_Y$	gives	$Y'M'$,	hence	M'.

In case this intersection is at a small angle or if YNM are in line, locate

M' by the proportion

$$\frac{OM'}{OY'} = \frac{NM}{NY}$$

The phorographs of points on any single link have the same *relative* positions as the original points. The size to which the link is phorographed is inversely as the angular velocities of the two links involved, as stated in the previous article. Links, such as F and D, having different angular velocities, will phorograph on B to different scales.

Some of the relations that can be obtained now from Fig. 5–6 are:

$$\omega_F = \omega_B \frac{N'M'}{NM}, \qquad \omega_D = \omega_B \frac{S'R'}{SR},$$

$$V^-M_A = (OM')\omega_B, \qquad V^-R_A = (OR')\omega_B,$$

$$V^-M_R = (M'R')\omega_B, \qquad V^-Y_T = (Y'T')\omega_B.$$

The linear velocities are, in each case, normal to the line of the two phorographs involved, and the sense depends on the sense of rotation of the phorograph link.

5–6. **Rubbing Velocities in Journal Bearings.**—The journal bearing, when operating under the best conditions, transmits its load from one link to another through a thin film of oil that separates the metal surfaces. The relative speed at which these surfaces pass each other, shearing the oil film, is called the rubbing velocity. It is of interest to designers as one of the factors controlling bearing performance.

Rubbing velocity is the algebraic difference between the angular velocities of the two links which the bearing connects, multiplied by the radius of the bearing surface.

The mechanism selected as an example is that of the V-type engine used largely in airplanes,[1] also in automobiles and air compressors. The two cylinders (or banks of cylinders) are, in this case, at 60°, Fig. 5–7. The main connecting rod C joins the piston at R to the crank pin P, and the small connecting rod D joins the piston S to the main rod at the knuckle bearing T.

We shall first phorograph the mechanism on the crank link B.

For R', direction of V^-R_A gives OR',
direction of V^-R_P gives PR'.
For T', direction of V^-T_R gives $R'T'$,
direction of V^-T_P gives PT'.
For S', direction of V^-S_A gives OS',
direction of V^-S_T gives $T'S'$.

[1] The Curtiss D-12 has dimensions approximating those of Fig. 5–7.

Suppose this engine runs clockwise at the constant speed of 2400 rpm. The dimensions of the links can be scaled from the drawing and the bearing diameters are: at O, 3 in.; at P, $2\frac{1}{2}$ in.; at T, 2 in.; at R and S, $1\frac{1}{2}$ in.

$$\omega_B = \frac{2400}{60} \times 2\pi = 251 \text{ radians per sec clockwise,}$$

$$\omega_C = \frac{PR'}{PR}\,\omega_B = \frac{0.66}{2.5} \times 251 = 66.3 \text{ counterclockwise,}$$

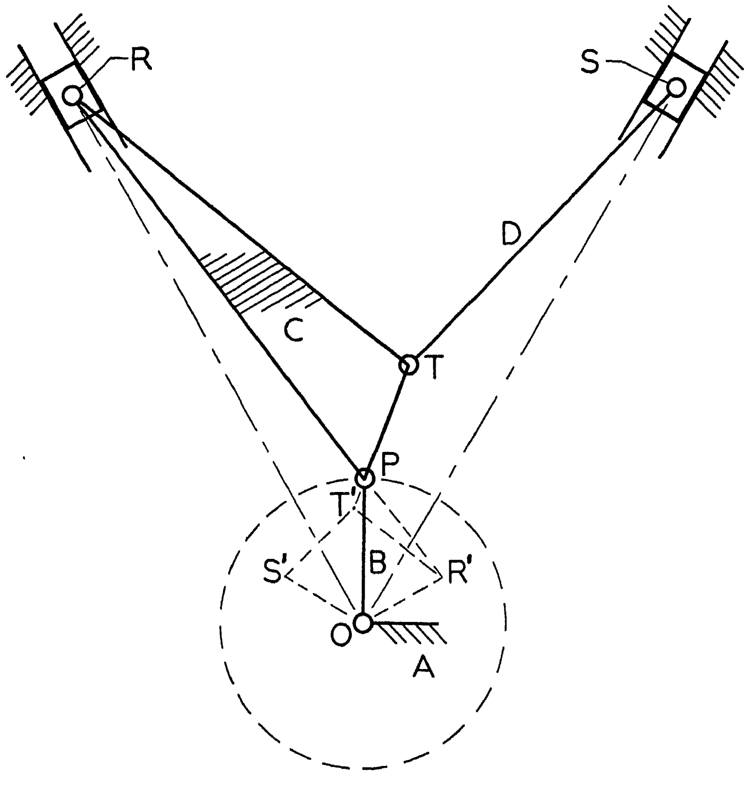


FIG. 5–7. V-Type Engine Mechanism. Scale, 1 in. = 4 in. (Pistons not to scale.)

using lengths on the drawing which are proportional to lengths on the machine.

$$\omega_D = \frac{S'T'}{ST} \times \omega_B = \frac{0.50}{2} \times 251 = 62.8 \text{ counterclockwise.}$$

Then the rubbing velocity at the main bearing O, using r for radius, is

$$(\omega_B - \omega_A)r_O = (251 - 0)\,\frac{1.5}{12} = 31.4 \text{ ft per sec.}$$

At the main crank-pin bearing P,

$$(\omega_B - \omega_C)r_P = [251 - (-66.3)]\frac{1.25}{12} = 33.1.$$

At the knuckle bearing T,

$$(\omega_C - \omega_D)r_T = (66.3 - 62.8)\frac{1}{12} = 0.3.$$

At the piston-pin bearing R,

$$\omega_C \times r_R = 66.3 \times \frac{0.75}{12} = 4.1.$$

At the piston-pin bearing S,

$$\omega_D \times r_S = 62.8 \times \frac{0.75}{12} = 3.9 \text{ ft per sec.}$$

A complete picture of these rubbing velocities would require the plotting of values for a succession of phases.

The sense of rotation of any link can generally be seen from inspection when the sense of rotation of the driving link is given, but if there is any doubt about it the phorographs will give the answer. Consider, for example, link D. Since ω_B is given as clockwise, $V^{-}S'_{T'}$ is toward the left. Therefore $V^{-}S_T$ is toward the left which indicates that ω_D is counterclockwise.

5–7. **Velocity Diagrams.**—It is frequently necessary in the development of machines to have complete information on the velocity of parts of the mechanism for a cycle. The velocity curve or diagram presents this information in most usable form. Velocity solutions are made for the mechanism in a succession of phases, vectors plotted on some convenient base, and a curve drawn through the ends of the vectors.

The drag-link mechanism is used on slotters and shapers for the purpose of effecting a quick return of the cutting tool on its idle stroke. The dimensions of the links, Fig. 5–8, must satisfy the relations given with Fig. 3–23. It is possible to obtain high ratios of return speed to cutting speed, just as in the cases of the Whitworth and oscillating-beam mechanisms.

In Fig. 5–8, B is the driving link and may be assumed to rotate at uniform speed. The drag link C drives crank D at varying speed, and, through the connecting rod F, the ram T which carries the tool is given an efficient program of motion.

Using B as phorograph link, we can find the velocity of T for the phase shown, in terms of ω_B.

For R', the direction of $V^{-}R_S$ gives OR',
$V^{-}R_P$ gives PR'.
For T', $V^{-}T_A$ gives OT',
$V^{-}T_R$ gives $R'T'$.

Now $V^{-}T = V^{-}T' = (OT')\omega_B$. If ω_B is given as 10 radians per second and OT' measures 0.55 inches on the drawing, the scale of the drawing being 1 in. = 2 in., then $V^{-}T = 10 \times 0.55 \times 2 = 11$ in. per sec. Assuming that velocities of T toward the right are to be plotted upward, this value, being toward the left, will be plotted downward to some convenient scale as Tb. A curve drawn through the ends of a sufficient number of these vectors gives a diagram similar to that marked $V^{-}T$.

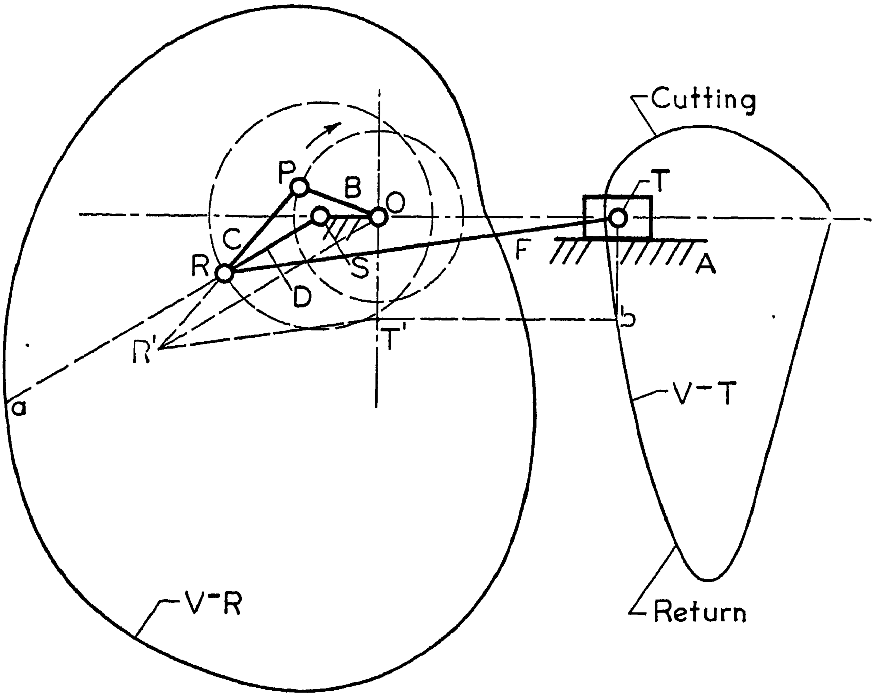


FIG. 5–3. Velocity Diagrams on the Drag-Link Mechanism. Velocity-Space Diagram of $V^{-}T$, Polar Diagram of $V^{-}R$.

It is called a *velocity-space* or velocity-travel diagram. The vectors are normal to the directions of the velocities represented.

If the distance OT' is used directly as the plotted vector Tb, as in Fig. 5–8, the velocity scale becomes ω_B multiplied by the space scale of the drawing. This makes the diagram conveniently applicable to any speed of the phorograph link.

Polar velocity diagrams are suitable to represent the velocities of points that travel in circular or curved paths. The ray vectors are measured outward from the paths and normal thereto. In Fig. 5–8, *R* travels on a circle about *S*, and its varying velocity is represented by the polar diagram marked *V–R*. Since *R'* as a point on *B* has the same velocity as *R* on *D*, for the phase shown, *OR'* plotted as *Ra* gives point *a* of the diagram. The scale is the same as for the space-velocity curve, namely: one inch of vector represents ω_B multiplied by the space scale of the drawing. The space scale for this purpose is the length represented by one inch on the drawing.

5–8. **The Floating Link.**—A situation sometimes occurs in mechanisms of many links, where the direct procedure for locating centros or

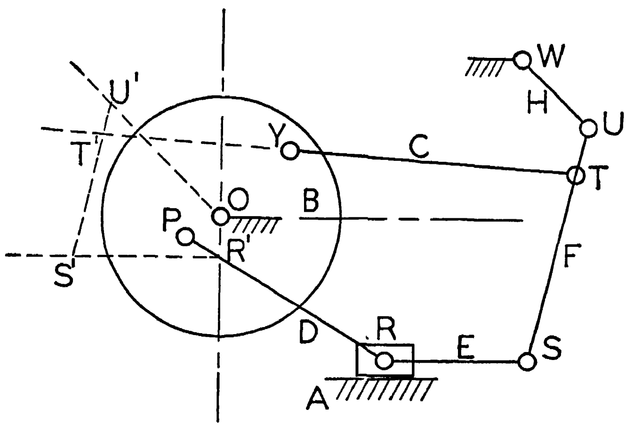


Fig. 5–9.

finding velocities is insufficient for solution. Link *F* in Fig. 5–9 is an illustration, and in the absence of a better name, will be called a floating link.

Cranks *C* and *D* are driven by the crank-shaft link *B* which, for convenience, is represented by the circle. After locating *R'* on the phorograph link *B*, the regular method breaks down when the location of *S'*, *T'*, or *U'* is attempted, because the velocities of these three points are interdependent. The facts on which solution must be based are these:

S' is on a line through *R'* parallel to *SR*.

T' is on a line through *Y* parallel to *TY*.

U' is on a line through *O* parallel to *WU*.

S', *T'*, and *U'* must be on a line parallel to *F* and must be spaced in proportion to the spacing of *S*, *T*, and *U*. There is only one location for

$S'T'U'$ that will fit these facts and that can be found by trial, moving $U'S'$ parallel to US.

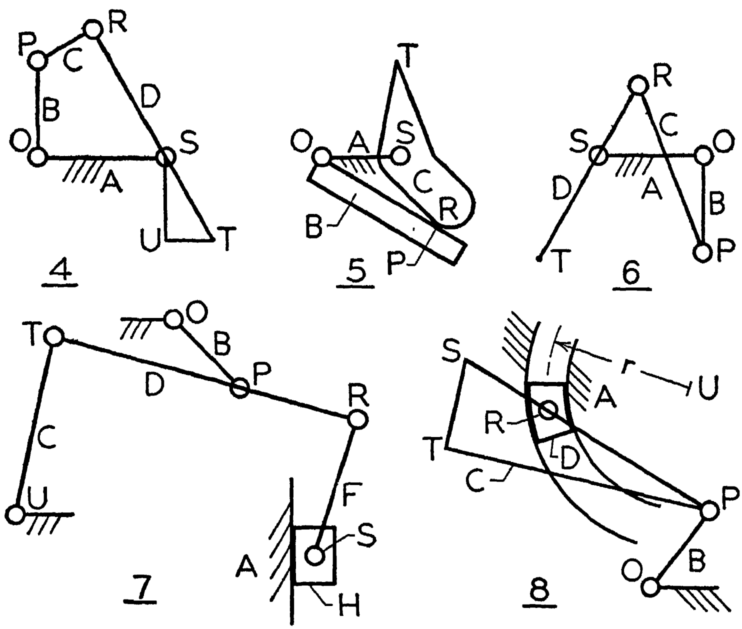
$$\frac{UT}{TS} = \frac{0.25}{1} = \frac{U'T'}{T'S'} = \frac{0.16}{0.64}$$

Application of this method will be found in Prob. 17 since the "lap and lead" lever F of the Walschaert valve gear is a floating link.

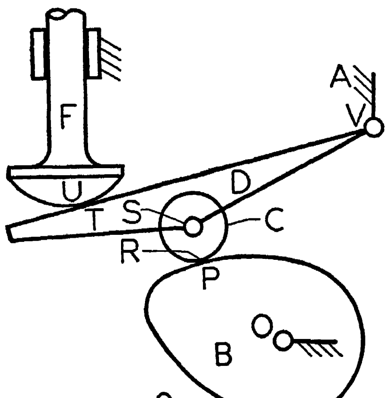
QUESTIONS AND PROBLEMS

1. A vertical disk of 4 in. diameter rotates clockwise on its axis at 20 radians per sec relative to the fixed link. Locate points on (or of) this disk that have the following velocities in in. per sec:
   P, 40 vertically downward,
   R, 30 horizontally toward the right,
   S, zero,
   T, 50 in direction 30° above the right horizontal.
2. Define the term phorograph, and indicate how best to choose a phorograph link in a mechanism.
3. In the direct-contact drive of Fig. 5–4, find the phorograph of P on link C.

4 to 10. Draw these mechanisms approximately three times the size shown and phorograph them completely on the link B in each case.



(*Problems continued on next page.*)



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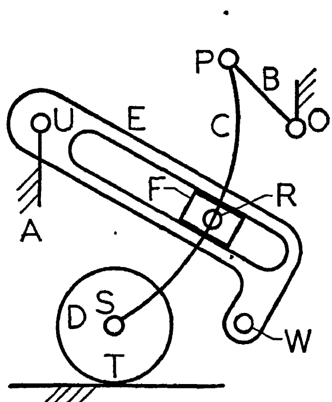

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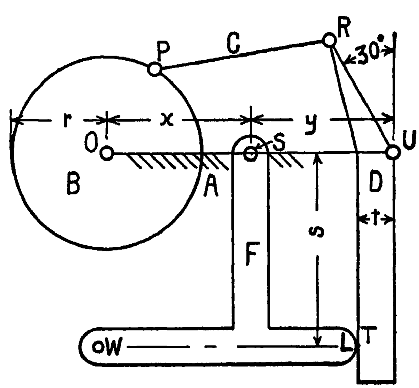
11. In the slider-crank mechanism, Fig. 5–5, the crank is 8 in. long and the connecting rod 16 in. If the crank revolves at 200 rpm, what is the speed of the crosshead in ft per min when it is 3 in. from head-end dead center; also what is the speed of the midpoint of the connecting rod? Ans. 785 and 740 ft per min.

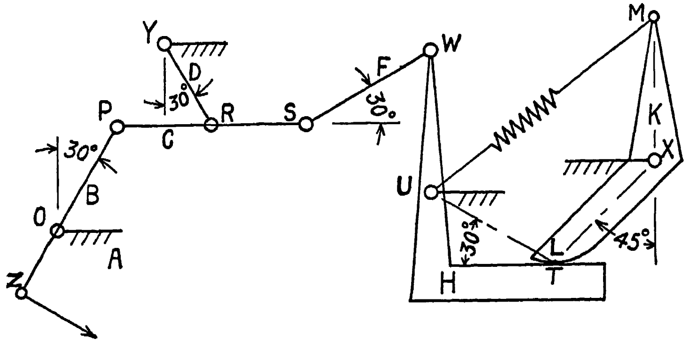
12. In the above mechanism, does the crosshead ever go faster than the crank pin, and if so, over what crank angles?

13. Using the data of Prob. 11, find
    (*a*) the rubbing velocity at the crank-pin bearing in the phase indicated if the diameter of the crank pin is 3 in.
    Ans. 219 ft per min.
    (*b*) the maximum velocity of rubbing at the crank-pin bearing for any phase.
    (*c*) the maximum velocity of rubbing at the crosshead bearing if the crosshead pin is $1\frac{1}{4}$ in. in diameter.

14. The driving wheel *B* of this mechanism revolves at 60 rpm, and *F* is held in contact with *D* by gravity. What is the velocity of *W* in in. per sec if $r = 1\frac{1}{2}$, x and $y = 2\frac{1}{4}$, $s = 3$, $t = \frac{5}{8}$, $C = 2\frac{7}{8}$, $RU = 2$, and $LW = 4$, all dimensions being inches?



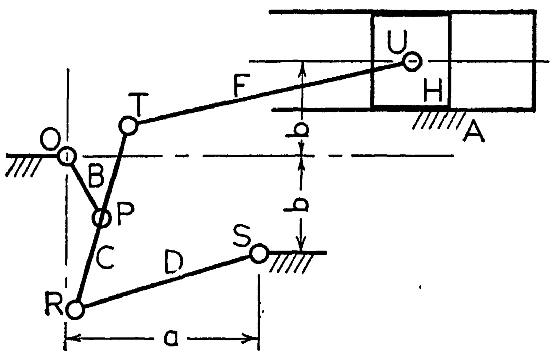
15. Given the velocity of Z as 2 ft per sec, find the velocity of M on link K. $OZ = \frac{3}{4}$, $OP = 1\frac{1}{4}$, PR, RS, and D each $= 1$, $F = 1\frac{1}{2}$, also WU, UT, TX, and $XM = 1\frac{1}{2}$.



16. This mechanism, used on pumps and compressors, gives a long stroke compared to the space required to house the linkage or the necessary length of connecting rod. The dimensions are: $B = 3$ in., $C = 8$ in. with P at the center, $D = 8$ in., $F = 12$ in., $a = 7\frac{1}{2}$ in., $b = 4$ in.

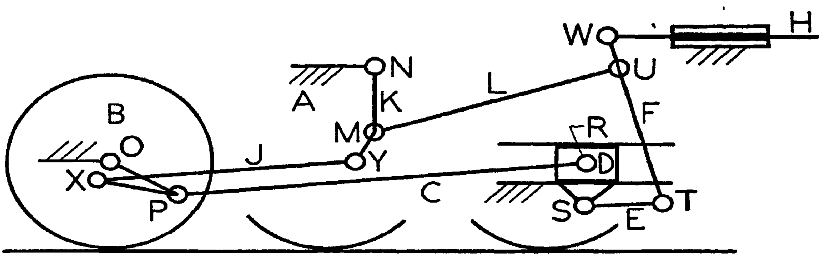
(a) Plot a velocity-space diagram for the travel of the piston for $\omega_B = 10$ radians per sec. Use 30° increments of crank position.

(b) Plot a polar velocity-space diagram for the point R.



17. Illustrated below is the essential mechanism of the Walschaert valve gear as applied to the steam locomotive. To avoid complications in presenting this problem, the reversing links have been omitted. The student is advised to observe this mechanism in action at the first opportunity.

The main driving wheel B carries the crank pin P which is driven by the main connecting rod C. The unique feature of this valve gear is that the bar PX is fastened solidly to the outer end of crank pin P and projects back to X where it carries the small crank pin that drives the valve connecting rod J. In this way, C passes inside J and K without interference. The valve rod H drives the valve which is above the steam cylinder.



Draw the mechanism to approximately double the size shown. Take a vector 2 in. long for $V{-}P$ with B turning clockwise, and find the velocity of the valve $V{-}H$ for the phase shown. For this phase it is better to phorograph on link K since a larger diagram of phorographs is obtained than on B.

CHAPTER VI

CENTRO AND COMPONENT METHODS FOR VELOCITY

6–1. **Two Controlling Principles.**—It was demonstrated in Chapter I that the *only possible instantaneous motion* of any body with respect to another is rotation about the centro of the two, translation being logically considered as a particular case of rotation where the centro is at an infinite distance. This fact leads to two fundamental principles governing the motion of any point in a body moving with respect to any other body.

Principle (1),—*The direction of instantaneous motion of any point in a body with respect to a second body is normal to the radius from the point to the centro of the two bodies.*

Principle (2),—*The magnitude of the velocity of any point in a body with respect to a second body varies directly as the radial distance from the point to the centro of the two bodies.*

These principles, applied to a point on the radial arm of a wheel revolving on a fixed axis, simply mean that its motion with respect to the axis is normal to the arm and proportional to the distance from the axis.

6–2. **Transfer Method Using Centros.**—This method is the most direct application of the above principles. In Fig. 6–1, assume that the velocity of any point of the quadric chain is known, for example the point N on the link B. It is to be assumed unless otherwise stated that all velocities dealt with are with respect to the fixed link A. It is required to find · the velocity of the point T on link D.

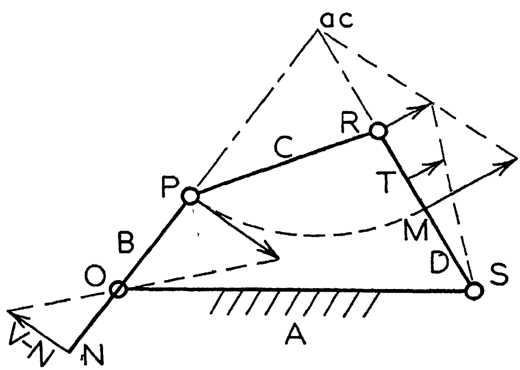


FIG. 6–1. Velocities on the Quadric Mechanism.

Lay off to scale the given velocity of N, calling it $V^{-}N$. All vectors at the turning-pair joints are to be measured from the center of the turning-pair circle, although, for convenience, they are drawn only to the circumference of the circles. The direction of this vector must be normal to the radius NO, according to

principle (1) above. Transfer will first be made to P. Both N and P are in line with the axis O, so a proportionality line through O and the end of vector $V^{-}N$ indicates, according to principle (2), the magnitude of the velocity of every point on the line NP and gives $V^{-}P$ as the velocity of P.

Now P is also a point on link C. A transfer to R is the next step since R is a point on both C and D. As all velocities dealt with are relative to the fixed link A, it is necessary to find the centro about which C has instantaneous motion with respect to A, namely ac. Since P and R are not in line with ac, a proportionality line cannot be applied directly. It is necessary first to describe an arc about ac through P giving M in line with R and ac. Note that we are dealing with points on C moving about ac, so M is on C, though it appears to be on D. As a point on C, M has the same radius as P and will therefore have the same velocity in magnitude, so at M lay off a vector of length $V^{-}P$ normal to the radius M–ac. Then a proportionality line to ac gives $V^{-}R$. Now for the first time we have the velocity of a point on D. The centro ad is at S, and as R and T are in line with S, a proportionality line gives $V^{-}T$, the required vector.

This is called the transfer method because we transfer velocities, using principle (2), from point to point through the linkage, working from the point of known velocity to the point the velocity of which is required. The location of certain centros is necessary both to determine the direction of the vectors and their magnitudes by proportionality lines drawn to the centros. Generally the velocities required are those relative to the fixed link, in which case centros of the moving links relative to the fixed link are alone required.

Little difficulty should be encountered with this method if one is careful to relate each point to the link to which it really belongs. For example, if the point M in the above solution is mistakenly assumed to be a point of link D, the operation will be a total failure.

6–3. Transfer Method for Velocities on Pump Mechanism.—Deep-well pumps are frequently operated by the mechanism shown in Fig. 6–2. The crank shaft B is generally driven through reduction gears (not shown), by electric motor, or oil engine. The lever F is made sufficiently long to give the required long stroke to the pump shaft H.

Given the rotative speed of the crank B, the velocity of the pump shaft H is required for the phase shown. $V^{-}P$ ($= B\omega_B$) is laid off to scale. With ac as center, an arc is drawn through R, locating M on ac–P which has the same magnitude of velocity as R. $V^{-}M$ is obtained using the proportionality line through ac, and the vector transferred to R where its direction must be normal to ac–R. Now R is a point on F,

so a proportionality line through *af* gives $V^{-}S$. All points of link D move relative to A about centro *ad*. Vector $V^{-}S$ is therefore transferred to N, a point of D in line with *ad* and T. Finally, a proportionality line through *ad* and the end of vector $V^{-}N$ gives $V^{-}T$, the required velocity.

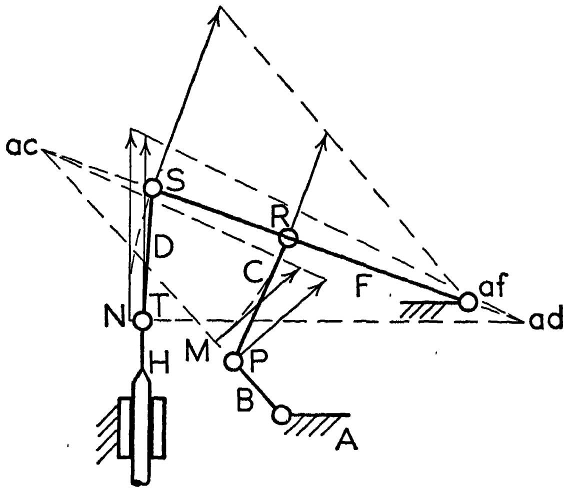


FIG. 6–2. Deep-Well Pump Mechanism.

It may be noted that this mechanism gives some degree of quick-return motion on the downward stroke. This is an advantage for a single-acting pump.

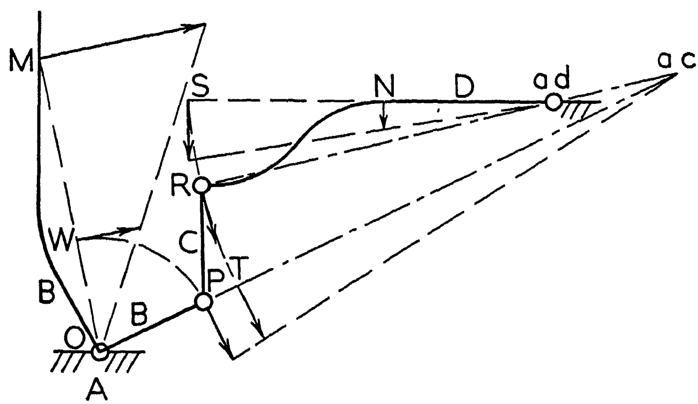


FIG. 6–3. Velocities on a Clutch-Operating Mechanism.

6–4. Other Examples of the Transfer Method Using Centros.—Fig. 6–3 shows a linkage used by the Colburn Machine Tool Co. on boring

mills. The bell crank B, with a bent handle having the grip at M, operates the lever D through the connecting link C. Attached at N is a small roller that operates the spool of a clutch. It is desired to find the relation between the motion of the hand at M and the clutch spool at N.

If the velocity at M is given, $V^{-}P$ is found by use of the centro ab at O. To transfer from P to R, the centro ac must be used. From R to N, centro ad governs, and since R and N are not in line with ad, it is necessary to transfer through S. A point with the same radius as N, between R and ad, could have been used.

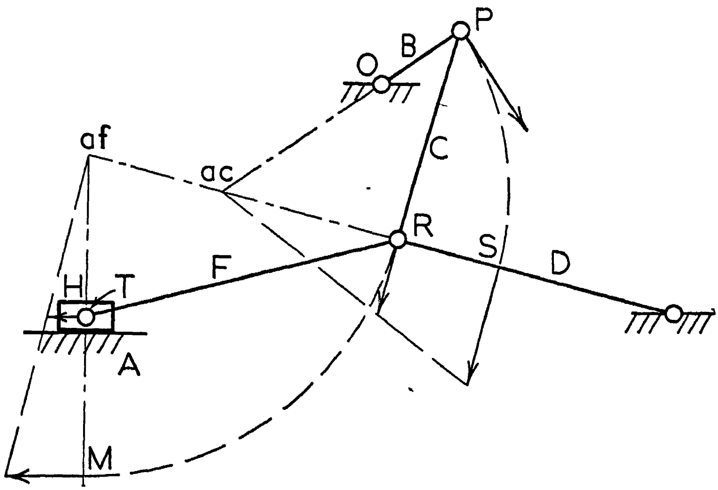


FIG. 6-4. Velocities on a Toggle Mechanism.

A toggle mechanism such as used on rock crushers, bull dozers, and press riveters, is shown in Fig. 6-4. The driving crank B has a large mechanical advantage at all phases, but particularly at the beginning and end of forward and backward strokes of the reciprocating link H.

To find $V^{-}H$ corresponding to a given $V^{-}P$, locate centros ac and af. Transfer vector $V^{-}P$ about ac giving $V^{-}S$. Point S is on C, not D. It is in line with R and ac, so $V^{-}R$ is obtained by proportion. Next we transfer through F, operating about centro af to obtain $V^{-}M$ and finally $V^{-}T$.

6-5. **The Direct-Centro Method for Velocity.**—A centro was defined as the common point of two bodies having the same velocity in each relative to any third body. This property of the centro suggests a method for velocities which is often convenient. It consists in finding the velocity of the centro itself. The centro used will, of course, be that of the link of known velocity and the link of required velocity.

The Whitworth quick-return mechanism, § 3–3, will be used to illustrate the method. The driving crank B, Fig. 6–5, rotates at known speed. It is required to find $V^{-}T$. Since T is on C, and since B is the link of known velocity, the centro bc is required. It is readily found using A and D as third bodies. $V^{-}bc = (P - bc)\omega_B$, and is plotted to a convenient velocity scale as vector $bc - L$ normal to radius $bc - P$.

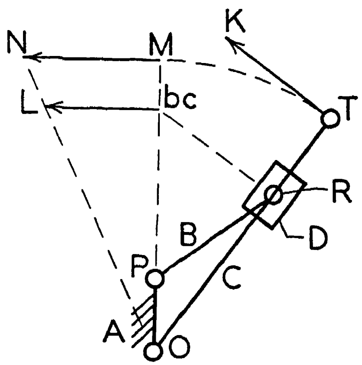


FIG. 6–5. Velocities on Whitworth Mechanism.

Now bc is also a point on C, and has instantaneous motion relative to A about O in common with all other points of C. An arc through T about center O gives M in line with bc and O. The proportionality line OLN determines $V^{-}M$ as vector MN. Then $V^{-}T$ is drawn normal to TO and of magnitude equal to $V^{-}M$.

An alternative construction is to use an arc through bc locating a point in line with O and T having the same velocity in magnitude as bc, followed by a proportionality line from O to K.

6–6. The Direct-Centro Method for Velocities of Bodies in Direct Contact.—The transfer method cannot be used in the absence of connecting links; however, the centros of bodies in direct contact can always be found as in § 2–10, and the direct-centro method used. Of course, the phorograph (Chapter V) and the component method (to be presented later) are also applicable to this case.

Fig. 6–6 shows a portion of a cam-operated trip mechanism that will serve as an example. First, the centro bc is located at the intersection of the line of centers and the normal to the surfaces at the contact point. Knowing the rotative speed of the driving cam B, it is possible to plot $V^{-}bc = (O - bc)\omega_B$ normal to $O - bc$. But bc is also a point on link C.

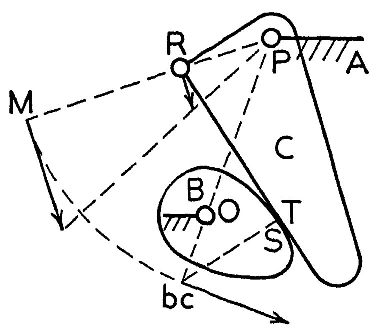


FIG. 6–6.

Using principles (1) and (2), § 6–1, we get $V^{-}R$ from $V^{-}bc$ operating about centro ac which is at P. $V^{-}M$ equals $V^{-}bc$ in magnitude, and, because M, R, and P are in line, a proportionality line through P and the end of vector $V^{-}M$ gives the magnitude of $V^{-}R$. Its direction must be normal to the radius PR.

6–7. **The Component Method.**—This method is based on two principles. The first is the use of rectangular components, sometimes called orthogonal components. **Rectangular components** of a vector are two components, normal to each other, which, added vectorially, produce the vector. Knowing the *direction* of a vector and the direction and magnitude of one rectangular component, the other rectangular component can be erected and the magnitude of the vector determined. This is the principle of rectangular components, § 1–9.

Secondly, it was demonstrated in § 5–2 that two points in the same body can have no component of *relative* motion in the direction of the line joining them. This means that if one point has any component of motion relative to a third body, in the direction of the line joining the two points, the other point must have the same component in that direction relative to the third body. If not, the first body would be ruptured.

Two points in the same body must have the same component of velocity relative to any third body, in the direction of the line joining the two points.

With these two principles in mind we shall apply the method to the quadric mechanism, Fig. 6–7. $V^{-}M$ is known and $V^{-}N$ is to be found. All velocities are relative to the fixed link A which is therefore the third body in each step. $V^{-}M$ has no component in the direction of the radius MO, but, as in all such cases, a proportionality line will give the velocities of all points of B on the line MP. In this manner $V^{-}P$ is obtained as the vector PT. P and R are two points on the link C. The component of $V^{-}P$ in direction PR is PU. Therefore RW, laid off equal to PU, must be the component of the motion of R in direction PR. Now the actual motion of R with respect to the fixed link must be normal to its radius to the centro ad at S. $V^{-}R$ must be of such length as to have RW as one rectangular component. WX is therefore drawn normal to RW to intersect the known direction of $V^{-}R$ at X, giving $V^{-}R = RX$.

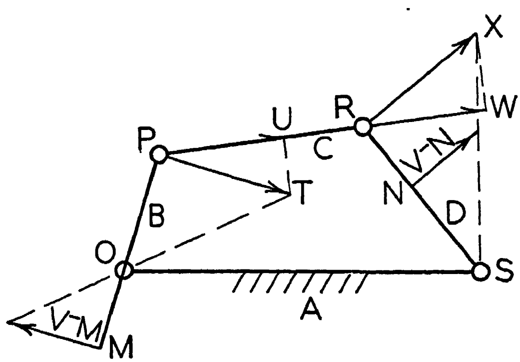


FIG. 6–7. Quadric Chain Velocities by Component Method.

The most common error in the use of this method arises from failure to recognize which vectors are components and which are total actual velocities. In the illustrations, therefore, components are represented with one ear of the arrow missing.

6–8. **Component Method for Velocities of Bodies in Direct Contact.**—In Fig. 6–8, link B drives C, the contact points for the phase shown being P on B, and R on C. $V^{-}P = (OP)\omega_B$ and is plotted normal to OP as vector PT. The next step is to determine PU, the component of PT that is normal to the surfaces at the contact point. The length of PU is found by drawing through T a line parallel to the tangent PK to the surfaces at the contact point.

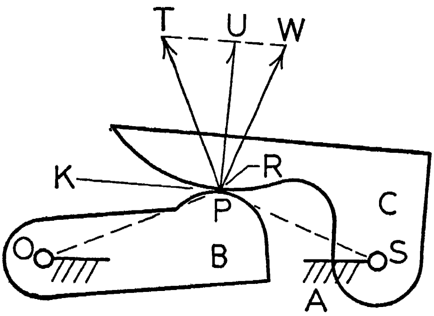


FIG. 6–8.

Now PU is the component of $V^{-}P$ normal to the contact surfaces, and, for continued transmission, $V^{-}R$ must have the same normal component.

When one body drives another by direct contact, their two contact points must have equal components of velocity normal to the surfaces at the contact point, for if not the bodies will either crush or separate.

The direction of $V^{-}R$ must be normal to radius SR, and, by continuing TU to the intersection at W, $V^{-}R$ is obtained as vector RW.

The principle used here is closely related to that of the previous article where transmission was through a connecting link. Here, equal components normal to the contact surfaces take the place of equal components in the direction of the connecting link.

6–9. **Velocities on an Injection-Pump Mechanism.**—The arrangement shown in Fig. 6–9 is a type frequently used to actuate valves and the pistons of injection pumps where the initial lift should be at low velocity. It may be also noted that there is very little sliding at the contact between D and E, with consequent advantages in efficiency and wearing characteristics. F is the stem of the pump piston or valve.

B is the driving crank, and, assuming that the velocity of bc is given, we use components $bc - N = cd - O$ to get $V^{-}cd$ as $cd - S$. Next, using the proportionality line $ad - S$, $V^{-}K$ is found, K and P being equidistant from ad. Vector KL is now swung about ad into position PW giving $V^{-}P$, the velocity of the contact point on D.

The component of $V^{-}P$ normal to the surfaces at the contact point is PY. This must also be the normal component of $V^{-}H$, where H is the contact point on E. The direction of $V^{-}H$ must be normal to $ae - H$. These two facts determine $V^{-}H$ as HX.

$V^{-}H$ is next swung about ae to give the velocity of a point in line with ae and R so that a proportionality line can be used through $ae - Z$ to

give V^-R as RU. It happens that RU is its own rectangular component, so a line through U parallel to the surfaces of contact at R yields the required velocity of the pump piston V^-F as the vector RT.

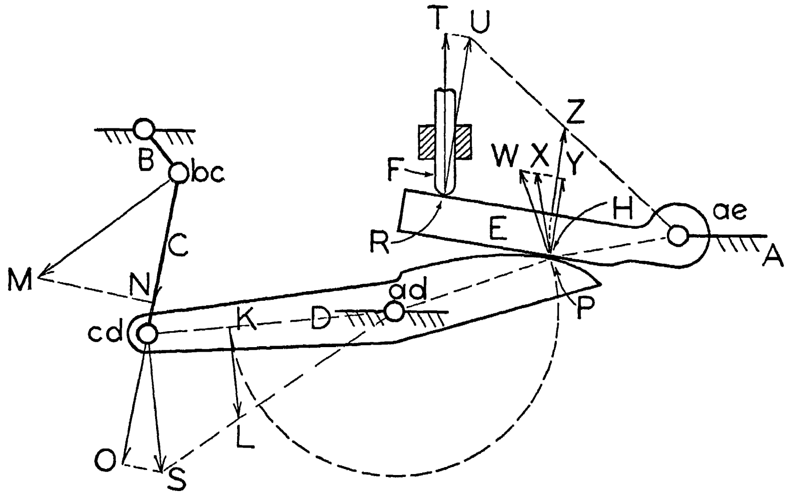


Fig. 6–9.

6–10. **Checking Velocities by Different Methods.**—When mechanisms are drawn accurately to scale it is possible to obtain numerical values for the velocities of any points desired, using any of the foregoing methods that would seem to be most advantageous for that particular problem. For important work one must be sure of results, and, except for an independent check by a colleague, there is no better procedure than to get the results by entirely different methods.

A variety of methods for velocity has been presented here mainly as an aid in the mastery of kinematics, but ability to check one's results, both while learning and later in practice, is desirable. A large scale drawing is advisable where accuracy is essential, but of greater importance is the use of thin sharp light lines for all construction. Probable error can be held under two percent with reasonably careful work.

Consider Fig. 6–10 which shows a linkage used for molding presses. F is the squeezing plunger and B the crank attached to the driving shaft. When P is near its lowest position, and B and C are nearly in line, D and E are also nearly in line, giving a doubly magnified mechanical advantage to the press.

Crank B is 1/2 in. long on the drawing or 8 in. on the machine. It revolves at 10 radians per sec so that $V^-P = 8 \times 10 = 80$ in. per sec. To the velocity vector scale of 1 in. $= 80$ in. per sec, V^-P is plotted as a

vector $P2$, of length 1 in. Component $P3 = R4$, together with the known direction of $V^{-}R$ give $V^{-}R$ as $R5$. The component of $V^{-}R$ in direction RT is $R6 = T7$. $V^{-}T$ is vertical and of such magnitude as to have $T7$ as rectangular component. $V^{-}T = T8$, scales 0.91 in. giving a velocity for the plunger in this phase of $0.91 \times 80 = 72.8$ in. per sec.

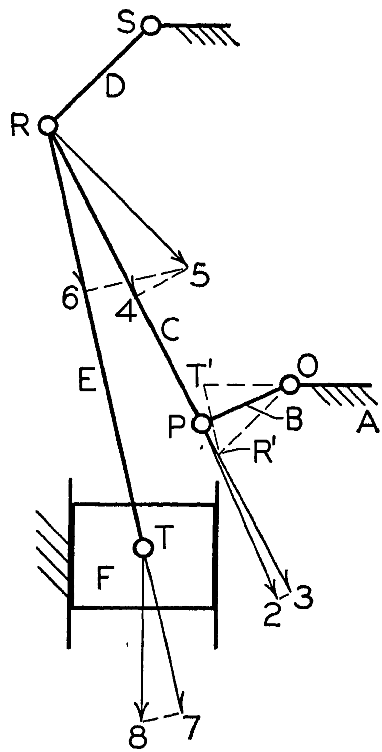


FIG. 6–10. Molding Press.
Scale of mechanism, 1 in. = 16 in.
Scale of velocity, 1 in. = 80 in. per sec.

A check will now be made using the phorograph. The obvious choice for phorograph link is B which rotates at known speed. R' is found on a line through O parallel to SR, and on a line through P parallel to PR. The location of T' is on a horizontal through O, and on a line through R' parallel to RT. Now $V^{-}T = V^{-}T' = (OT')\omega_B = 0.45 \times 16 \times 10 = 72.0$ in. per sec.

What is the rubbing velocity of the bearing between links D and E at R for the phase shown, the bearing diameter being 4 in.?

The angular velocities of D and E can be found directly from the vectors.

$$\omega_D = \frac{(R5)80}{(RS)16} = \frac{1.04 \times 80}{0.75 \times 16}$$

$$= 6.935 \text{ rad per sec}$$

counterclockwise.

$$\omega_E = \frac{[(6, 5) + (7, 8)]80}{(RT)16}$$

$$= \frac{[0.55 + 0.22]80}{2.25 \times 16} = 1.710$$

and is clockwise.

Rubbing velocity $= [6.935 - (-1.71)]2 = 17.29$ in. per sec.

Using the phorographs as a check, we proceed according to § 5–4.

$$\omega_D = \omega_B \times \frac{OR'}{SR} = 10 \times \frac{0.52}{0.75} = 6.935 \circlearrowleft$$

$$\omega_E = \omega_B \times \frac{R'T'}{RT} = 10 \times \frac{0.38}{2.25} = 1.69 \circlearrowright$$

Rubbing velocity $= [6.935 - (-1.69)]2 = 17.25$ in. per sec.

The values that had to be obtained from the drawing were scaled from a thin-lined pencil drawing double the size of Fig. 6–10.

6–11. **Velocity Diagrams for an Oscillating-Beam Mechanism.**—This quick-return mechanism, § 3–2, is used on shapers and slotters to reduce the time of the idle return stroke. It is quite important, however, that

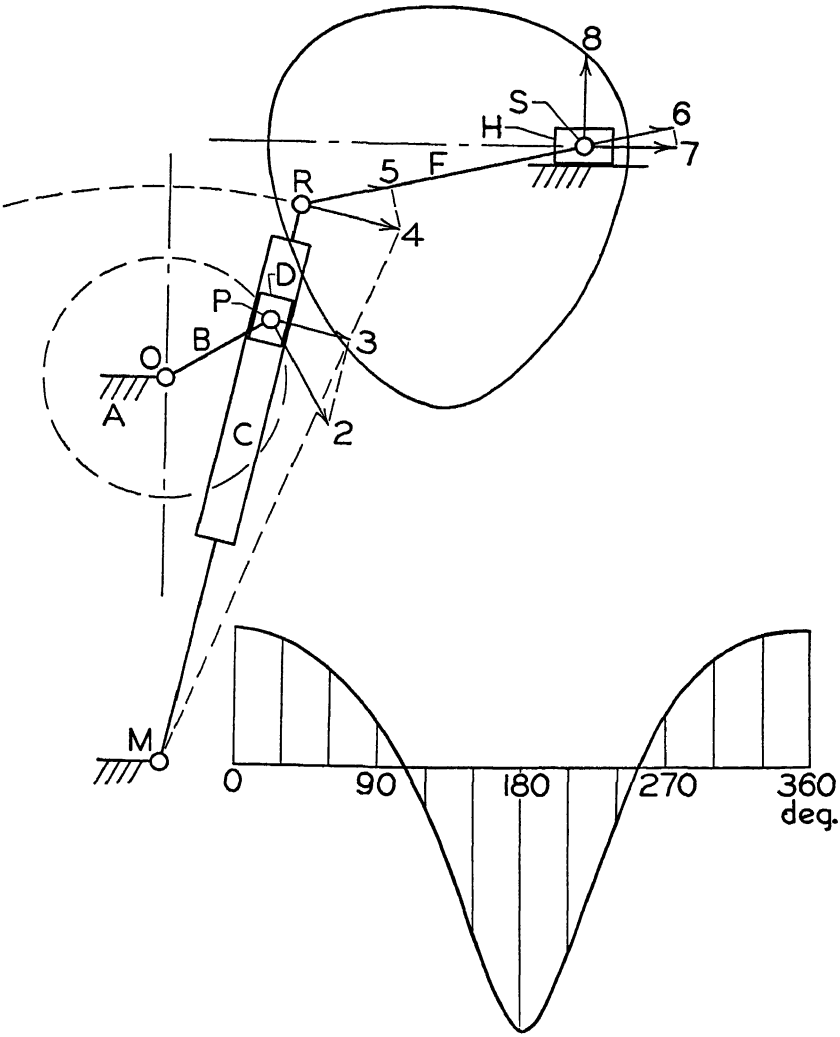


Fig. 6–11. Velocity-Space and Velocity-Time Diagrams for the Oscillating-Beam Quick-Return Mechanism.

Space scale, 1 in. = 8 in.
Velocity scale, 1 in. = 4 ft per sec.

the allowable cutting speed of the tool should not be exceeded on the forward stroke. To arrive at the best dimensions of the mechanism to suit a particular design, velocity diagrams for the complete cycle are useful.

In Fig. 6–11, the component method is used to find the velocity of the ram H at one point in its travel. The driving link B (bull gear) turns clockwise at 6 radians per second. OP is $\frac{5}{8} \times 8 = 5$ in. long on the machine. $V^{-}P = \frac{5}{12} \times 6 = 2.5$ ft per sec. This plotted to the velocity scale of 1 in. = 4 ft per sec gives vector $P2$ of length $\frac{5}{8}$ in. The point of link C that has the same position as P must move in direction normal to MP, and must have magnitude of velocity equal to the rectangular component of $V^{-}P$, § 6–8. This gives $P3$ which is increased in the proportion MR/MP to give $V^{-}R$ as $R4$. Then the component $R5$ equals $S6$ yielding $S7$ as $V^{-}S$.

To give a diagram on travel (or space) base, such vectors as $S7$ must be turned through 90°. Further, a convention must be adopted to govern sign. Assuming velocities toward the right positive, $S8$ (= $S7$) is plotted upward. It scales 0.49 in., indicating a speed of 1.96 ft per sec for the cutting tool in this phase.

It is often desirable to have such curves of velocity on a time base. The driving crank B will turn at approximately constant speed; therefore a base indicating degrees of displacement of B is a convenient time base. This is used for the lower curve of Fig. 6–11. Zero degrees corresponds to the vertically upward position of crank B.

If the time scale is required, it can be derived from the data given. Since B turns at 6 rad per sec, one revolution or 360° requires $2\pi/6$ sec. This is represented by 3 in. on the drawing, or the time scale is 1 in. = $\pi/9$ sec. The degree base has the advantage of easy subdivision since 360 is a highly factorable number.

It can be seen from the velocity-time curve that the fast return stroke requires 144° or 40% of the time of a cycle.

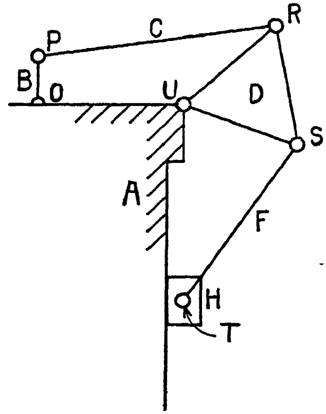
QUESTIONS AND PROBLEMS

1. What are the two principles that are basic to the transfer method using centros?
2. Cite the basis for the direct-centro method for velocity.
3. (a) What is the "principle of rectangular vectors" relative to velocity?

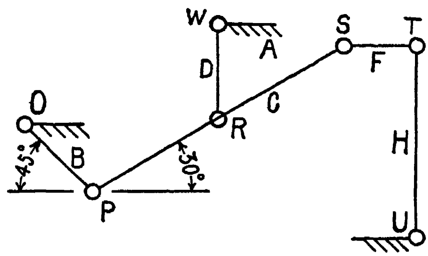
   (b) On what other principle does the component method for velocities depend?
4. For what special error must one be on guard in using the transfer method?

5. What error must be carefully avoided in the use of the component method?

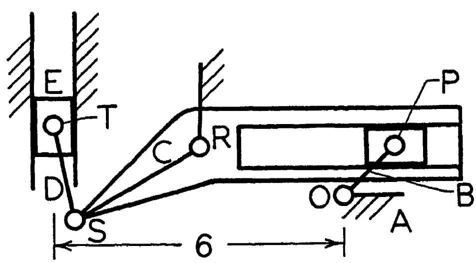
6. This bell-crank mechanism is driven by the crank B, revolving at 40 rad per sec. Find the velocity of the slider H in in. per sec, using each of the three methods of this chapter in turn with a separate drawing for each. The dimensions in inches are: $B = \frac{1}{2}$, $C = 2\frac{1}{2}$, all sides of $D = 1\frac{1}{4}$, $F = 2$, and $OU = 1\frac{1}{2}$. Ans. 45.



7. If B is rotating through the phase shown at 10 rad per sec, what is the instantaneous velocity of the midpoint of H? Make B and $D = 1$, PR and $RS = 1\frac{1}{2}$, $F = \frac{3}{4}$, and $H = 2$.



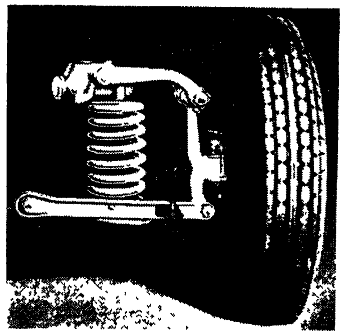
8. This mechanism is used for a vertical shaper. Dimensions in inches are: $OP = 1\frac{1}{2}$ and inclines at 45° in the phase shown, $PR = 4$, $RS = 3$ and is 30° below the line of PR which is horizontal, $ST = 2$. When crank B revolves at 8 rad per sec, what is the cutting speed of the tool in the ram E? Solve by the component method and check by one other method.



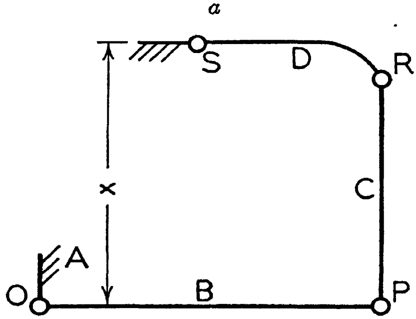
9. Draw the mechanisms of problems 4 to 10 inclusive, Chap. V, to approximately three times the size shown, and, from the data that follow, solve by the transfer method using centros and check by the component method. Assume that, in each case, the mechanism is driven by B revolving as indicated at 10 radians per second. Find the velocities of the points in in. per sec.

(4) V^-U for $\omega_B \circlearrowright$
(5) V^-T for $\omega_B \circlearrowleft$
(6) V^-T for $\omega_B \circlearrowright$
(7) V^-H for $\omega_B \circlearrowright$
(8) V^-T for $\omega_B \circlearrowleft$
(9) V^-F for $\omega_B \circlearrowright$
(10) V^-W for $\omega_B \circlearrowleft$, and the rubbing velocity of a bearing of 3/4 in. diameter at S.

10. The linkage of the front wheel suspension of the Pontiac automobile is pictured at (*a*) and the essential mechanism at (*b*). Assume that (*b*) shows the position for normal load with link C vertical, B horizontal, and that B can move 10° above and 10° below this position. $OP = 18$ in., $PR = 13$, $RS = 10$, $x = 15$.
   (*a*) Plot the angular position of the wheel link C, on the angular position of B as abscissa. Use 5° increments in the position of B.
   (*b*) Discuss the significance of this curve in relation to the ability of the wheel to surmount an obstacle when rounding a curve, this being the outside wheel and therefore carrying the heavier load.

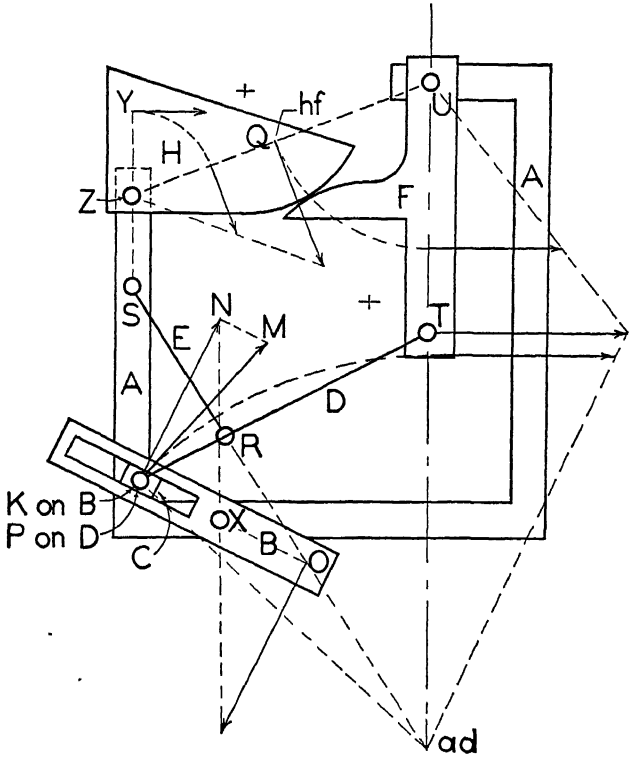


a



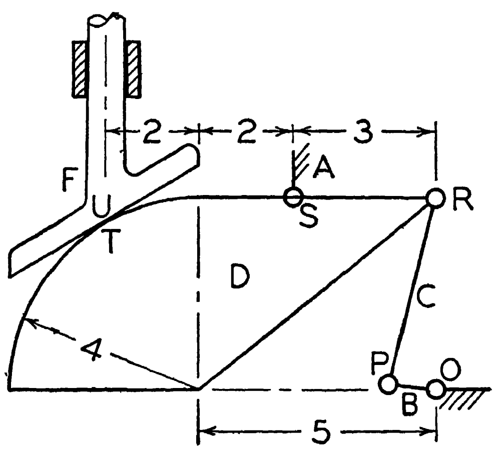
b

11. In this mechanism V^-Y is given as a vector 3/8 in. long and solution for V^-O is illustrated using the transfer method. Find V^-O using (*a*) the component, (*b*) the phorograph method.



12. Draw the ellipsograph mechanism, Fig. 3–13, making $a = 3\frac{1}{2}$ in. and $b = 1\frac{1}{2}$ in. When link C is 30° from the horizontal and W is being moved in its circular path at 10 in. per sec, what is the speed of the tracing point T?

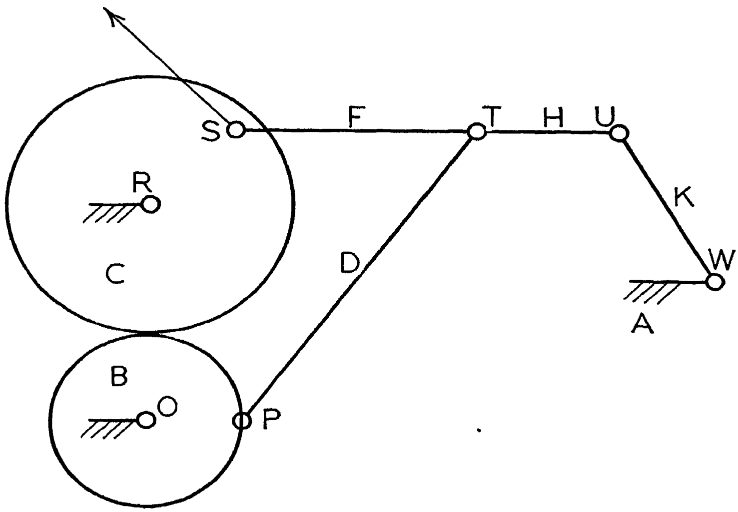
13. In this cam mechanism $C = 4$ in., and crank B which is 1 in. long revolves at 20 rad per sec. The face of cam follower F is 30° from the horizontal. Find: (*a*) V–F for the phase shown, (*b*) the component of rubbing velocity at the cam sur-



faces, (*c*) the angular position of *B* for least rubbing at the cam surfaces while *F* is moving.

Ans. (*a*) 29.5 in. per sec, (*b*) 4.2 in. per sec, (*c*) 54.5° left of lower vertical.

14. Reproduce this mechanism at exactly double the size shown, scaling dimensions and angles. Wheels *C* and *B* roll together without sliding. $V^{-}S$ is 20 in. per sec and may be represented by a vector 2 in. long. Find: (*a*) $V^{-}U$; (*b*) Rubbing velocity at bearing *U* if it is $1\frac{1}{2}$ in. in diameter.



CHAPTER VII

ACCELERATIONS IN MECHANISMS

7–1. **The Influence of Acceleration Forces on Design and Performance.**—Before developing the technique of determining accelerations in a mechanism, it is in order to appraise the importance of acceleration forces in certain problems of design and operation. The acceleration of any mass requires the application of an external force equal to the product of the mass and the acceleration. Most mechanisms, from their nature, impose relative and therefore absolute accelerations on some of their parts every cycle. When the links of such a mechanism are given mass to become a machine, its operation entails the action of these acceleration forces which appear as stress loads in the connecting links, in the links being accelerated, and in the bearings.

In a modern automotive or aircraft engine, operating at its power peak, the stresses imposed on the connecting rod as well as the bearing loads, that arise directly from accelerations, are considerably greater than those that result from the pressure of the burning gas. Acceleration diagrams are therefore fundamental, both to stress analysis in such an engine and to problems of bearing lubrication. The design of rotors for steam turbines, centrifugal pumps, high speed blowers, and electric dynamos is largely influenced by acceleration stresses.

7–2. **Normal and Tangential Components.**—The formulas expressing the common relations of displacement, time, velocity, and acceleration were tabulated in § 1–8 and should be reviewed at this point.

Consider a point P on a link revolving about the fixed center O, Fig. 7–1. In the general case, $V^{-}P$ will vary as P moves with constant radius r. When P is at P_1, let $V^{-}P$ be represented by the vector P_1R_1. An instant later when P has travelled the distance ds, $V^{-}P = P_2W$. Its change of velocity is represented by the vector difference R_2W.

Resolve R_2W into the rectangular components R_2T and TW. R_2T is called the **normal component** because it is *normal to the path of the point considered.* In dealing with this normal component we shall call R_2T, for brevity, dV. In the limit, as $d\theta$, ds, and dt become small, OP_1P_2 and P_2R_2T may be considered triangles and are similar. Then

$$\frac{dV}{V} = \frac{ds}{r} \tag{1}$$

Dividing by dt

$$\frac{dV}{dt} = \frac{V}{r}\frac{ds}{dt} = \frac{V^2}{r} \tag{2}$$

Also $\dfrac{V}{r} = \omega$; therefore

$$\frac{dV}{dt} = a_N = r\omega^2 \tag{3}$$

where a_N is the **normal component** of the acceleration of P relative to O and ω is the angular velocity of the revolving radius of P.

The other component of acceleration of P is a_T, the tangential component. Its value is TW/dt and from inspection

$$a_T = r\alpha \tag{4}$$

where α is the angular acceleration of the revolving radius.

Note that the normal component of acceleration is entirely independent of the angular acceleration of the radius or body on which the point is located. The normal component of acceleration depends solely on the distance r from the point considered to the point of reference, and on ω, the attained angular velocity of the radius joining the points, and is independent of whether ω is increasing, decreasing, or uniform.

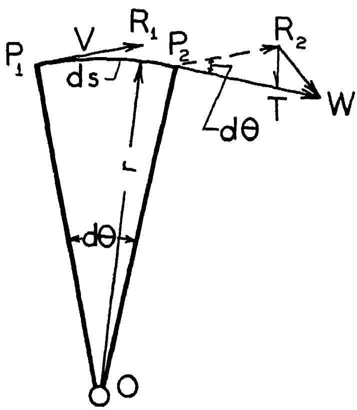


FIG. 7–1.

7–3. The Elements of Relative Acceleration.—Acceleration is a vector quantity having magnitude, direction, and sense. It is the time rate of change of another vector quantity, velocity. Acceleration is absolute. If a body has no unbalanced force acting upon it and no unbalanced moment, its absolute acceleration is zero. In determining absolute accelerations in mechanisms we deal largely with the relative accelerations of points. The relative acceleration of two points is the difference between their absolute accelerations.

In Fig. 7–2 is shown a straight rod B revolving about a fixed center O. As B goes through the position shown, its angular velocity is ω radians per second. Due to this velocity, P is accelerated with respect to O, in a direction and sense toward O, and in magnitude $(OP)\omega^2$ ft per sec per sec, if OP is measured in ft. This is the *normal acceleration* of P with respect to O, because it is normal to the path of P moving with respect to O. The normal acceleration keeps P on its circular path, and,

if it ceased, P would at once take the tangent as its further path. The normal acceleration necessitates a tension in B, and, if ω were sufficiently increased, B would be fractured in tension.

Now the normal acceleration of O relative to P is the same in magnitude, namely $(OP)\omega^2$, but opposite in sense to the normal acceleration of P relative to O. Further, since the normal acceleration of R relative to O is $(OR)\omega^2$, it follows by addition that the normal acceleration of R relative to P is $(RP)\omega^2$, in direction and sense from R to P. The remarkable thing about this last value is that it is independent of the location of O but depends solely on ω and the distance between the points in question.

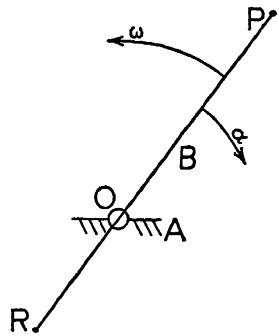


FIG. 7–2. Accelerations on a Straight Rod.

Consider now the possibility of acceleration of P relative to O, in direction tangent to the path of P with respect to O. This is the *tangential acceleration* and depends on angular acceleration of the body which can be positive, negative, or zero, regardless of the value of ω. If, at the instant the rod is going through the position shown, it is subject to an angular acceleration clockwise of α radians per second per second, the tangential acceleration of P relative to O would be $(OP)\alpha$ ft per sec per sec. The total instantaneous acceleration of P relative to O would be the vector sum of the two rectangular components, the normal and the tangential.

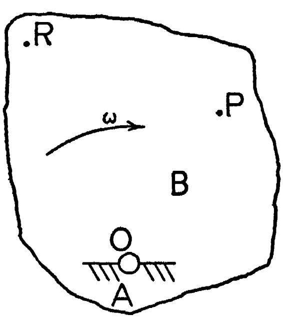
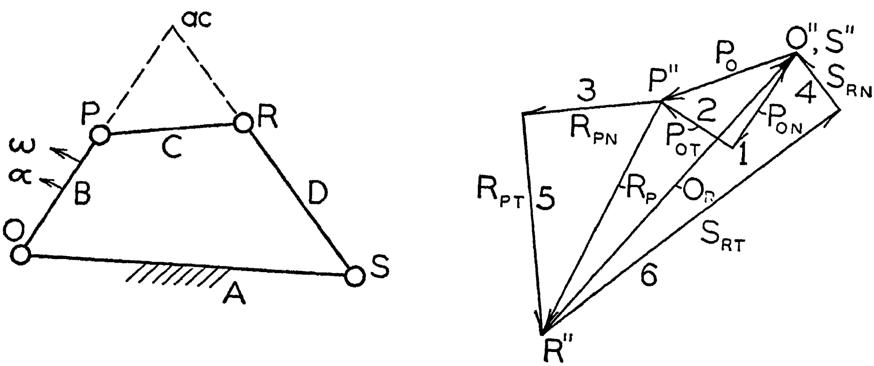


FIG. 7–3. Relative Accelerations on a Body.

The tangential acceleration of P relative to R would be $(PR)\alpha$. Since α is opposite in sense to ω, the latter is decreasing.

The body B, in Fig. 7–3, is rotating relative to A at ω rad per sec. The normal acceleration of P relative to O is $(OP)\omega^2$, and of R relative to O is $(OR)\omega^2$. What is the normal acceleration of P relative to R? It must depend on the distance between the points and the rate of turning of the line joining them. Obviously every line that could be drawn on the body is turning at ω radians per second; therefore the answer is $(PR)\omega^2$; the direction is from P toward R. To avoid uncertainty concerning the sense of normal accelerations, remember that any two points in the *same* revolving body are *always* accelerated toward each other.

7–4. **Accelerations on the Quadric Chain.**—In the quadric mechanism of Fig. 7–4, the point P will have a certain total acceleration relative to O, and it will be the resultant of a normal component depending on ω_B and a tangential component depending on α_B. Similarly, R will have a certain total acceleration relative to P composed of two components. The vector sum of these two *total* accelerations is the total acceleration of R relative to O. If now we add the total acceleration of S relative



(*a*)
Scale, 1 in. = 1 in.

(*b*)
Acceleration scale,
1 in. = 2000 in. per sec².

FIG. 7–4.

to R, and of O relative to S (the latter being zero), the resultant must be the acceleration of O relative to O which is zero, and therefore the total vector diagram must close.

This outlines the method that we shall use in general for obtaining accelerations in link mechanisms. The principle is quite general. In fact the statements of the previous paragraph would be true if O, P, R, and S were points on separate bodies anywhere in space.

The following system of symbols will be used.

Let P_{ON} be the normal acceleration of P relative to O,
P_{OT}, the tangential acceleration of P relative to O,
R_{PN}, the normal acceleration of R relative to P,
R_{PT}, the tangential acceleration of R relative to P,
S_{RN}, the normal acceleration of S relative to R,
S_{RT}, the tangential acceleration of S relative to R.

Then $P_{ON} \mathrel{+\!\!\!\rightarrow} P_{OT} \mathrel{+\!\!\!\rightarrow} R_{PN} \mathrel{+\!\!\!\rightarrow} R_{PT} \mathrel{+\!\!\!\rightarrow} S_{RN} \mathrel{+\!\!\!\rightarrow} S_{RT} = 0$. The frame A is assumed not to be accelerated in any way, so the total acceleration of O relative to S, O_S, is zero.

Suppose that crank B is being accelerated counterclockwise at $\alpha_B = 1200$ radians per sec^2, and, when passing through the position shown, has attained an angular velocity ω_B of 40 rad per sec.

$$P_{ON} = B\omega_B{}^2 = 0.75 \times 40^2 = 1200 \text{ in. per sec}^2$$

where the name of the link is used to represent its length. The direction and sense of P_{ON} is from P toward O and it is plotted as a vector from O'' in Fig. 7–4 (*b*). To distinguish component vectors from total acceleration vectors, the former will be given one-eared arrows as in the case of velocities.

$$P_{OT} = B\alpha_B = 0.75 \times 1200 = 900$$

which is plotted to the given scale in the direction of the tangent to the path of P relative to O and in the given sense. The sum of these two components is the total acceleration of P relative to O, P_O, locating P''.

To determine R_{PN} requires ω_C. Using centro *ac*,

$$\omega_C = \frac{B}{ac\ P}\,\omega_B = \frac{0.75}{0.68} \times 40 = 44.1$$
$$R_{PN} = C\omega_C{}^2 = 0.75 \times 44.1^2 = 1459$$

and is plotted from P'' in direction R to P.

A change in procedure is now necessary. R_{PT} and S_{RT} depend on α_C and α_D, neither of which can be readily evaluated at this point. *In any vector diagram known to close, two vectors, unknown in magnitude, can be determined if their directions are known.* Leaving R_{PT} and S_{RT} to be so evaluated, there remains only one component vector to be included, S_{RN}, which depends on ω_D.

$$\omega_D = \frac{ac\ R}{D}\,\omega_C = \frac{0.62}{1} \times 44.1 = 27.3$$
$$S_{RN} = D\omega_D{}^2 = 1 \times 27.3^2 = 745$$

The order in which the two components of a total acceleration are plotted does not affect the result, so S_{RN} is plotted as the final vector terminating at O'' and having direction S to R. The order of plotting is indicated by the numbers 1 to 6.

From the end of R_{PN} draw R_{PT} normal to C, and through the initial point of S_{RN} draw S_{RT} normal to D, intersecting at R''. The triangle $O''P''R''O''$ is the final diagram representing the total accelerations of all points of the mechanism, both absolute and relative. The absolute acceleration of R is R_O, represented by the vector $O''R''$ which measures 1.98 in., giving $R_O = 3960$ in. per sec^2 with direction and sense O'' to R''. The acceleration of R relative to P, R_P, is given completely by the vector

$P''R''$, and P_R is given by $R''P''$. When uncertain about the sense of any acceleration, it is a good plan to return to the first total vector, in this case P_O. It is represented by the vector from O'' to P'' with sense $O''P''$. O'' may be called the *pole point* of the diagram, it being the point of zero absolute acceleration. Each point of the mechanism has absolute acceleration proportional to the distance that its double-primed namesake on the diagram lies from O''.

7–5. **Accelerations on the Slider-Crank Mechanism.**—The example considered in the previous article illustrated the more general case where the driving link is subject to angular acceleration at the instant considered. In practice, the most useful determinations of accelerations in machines are for the maximum operating speeds. At such speeds, the angular velocity of the principal rotating link is generally so nearly uniform that its α can be considered zero.

On the slider-crank mechanism, the **ratio of the lengths of connecting rod to crank** has an important effect on the acceleration of the piston. On engines for the following services, the ratios are:

Aircraft	3.25 to 3.6,
Automobiles	3.6 to 4.2,
Trucks	4.35 to 4.55,
Stationary steam	4.5 to 5.5,
Steamships	4.0 to 4.55,
Locomotives	6.0 to 7.0.

In Fig. 7–5 the length of the connecting rod is 12 in. and of the crank 3.6 in. If the speed ω_B is taken as 200 radians per sec, slightly more than 1900 rpm, conditions will be representative of those on an aircraft engine at maximum speed.

It is convenient to use O as pole point of the acceleration diagram. It could be located elsewhere as O''.

$$P_{ON} = B\omega_B{}^2 = \frac{3.6}{12} \times 200^2 = 12{,}000 \text{ ft per sec}^2$$

The foot is a more convenient unit here and we must be careful to use lengths on the machine rather than on the drawing. Using the phorograph R' to determine ω_C,

$$\omega_C = \frac{R'P}{C}\,\omega_B = \frac{0.4}{1.5} \times 200 = 53.3$$

where a ratio is expressed, lengths on the drawing will serve.

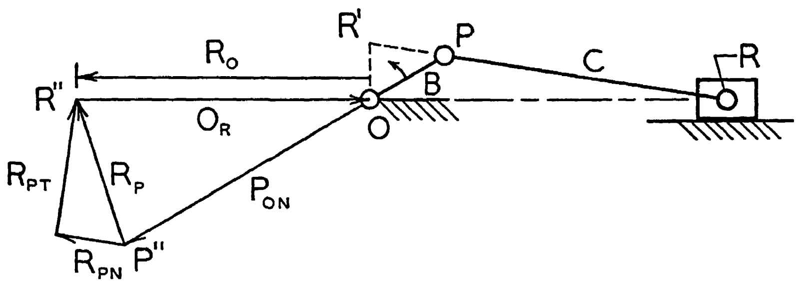
$$R_{PN} = C\omega_C{}^2 = \frac{12}{12} \times 53.3^2 = 2844$$

Having plotted P_{ON} in direction P to O, and R_{PN} in direction R to P, there remain only R_{PT} normal to C, and O_R horizontal, to close the diagram.

Two comments on O_R seem necessary. First, having started around the mechanism in clockwise sense, the resulting order must be preserved,

$P_O \nrightarrow R_P \nrightarrow O_R = 0$ gives a closed diagram.

$P_O \nrightarrow R_P \nrightarrow R_O \neq 0$ will not give a closed diagram, because the clockwise order has not been preserved. Second, although O is a stationary point, it can and does have acceleration *relative* to R, it being the reverse of R_O as shown in Fig. 7–5.



Space scale, 1 in. = 8 in.
Acceleration scale, 1 in. = 8000 ft per sec².

FIG. 7–5.

R_O scales 12,320 ft per sec² for the phase shown. It is the acceleration imposed on the piston and the small end of the connecting rod, and, when known for all phases, is most important information for design and balancing.[1]

7–6. Slider-Crank Velocities and Accelerations by Analytical Methods. —The displacement s, of the reciprocating link, Fig. 7–6, is conveniently measured from its head-end dead-center position.

$$s = fg + gh = B(1 - \cos\theta) + C(1 - \cos\beta) \tag{5}$$

The ratio of the lengths of connecting rod to crank, C/B, will be called r. Next we eliminate β in terms of θ.

$$\frac{C}{B} = \frac{\dfrac{Pg}{B}}{\dfrac{Pg}{C}} = \frac{\sin\theta}{\sin\beta} = r \tag{6}$$

[1] See Appendix II for the Ritterhaus Construction, and Appendix III for Klein's Construction, for acceleration on the slider-crank mechanism.

and

$$\cos \beta = \sqrt{1 - \frac{\sin^2 \theta}{r^2}} \tag{7}$$

From (5) and (7)

$$s = B[1 - \cos \theta + r - \sqrt{r^2 - \sin^2 \theta}] \tag{8}$$

$$V^{-}R = \frac{ds}{dt} = B\omega \left[\sin \theta + \frac{\sin 2\theta}{2\sqrt{r^2 - \sin^2 \theta}} \right] \tag{9}$$

where

$$\omega = \omega_B = \frac{d\theta}{dt} \qquad \text{and} \qquad \sin 2\theta = 2 \sin \theta \cos \theta$$

$$R_O = \frac{d^2s}{dt^2} = B\omega^2 \left[\cos \theta + \frac{\cos 2\theta(r^2 - \sin^2 \theta) + \sin^2 \theta \cos^2 \theta}{(r^2 - \sin^2 \theta)^{3/2}} \right] \tag{10}$$

$$R_O = B\omega^2 \left[\cos \theta + \frac{\cos 2\theta(r^2 - 1) + \cos^4 \theta}{(r^2 - \sin^2 \theta)^{3/2}} \right] \tag{11}$$

Examining the expression for velocity, (9), the first term, $B\omega \sin \theta$, is the velocity the piston would have if its motion were harmonic as on the Scotch yoke, Fig. 3–17. The second term accounts for the effect of connecting rod angularity, an effect given graphically in Fig. 7–6 by the ratio OR'/OT since T would be the phorograph of R for harmonic motion.

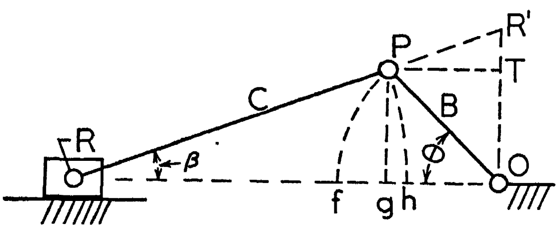


FIG. 7–6.

Expressions (9) and (11) are so ponderous that approximations[1] have been used considerably. Dropping the $\sin^2 \theta$ from the denominator of (9) gives the **approximate equations,**

$$V^{-}R = B\omega \left[\sin \theta + \frac{\sin 2\theta}{2r} \right] \tag{12}$$

and

$$R_O = B\omega^2 \left[\cos \theta + \frac{\cos 2\theta}{r} \right] \tag{13}$$

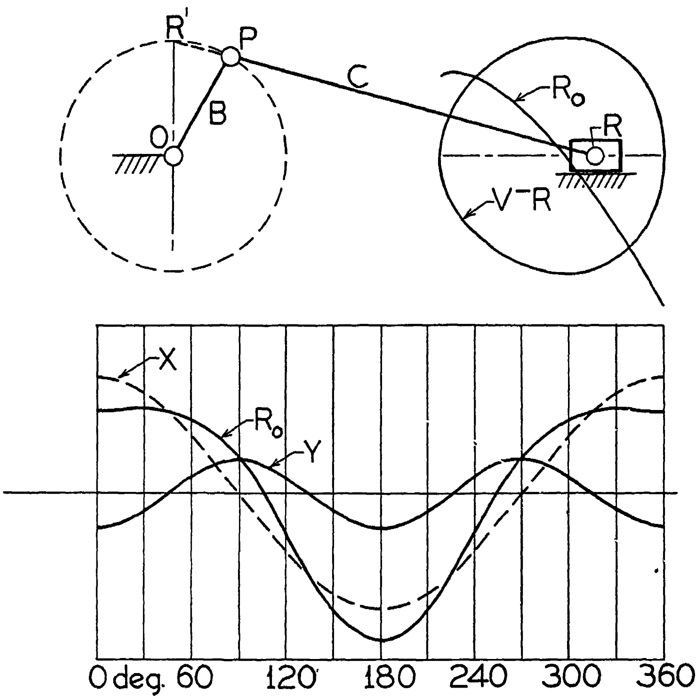
Sharp-lined large-scale graphical work will yield better accuracy than the approximate formulas, especially if r is small.

7–7. **Velocity and Acceleration Diagrams for the Slider-Crank Mechanism.**—The dimensions of the mechanism, Fig. 7–7, and the speed used,

[1] For tables of factors for both exact and approximate solutions see Kent's Power M. E. Handbook, p. 14–63.

$\omega_B = 200$, are the same as used in § 7–5, so the values of velocity and acceleration for the whole cycle represent those to be found on a particular size of aircraft engine at maximum speed.

The convention governing sign is that velocity and acceleration toward the right are positive. *The sense of acceleration is the same as the sense of the force causing it.*



Space scale,	1 in. = 6 in.
Velocity scale,	1 in. = 100 ft per sec
Acceleration scale,	1 in. = 20,000 ft per sec².

FIG. 7–7. Velocity and Acceleration Diagrams for an Aircraft Engine.
Above, V^-R and R_O are velocity-space and acceleration-space curves for the piston. Below, R_O is the acceleration-time curve, X is the acceleration-time curve for harmonic motion of the piston, Y is the second harmonic or difference curve.

The velocity scale used is such that the length of the crank represents the crank-pin velocity. Hence OR' is the velocity ordinate for each phase, which is quite convenient. On the travel or space base, the velocity curve is symmetrical about the base, while the acceleration curve retraces itself on the return stroke. Determinations for each 15° of crank travel give sufficient points for curve drawing.

On the time, or degrees of crank travel, base (lower curves) zero degrees corresponds to head-end dead center. R_O is the acceleration curve for the piston of the engine. X is a true harmonic (sine wave), the acceleration curve for an equivalent mechanism of the Scotch yoke variety. Y is R_O minus X, so the Y curve is the connecting-rod angularity effect and it has double the frequency of the other curves. It is the so-called **second harmonic** and is a fundamental consideration in the problem of balancing multiple-cylinder engines.

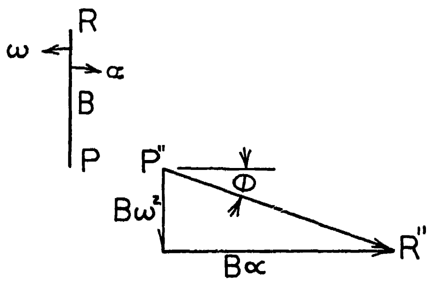


FIG. 7–8.

7–8. **The Acceleration Image.**—The bar B of Fig. 7–8 is revolving counterclockwise while being retarded as indicated by the sense of α. The relative acceleration of R and P is obtained from the components and is represented in magnitude, direction and sense by $P''R''$. The angle from PR to $P''R''$ is $90° + \tan^{-1}(\omega^2/\alpha)$ in the same sense as α. Note that the sense of this angular displacement is entirely independent of the sense of ω.

The length $P''R''$ is established by the relation

$$(P''R'') \times \text{acceleration scale} = B \times \text{space scale} \times \sqrt{\omega^4 + \alpha^2} \quad (14)$$

where $P''R''$ and B are lengths on the drawing in like units, and the acceleration and space scales are the quantities in congruent units repre-

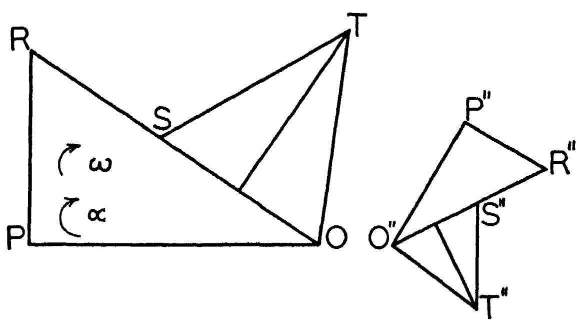


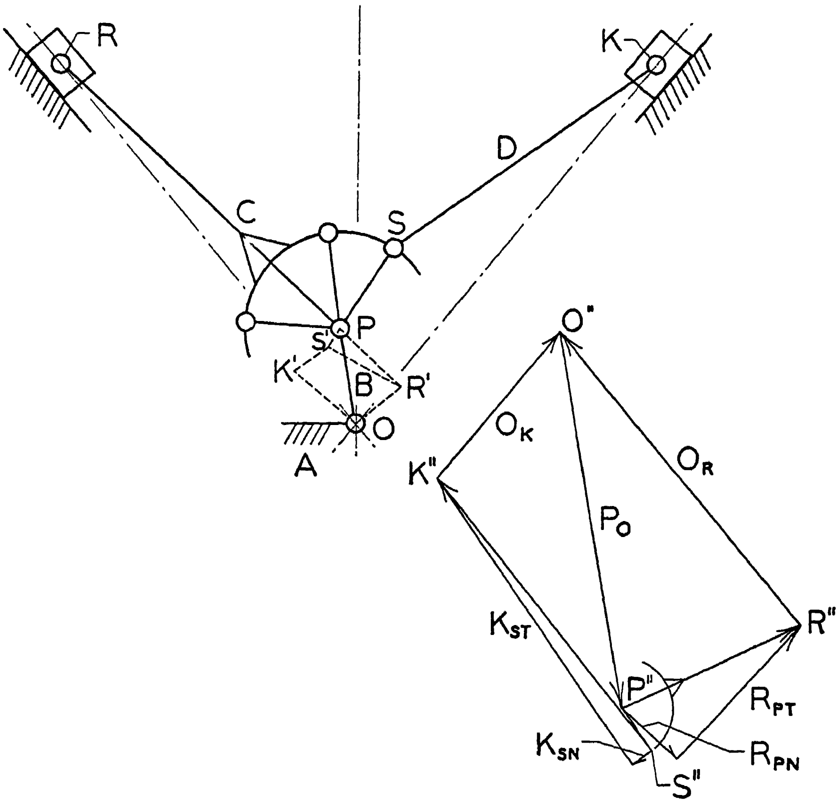
FIG. 7–9.

sented by one inch on the drawing. $P''R''$ may be called the acceleration image of PR. When a body rather than a line is considered, the value of this principle becomes more evident.

The irregular figure $OPRST$ shown in Fig. 7–9, could be treated in the same manner as the quadric chain, § 7–4, to obtain the complete acceleration diagram. But since all lines are on the same body, having

the same ω and the same α, it is only necessary to establish any two points on the acceleration diagram, such as $O''P''$, and complete the diagram using the principle of the acceleration image, having already established the angular position and the scale.

What is the acceleration of R relative to O? It is given by the vector $O''R''$ with the arrow at R''. What is the acceleration of T relative to R?



Space scale, 1 in. = 6 in.
Acceleration scale, 1 in. = 5000 in per sec².

FIG. 7–10. Radial-Engine Accelerations.

It is $R''T''$. What is the absolute acceleration of T? We do not know until the absolute acceleration of some point of the body is given.

7–9. **Accelerations on a Radial Engine.**—The usual cylinder arrangement on this type is nine in one plane normal to the shaft axis and equally spaced. The center lines of adjacent cylinders will therefore make an angle of 40° as indicated on Fig. 7–10. An odd number of cylinders must be used to give uniform torque with four-cycle operation.

There must be one master connecting rod, in this case C, which connects with the crank shaft through a large bearing at P. The eight small connecting rods join the master rod through turning pairs at such points as S, spaced at 40° intervals around P. The mechanisms for two cylinders only will be treated, one of which must have the master rod. The small rod SK is 10 in. long; the master rod PR is 12 in. The crank B is 3 in. and revolves at $\omega_B = 200$ rad per sec, the maximum speed, which is uniform.

$$P_O = B\omega_B^2 = \frac{3}{12} \times 200^2 = 10000 \text{ ft per sec}^2$$

$$\omega_C = \frac{PR'}{PR}\omega_B = \frac{0.44}{2} \times 200 = 44$$

$$R_{PN} = C\omega_C^2 = \frac{12}{12} \times 44^2 = 1936$$

R_{PT} and O_R are determined by closing the diagram, as in Fig. 7–5, giving the point R''. The image of PR is $P''R''$ so S'' is located by completing the image of link C.

Now $OPSKO$ must give another closed acceleration diagram, and since it fulfils the definition of a mechanism, all points must have definite motion for a given phase and therefore definite acceleration.

$$\omega_D = \frac{K'S'}{KS}\omega_B = \frac{0.23}{1.67} \times 200 = 27.5$$

$$K_{SN} = D\omega_D^2 = \frac{10}{12} \times 27.5^2 = 630$$

There remain K_{ST} normal to D, and O_K parallel to KO, to close this loop of the diagram and locate K''.

The most important results are R_O which is given by the vector $O''R''$ and scales 9950 ft per sec², also K_O given by the vector $O''K''$ which scales 4950. Suppose it is desired to know the acceleration of K relative to R. It is given by the vector $R''K''$, has magnitude 10300, direction and sense R'' to K''. What is the angular acceleration of the small rod D for the given phase? It is

$$\alpha_D = \frac{K_{ST}}{D} = \frac{9080 \times 12}{10} = 10900 \text{ rad per sec}^2$$

7–10. **Accelerations on Compound Mechanisms.**—Compound mechanisms are those in which the links form more than one closed chain or loop. Examples are common—MacCord's Fig. 2–21, Peaucellier's Fig. 4–13, the radial engine of the previous article. The general method for finding the acceleration in such mechanisms has just been illustrated but a few comments are necessary.

Whenever a link has more than two elements of kinematic pairs, its mechanism has more than one kinematic loop. Consider Fig. 7–10. The master rod C has ten elements of pairs (not all shown). Consequently there must be at least nine kinematic loops. A mechanism of one loop cannot have more than four links. Therefore any mechanism having more than four links must have multiple loops, and the only possible way in which mechanisms of many links can be built up, is by the addition of more closed loops. Each of these component loops is a closed chain and will have a closed acceleration diagram.

It follows that the most formidable mechanism can have the acceleration of its every point determined by solving one loop at a time. That loop containing the driving link or the link having specified motions must first be solved and this will lead to the information for the solution of adjacent loops.

7–11. **The Coriolis Component.**—In Fig. 7–11, B represents a straight rod or wire revolving about the fixed center O at the uniform angular velocity ω. Sliding up the rod at the uniform velocity V_L (longitudinal velocity) is a point S which may be regarded as a bead, or a crosshead, or any body having translation along link B. The normal component of the velocity of T_1 is V_{N1} and equals ωr. V_{N1} is also the total velocity of S which is the point on B under T_1.

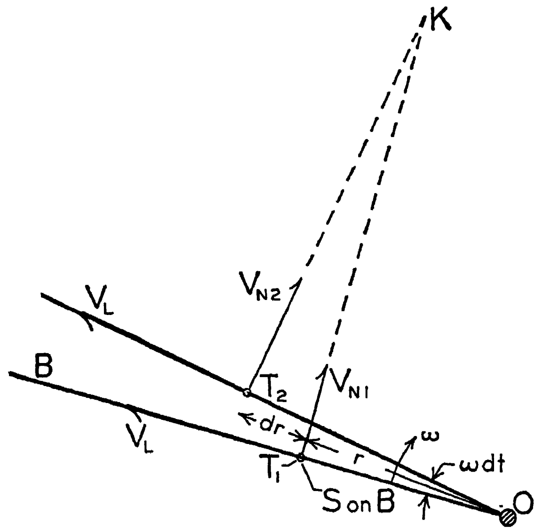


FIG. 7–11. Proof of Coriolis' Component.

After a differential rotation of B, in time dt, the bead is at T_2. V_{N2} differs from V_{N1} in two particulars. Its magnitude is increased due to the increased radius OT_2, and its direction is changed by the angle ωdt. From these two causes, two accelerations towards K result:

first, $$\frac{(r + dr)\omega - r\omega}{dt} = \frac{dr\omega}{dt} = V_L\,\omega \tag{15}$$

second,
$$\frac{V_L^2}{T_2K} = \frac{V_L^2}{\frac{dr}{\omega dt}} = V_L\,\omega \tag{16}$$

The development of (16) depends on the fact that the two triangles are similar by construction, so
$$\frac{T_2K}{r} = \frac{dr}{r\omega dt} \tag{17}$$

Further, the acceleration due to change of direction of V_N and V_L is the same as if T were moving on a circle about K for the instant.

Coriolis' component is the sum of (15) and (16), and, using the usual symbols, is expressed as
$$T_{SN} = 2V_L\,\omega \tag{18}$$

V_L is the velocity of sliding of the point on the revolving path, and ω is the angular velocity of the path.

Rule for the sense of the Coriolis Component: The sense of the Coriolis component is toward the direction in which the angular velocity of the path tends to turn the head of the sliding-velocity vector.

In this development, V_L and ω were both assumed to be constant. If ω changes, that is if α_B is not zero, S will have a component of acceleration normal to B. This will affect the absolute acceleration of T, but will not change the Coriolis component which is T_{SN}. If the sliding velocity V_L is variable, there is a tangential component of acceleration T_{ST}, but again T_{SN} is not affected. The value of T_{SN} depends only on the attained values of V_L and ω.

Assuming that O has zero absolute acceleration, the absolute acceleration of T for the general case where V_L and ω vary is given by the vector sum
$$T_O = r\omega^2 \mathrel{+\!\!\!\rightarrow} \frac{dV_L}{dt} \mathrel{+\!\!\!\rightarrow} r\alpha_B \mathrel{+\!\!\!\rightarrow} 2V_L\,\omega \tag{19}$$

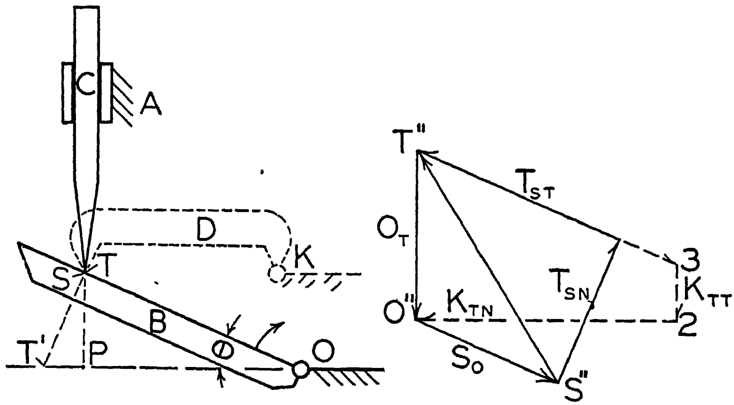
This is known as the **Coriolis law.**

The phorograph offers a convenient solution for both magnitude and sense of the Coriolis component. Since $V_L = (T'S)\omega = (T'T)\omega$, it follows from (18) that
$$T_{SN} = 2(T'T)\omega^2 \tag{20}$$

The Coriolis component is twice the distance from the phorograph of the sliding point taken on the rotating link, to the point itself, multiplied by the square of the angular velocity of the rotating link.

Phorograph Rule for Sense: The direction and sense of Coriolis' component is from the phorograph of the sliding point taken on the link carrying the rotating path, to the point itself.

Let us examine the evidence supporting the rules given for sense. The mechanism of Fig. 7–12 is convenient for this purpose. As B turns clockwise through the position shown, T is accelerated upward. In fact, assuming B infinitely long, $V^{-}T$ would become infinite before B reached vertical position. Both T_{ST} and T_{SN} must become infinitely large, and the sense of T_{SN} is manifestly from T' to T. Now consider B to turn counterclockwise through the position shown. $V^{-}T$ is being reduced.



(a) (b)

FIG. 7–12. Example Involving Coriolis' Component.

The acceleration is still upward, so its sense is properly represented as T' to T. When B has turned through the angle θ, and its contact side goes through horizontal position, $V^{-}T$ passes through its minimum value (ω_B constant), and T_O is reversed becoming downward. This is indicated by T' emerging on the upper side of T.

Consider the first rule given for sense. While T travels down toward P, the sliding-velocity vector V_L is toward O, and ω_B tends to turn the head of this vector upward. When T passes below P, the head of vector V_L is toward the left, and ω_B now tends to turn the head of the vector downward.

In Fig. 7–12 (b) is given the complete acceleration diagram (full-lined) corresponding to the full-lined mechanism at (a). $S_O = (SO)\omega_B^2$ giving the vector $O''S''$. $T_{SN} \doteq 2(T'T)\omega_B^2$. This is the Coriolis component and has the sense T' to T. There remain two vectors to close the diagram, T_{ST} parallel to the direction-controlling surface of B, and O_T which must be vertical. The complete diagram is $O''S''T''$ from which all relative accelerations can be read.

We shall now substitute for C another link D, having the same driving contact at T, but rotating about the fixed center K. The location of T'

is unchanged, therefore $V^{-}T_S$ is unchanged and T_{SN} is unchanged. Two new vectors appear, K_{TN} and K_{TT}. $K_{TN} = (TK)\omega_D^2$ and is plotted from 2 to O''. The diagram must close with K_{TT} vertical, and T_{ST} of known direction. The effect is to reverse T_{ST}, and the new position of T'' is at 3, giving O'' 3 as the new absolute acceleration of T.

A Coriolis component of acceleration will be encountered in a mechanism when, and only when, a point of one link moves along a rotating path on another link. Such relative motion can be produced by sliders (crossheads) and rollers as well as by direct contact.

7–12. **Acceleration from Rolling Contact—Equivalent Mechanisms.**—Fig. 7–13 shows a wheel or disk revolving about a fixed center and in contact with a stationary point R, past which it slides. This is not rolling contact, but the relative acceleration of the contact points is important and throws light on the rolling-contact problem.

Since R and O are fixed and have no relative acceleration, P_O must be equal and of opposite sense to R_P or $P_O \mathrel{+\!\!\rightarrow} R_P = R_O = 0$. If ω_B is uniform, $P_O = P_{ON} = (PO)\omega_B{}^2$, in direction P to O. Therefore

$$R_{PN} = (PO)\omega_B{}^2 \text{ in direction } O \text{ to } P \tag{21}$$

In Fig. 7–14, B has pure rolling contact (no sliding) on A. First consider ω_B uniform. $V^{-}O_R$ is zero vertically and constant horizontally,

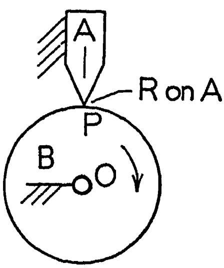


FIG. 7–13. Sliding.

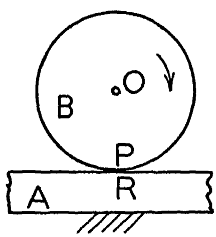


FIG. 7–14. Rolling.

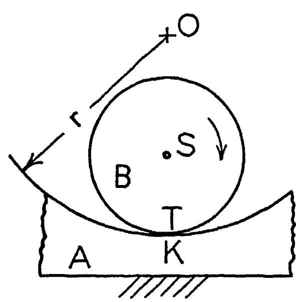


FIG. 7–15. Rolling.

therefore O_R is zero. $P_{ON} = (PO)\omega_B{}^2$ in direction P to O; consequently $P_R (= P_{RN})$ must have the same value and direction.

Next suppose there is an α_B in clockwise sense. Its effect will be to give to O a component acceleration toward the right relative to both P and R, but no change in P_R will result.

In Fig. 7–15, B has pure rolling on a cylindrical surface of A, having center of curvature at O. For the case of ω_B constant,

$$S_O + T_S + K_T = K_O = 0 \tag{22}$$

all vectors being vertical. Therefore

$$S_O + T_S = -K_T = T_K \tag{23}$$

$$(SO)\omega_{SO}^2 + (TS)\omega_B^2 = T_K \tag{24}$$

The two vectors are of like sign (upward) in this case. If B rolled on the outside of a cylinder, S_O would have opposite sign from T_S. As before, the presence of an α_B would only introduce a horizontal component into S_T and S_K, leaving T_K unchanged.

A roller is shown in contact with the plane surface of a link having translation in Fig. 7–16 (*a*). If link B and roller C were one body, as suggested by the broken lines, the relative motion and therefore the

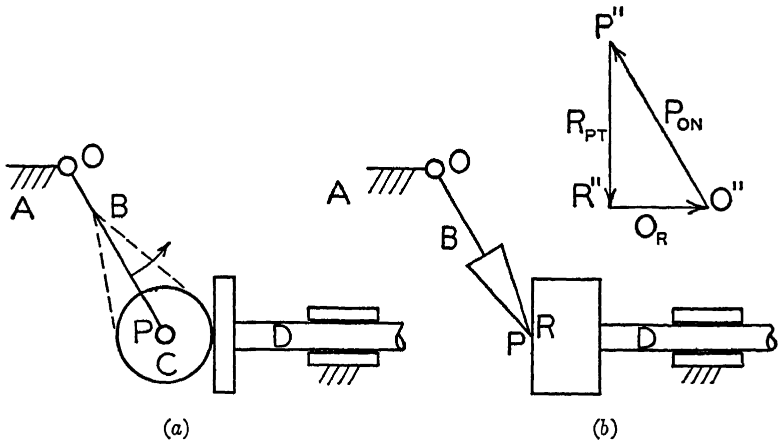


FIG. 7–16. Roller on Plane Surface of Translated Link.

accelerations would be unchanged. The most direct solution employs the **equivalent mechanism** shown at (*b*). This will be called the **slider-plane** mechanism since it was demonstrated in § 7–11 that the point-plane transmission is identical to the slider-plane as far as relative motions of the principal links are concerned.

At (*b*) the link D is shown extended so that its plane contact surface passes through P which is the center of the roller, or the center of curvature of the contact surface of B, if there is no rolling action.

Assuming ω_B constant, $P_O = P_{ON} = (PO)\omega_B^2$ which is plotted as $O''P''$. R_{PN} is zero since their horizontal relative motion is zero. This leaves R_{PT} and O_R to close the diagram.

Fig. 7–17 shows a roller in contact with a curved surface of a translated link. P and R must remain a fixed distance apart equal to the sum of the two radii just as if they were connected by a rod. This suggests the

slider-crank as an equivalent mechanism. It happens that the obvious approach gives the results with least work. The equivalent mechanism and the solution are shown at (*b*).

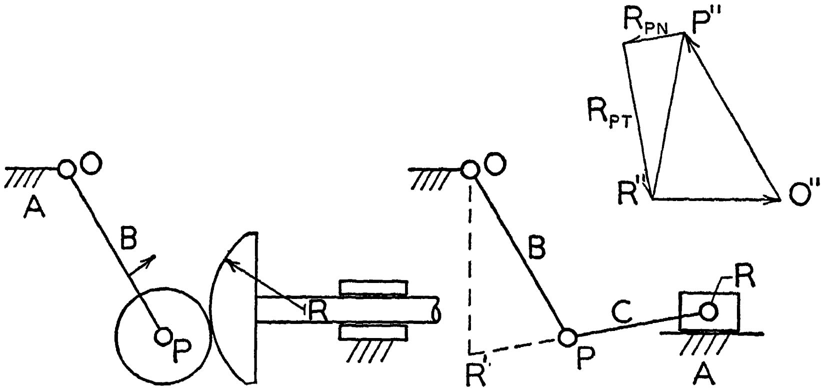


FIG. 7–17. Roller on Curved Surface of Translated Link.

In Fig. 7–18 (*a*), transmission occurs between a roller and the plane surface of a rotating link. The case resembles that of Fig. 7–16 and the slider-plane equivalent mechanism will be used. A point moving on a rotating path, however, is the signal of an approaching Coriolis component.

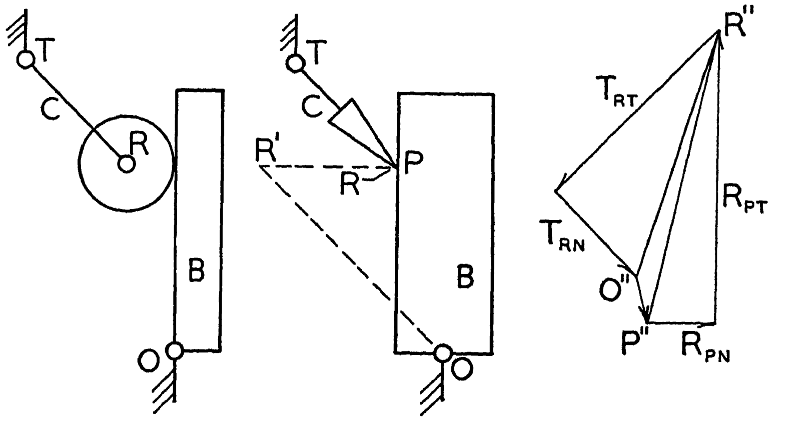


FIG. 7–18. Roller on Plane Surface of Rotating Link.

Assuming ω_B constant, $P_O = (PO)\omega_B^2$ which is plotted as $O''P''$ in Fig. 7–18 (*c*). $R_{PN} = 2(R'R)\omega_B^2$ in sense R' to R. Next, confronted by unknowns, we pass to one of the end components that yields to computa-

tion. $T_{RN} = C\omega_C^2$ and is plotted towards O''. The remaining two vectors of known direction, R_{PT} and T_{RT}, locate R'' and the final diagram appears as $O''P''R''$. Suppose the angular acceleration of C is required. $\alpha_C = T_{RT}/C$ and is counterclockwise.

The case of a roller in contact with a curved surface on a rotating link, Fig. 7–19 (a), is also easily solved by the use of an equivalent mechanism. The center of curvature of the active surface of B is S, and S moves as a point of B. The center of the roller R must move as if it were connected to S by a link. Therefore the equivalent mechanism is the crossed-

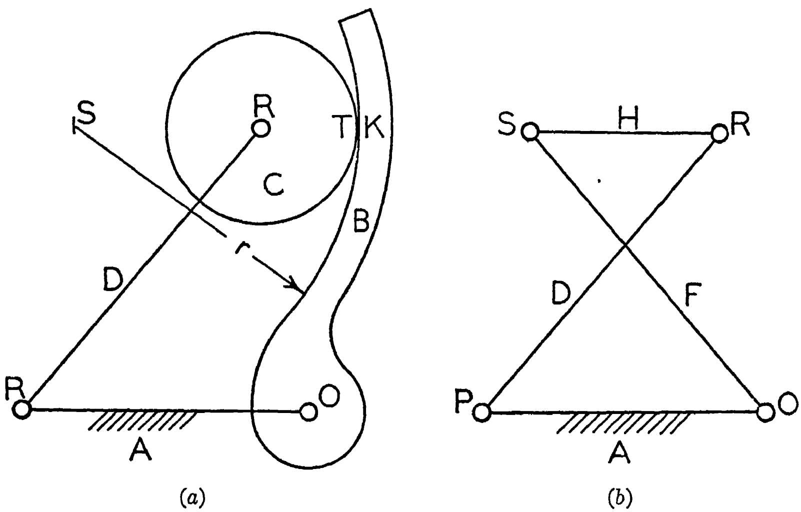
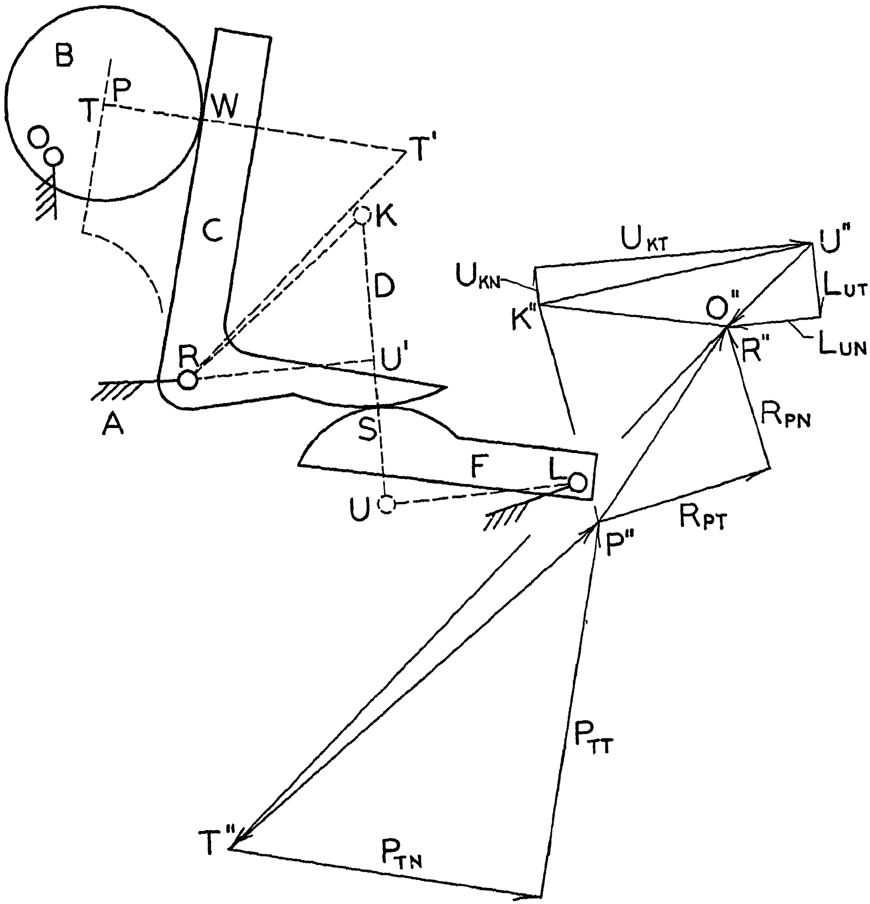


FIG. 7–19. Roller on Curved Rotating Link.

quadric or Tchebicheff mechanism shown at (b), giving the same relative motions to R and S as they receive at (a). The acceleration diagram for (b) can be produced as for the open quadric chain which is solved in § 7–4. This method cannot be used if the active surface of B is plane, Fig. 7–18 (a), for then S is at infinity.

7–13. **Acceleration from Direct Contact or Cam Transmission.**—All direct contact transmission can be classified under the general heading of cams which will be treated in Chapter VIII. Direct contact is closely related to rolling contact, because, as already demonstrated, it makes no difference to the motions of the principal links whether the contact is through a roller or through a surface of the principal link, provided the contacting surface has the same curvature as the roller. Hence the methods of § 7–12 can be applied directly to transmission by cams. An example follows.

In Fig. 7–20, an eccentric link B, rotates about O causing oscillation of the bell crank C which in turn drives F. The two most common cases of direct-contact or cam transmission are here illustrated, curved on plane surfaces at W, curved on curved surfaces at S.



Space scale, 1 in. = 2 in.
Acceleration scale, 1 in. = 20 in. per sec².

FIG. 7–20. Acceleration through Cam Surfaces.

All three curved surfaces have circular arc outlines. No matter how irregular the outlines may be, however, the contact for any phase will be at points or lines on the surfaces where, for each surface, the curvature can have only a single value. On these definite radii of curvature for each phase, the velocity and acceleration relations for the phase de-

pend. Therefore, Fig. 7–20 illustrates the general case of direct contact transmission.

For the B-to-C contact, we substitute a slider-plane contact at the center of curvature of the surface on B. Without further construction one can visualize the broken line as the new surface on C, also T as a slider or contact point. T is on B, and P on C.

For transmission through S, where both C and F present curved surfaces, the quadric equivalent mechanism is $RKUL$, K and U being the centers of curvature for the phase shown.

With ω_B given as constant at 10 rad per sec, $T_O = T_{ON} = 0.75 \times 10^2 = 75$. This is plotted from O'' to T''. Choosing C as the phorograph link,

$$V\text{-}T = (OT)\omega_B = (RT')\omega_C$$

or

$$\omega_C = 10 \times \frac{0.75}{3.31} = 2.265$$

The Coriolis component, as derived, would be $T_{PN} = 2(T'T)\omega_C^2$ in direction T' to T. Here we are solving the first loop in the order $OTPRO$ and require P_{TN} which will have opposite sense.

$$P_{TN} = 2 \times 3.19 \times 2.265^2 = 32.6$$

The final vector, $R_{PN} = (RP)\omega_C^2 = 3 \times 2.265^2 = 15.4$, and is plotted towards O''. The remaining two vectors for this loop, P_{TT} and R_{PT} are of known direction, so this loop is closed and P'' located.

K is on C, and K'' is located by using the acceleration image of link C. R_{PT} shows that α_C is counterclockwise. Turning the $\triangle RPK$ in this sense and reducing its size until RP fits $R''P''$ will locate K''. (Any proportionate reduction of a triangle can be easily obtained by a line parallel to one side.)

For the solution of loop $RKULR$,

$$\omega_D = \omega_C(KU')/(KU) = 2.265 \times 1.5/3 = 1.13$$
$$\omega_F = \omega_C(RU')/(LU) = 2.265 \times 1.96/2 = 2.22$$

The computed vectors are,

$$U_{KN} = D\omega_D^2 = 3 \times 1.13^2 = 3.83$$

and

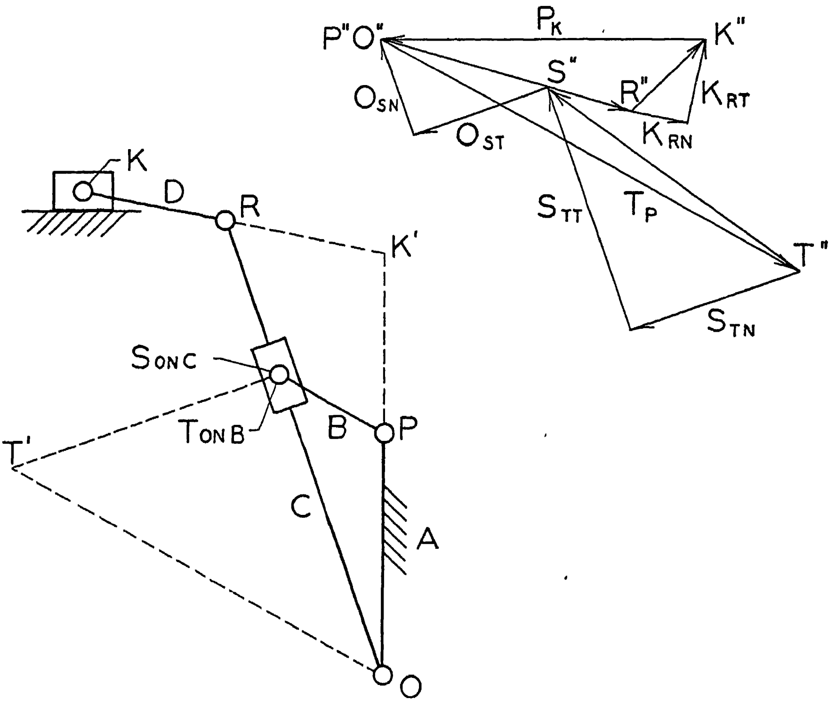
$$L_{UN} = F\omega_F^2 = 2 \times 2.22^2 = 9.86$$

With these the diagram is completed locating U'' and making it possible to answer quickly any question on the acceleration of the mechanism for this phase. $\alpha_F = L_{UT}/(LU) = 7.8/2 = 3.9$ rad per sec² and is clockwise. $\alpha_C = R_{PT}/RP = 18.7/3 = 6.2$ and is counterclockwise. If the accelerations of contact points or of other points on any link are required,

they can be found by the image method or by constructing the two component vectors for each point.

Further practice on cam accelerations will be given in Chapter VIII, but the methods so far given, while by no means inclusive of those available, will be found adequate for all plane-motion mechanisms.

7–14. **Accelerations on the Oscillating-Beam Mechanism.**—This solution, Fig. 7–21, illustrates the case of a compound mechanism with two



Space scale, 1 in. = 1 ft. Acceleration scale, 1 in. = 1 ft per sec².

FIG. 7–21.

sliding pairs, one of which yields a Coriolis component. The driving crank B rotates at the constant speed of 2 rad per sec and is $7\frac{1}{2}$ in. long. The oscillating beam C is 5 ft, and D is 18 in. The loop $PTSOP$ must be solved first. The foot will be used as the unit of length. All phorographs are taken on C.

$$V\text{-}T = B\omega_B = V\text{-}T' = (OT')\omega_C$$

or

$$\omega_C = \omega_B \frac{B}{(OT')} = 2 \times \frac{0.625}{2.21} = 0.566$$

$$V\text{-}K_R = D\omega_D = (K'R)\omega_C$$

or

$$\omega_D = 0.566 \times \frac{0.85}{0.75} = 0.64$$

$$T_{PN} = B\omega_B^2 = 0.625 \times 4 = 2.5 \quad \text{giving} \quad P''T''$$

on the acceleration diagram.

S_{TN} is the reverse of T_{SN}, the latter being the Coriolis component.

$$T_{SN} = 2(T'T)\omega_C^2 = 2 \times 1.46 \times 0.320 = 0.935$$

$S_{TN} = 0.935$ in direction T to T' and is so plotted from T''. Now we must pass to the final component for this loop, O_{SN}, which will be plotted to terminate at O''.

$$O_{SN} = (OS)\omega_C^2 = 1.66 \times 0.32 = 0.531$$

Closing this loop with T_{ST} and S_{OT} of known directions locates S''. Then R'' is located, using the acceleration image of C, by the proportion $(O''R'')/(O''S'') = (OR)/(OS)$. This is the first step on the second loop $PSRKO$.

$$K_{RN} = D\omega_D^2 = 0.75 \times 0.64^2 = 0.307$$

K_{RT} and P_K have known directions and complete the diagram as shown. K_P, the reverse of P_K, scales 1.69 ft per sec² and is towards the right. $\alpha_C = O_{ST}/(OS) = 0.73/1.66 = 0.44$ rad per sec², and $\alpha_D = K_{RT}/D = 0.44/0.75 = 0.59$. The total acceleration of K relative to T is given by the vector $T''K''$ with arrow at K''. It scales 1.3 ft per sec².

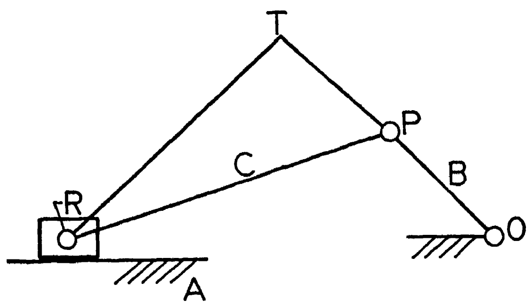
PROBLEMS

1. On what does normal acceleration depend? On what does tangential acceleration depend? Are these components independent?
2. A pulley 3 ft in diameter is having its speed reduced at the constant rate of 180,000 rpm per min, and, at the instant considered, is revolving clockwise at 600 rpm. What is the total acceleration of the point P, at the right end of a horizontal diameter, relative to the center O? What is the total acceleration of P relative to the top point? Draw an acceleration diagram for the three points showing all components.
3. Draw a quadric chain, Fig. 7–4 (*a*), making $A = 3$ in., $B = 1\frac{1}{2}$, C and D each $= 1\frac{1}{4}$, and the angle $POS = 40°$. Make $\omega_B = 10$ rad per sec and $\alpha_B = 60$ rad per sec² and draw to scale the complete acceleration diagram. What is the total acceleration of R with respect to the fixed link? What is the total acceleration of the midpoint of D with respect to P?

   Ans. 400 in. per sec²
   90 in. per sec².

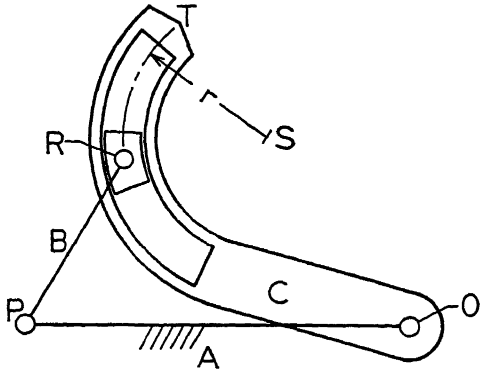
4. Using the data of the slider-crank mechanism of § 7–5 where the angle *POR* is 30°, draw the acceleration diagram for the phase with the crank 180° from the position shown and find: (*a*) R_O, (*b*) the absolute acceleration of the mid-point of the connecting rod, (*c*) the angular acceleration of the connecting rod.
5. Check your result for Prob. 4 (*a*) using the exact and approximate analytical expressions of § 7–6.
6. A propeller blade 4 ft long measured from shaft center line to tip is being accelerated clockwise at 400 revolutions per sec². When its speed has reached 600 rpm, the blade points vertically upward.
   (*a*) Compute the angle of its acceleration image.
   (*b*) Compute the length of the image for an acceleration scale of 1 in. = 400 ft per sec².
   (*c*) Find the total acceleration, relative to the fuselage, of the point where the blade enters the spinner, 8 in. from the axis.

   Ans. (*a*) 90° + 57.5°, (*b*) 4.86 in., (*c*) 312 ft per sec².
7. Leaving plenty of room at the top and right side, construct the acceleration diagram for this slider-crank mechanism for ω_B constant at 100 rad per sec. In inches, $B = 1\frac{1}{2}$, $C = 3\frac{1}{2}$, $PT = 1\frac{1}{2}$, $TR = 3$, angle $POR = 45°$.

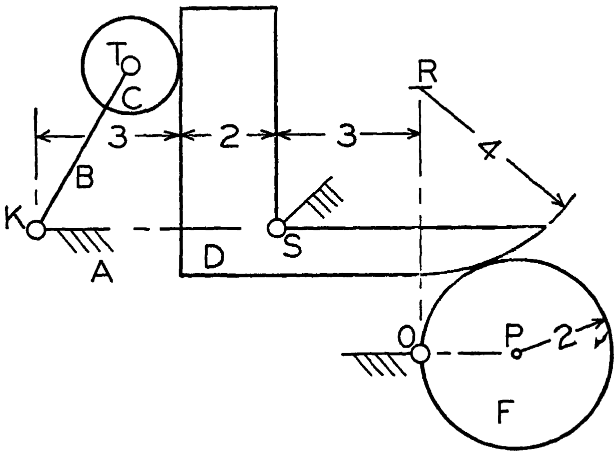


   (*a*) Find the absolute acceleration of *T*.
   (*b*) Find the acceleration of the centro *ac* for this phase, if it adhered to link *C*.
   (*c*) Locate the point on *C* that has zero acceleration.
8. Using the slider-crank mechanism of Prob. 7, dropping *T*, find the acceleration of the piston for the head-end and crank-end dead-center phases. Why are the values different?
9. Draw the grasshopper mechanism, Fig. 4–16, with *D* vertical and 2 in. long, *B* at 30° with the horizontal and 2 in. long. *C* is 4 in. and is connected to *B* at its midpoint. If centro *cd* is moving towards the right at the constant speed of 60 in. per sec, what is the acceleration of *P*? Ans. 7260 in. per sec².

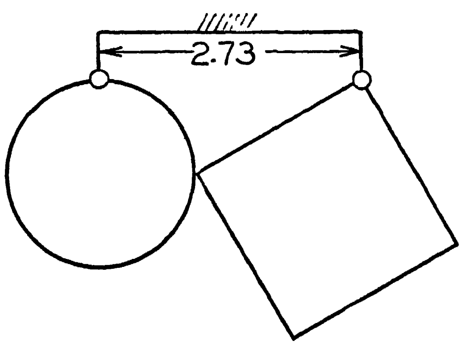
10. Make $A = 4$ in., $B = 2$, $r = 1\frac{1}{2}$, $OS = 2\frac{1}{2}$ and $\angle RPO = 60°$. Find the absolute acceleration of T which is in line with O and S, if ω_B is constant at 10 rad per sec.



11. Link F is circular and rotates counterclockwise about O at the constant speed of 6 rad per sec. B is 4 in., and roller C is 2 in. in diameter. SR is 4.25 in. Find the angular acceleration of B for the phase shown.



12. The 2 in. square, rotating clockwise at the constant speed of 10 rad per sec, drives the 2 in. circular disk. Find the angular acceleration of the disc, (a) for the phase shown, the center of the circle being vertically downward, (b) after the square has rotated through 30°.



13. Assume that the mechanism of Prob. 19, Chap. II, is driven by the crank B which, in the phase shown, has an angular acceleration of 40 rad per sec^2 and an angular velocity of 10 rad per sec, both clockwise. Find the acceleration of T.
14. In the Whitworth quick-return mechanism of Fig. 6–5 make $PO = 1\frac{1}{2}$ ft, $B = 3$ ft, $C = 6$ ft. With ω_B constant at 4 rad per sec, find the acceleration of T when B is 30° from the horizontal.

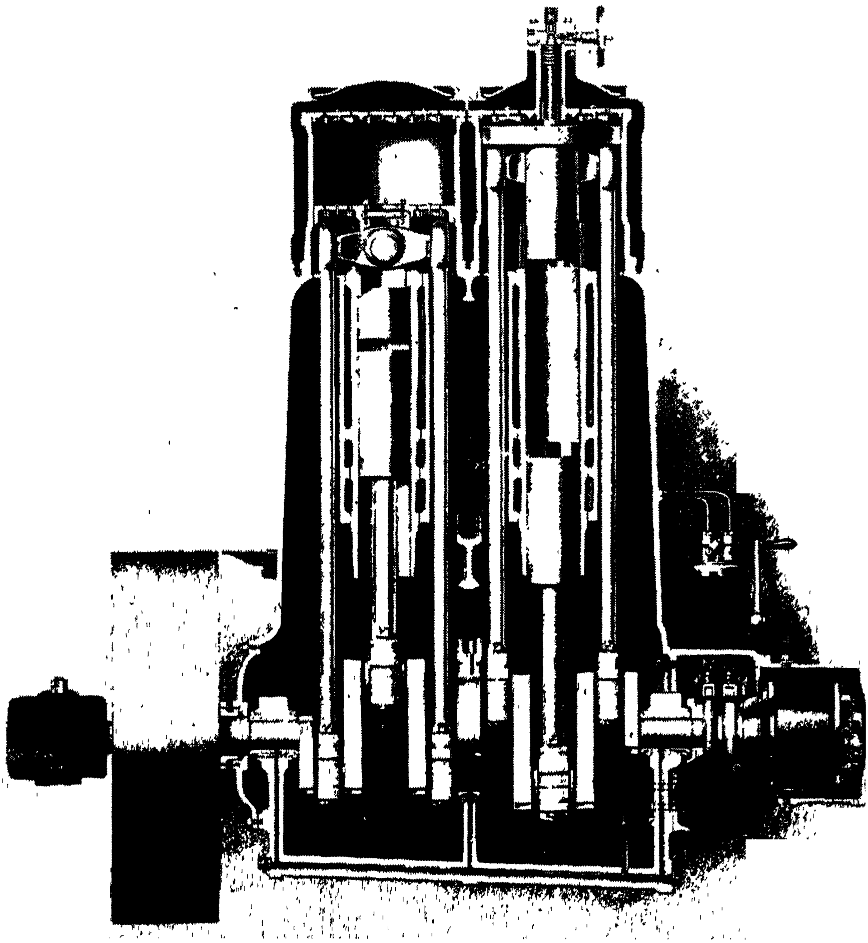


Fig. Prob. 16. Sun-Doxford Engine.
Sun Shipbuilding and Dry Dock Co., Chester, Penna.

15. On the radial engine of Fig. 7–10 find the acceleration of the two pistons, *R* and *K*, when the crank *B* is passing through the left horizontal position. Use the specifications given in § 7–9.
16. In this two-cycle opposed-piston compression-ignition engine, consider that the two cranks for one upper piston are 180° from the crank for the lower piston of the same cylinder. Each upper piston has a stroke of 50 in. and their connecting rods are 200 in. long. Each lower piston has a stroke of 44 in. and their connecting rods are 90 in. long. The operating speed is 92 rpm. On the same base of crank degrees, draw acceleration curves of the upper and lower pistons of one cylinder for a cycle. To what extent is it possible for the reciprocating masses only of one cylinder to balance each other as to accelerating forces? (Cylinder bores are 32 in.)
17. Construct the velocity and acceleration diagrams for the reciprocating parts of a horizontal slider-crank engine to run at 360 rpm. The stroke is 2 ft, and the connecting rod is 4 ft long. Use a space scale of 3 in. = 1 ft, a velocity scale such that the length of the crank represents the velocity of the crank pin, and an acceleration scale of 1 in. = 400 ft per sec^2. Draw the curves both on a travel base and on a time base. Call acceleration toward the right positive.

CHAPTER VIII

CAMS

8-1. **Classification.**—A cam is a device for transmitting motion to a follower by direct contact, including the case where a roller is interposed to reduce wear and friction. This is a very broad definition, including, as it does, gears of all kinds. A single gear tooth is properly considered a cam, but gears, having many teeth, constitute such an important class of direct-contact transmission equipment as to receive an entirely separate classification.

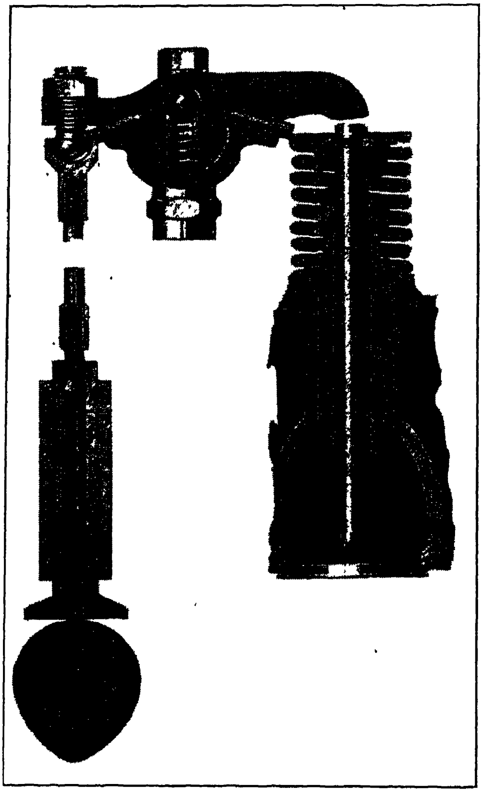


Fig. 8-1. Automobile Disk Cam.
The Nash Motors Co., Kenosha, Wis.

Cams are conveniently classified according to the characteristics of the driver as

(1) *Disk, plate,* or *radial,* in which the required motion of the follower results from a changing radius on the driver, Fig. 8-2 *a* and *b*. Automobile valve cams are regularly of this type. Fig. 8-1 illustrates a typical design.

(2) *Cylindrical cams,* in which the follower moves parallel to the axis of the driver as in Fig. 8-16 and Fig. 8-2 *d*. Fig. 8-2 *c* is also called an *edge cam.*

Based on the characteristics of the follower there are

(3) *V-edge followers,* such that the same contact surface on the follower is in action throughout the cycle, Fig. 8-3. These are used only where small forces are to be transmitted. They are not easy to lubricate properly and are subject to rapid wear.

(4) *Roller followers,* Fig. 8–5.

(5) *Flat-foot followers.* If circular, these are often called mushroom followers. This is the commonest type for automotive use, Fig. 8–1. They are cheaper than the roller variety, and, if properly designed and sufficiently lubricated, wear well.

(6) Followers with *curved-contact surfaces,* Fig. 8–23.

Again changing the basis of classification,

(7) *Positive-return cams* are those that do not depend on gravity or springs to give the follower its return stroke. Having the roller of the follower ride in a groove either in the side of a disk cam or on the face

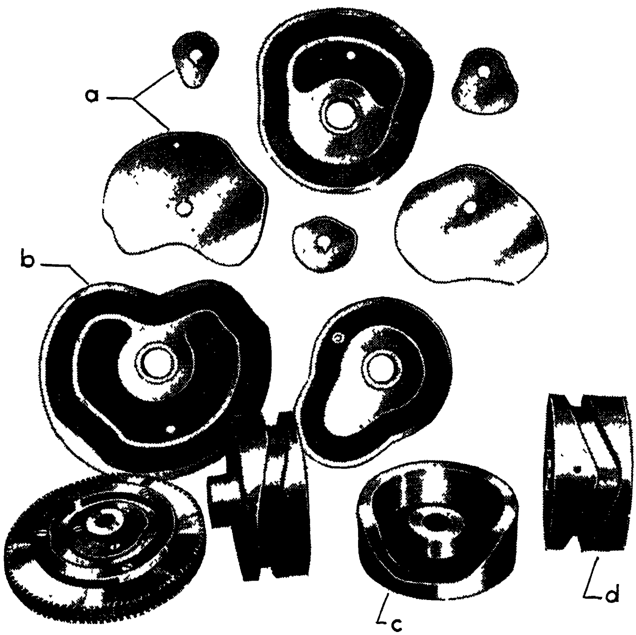


FIG. 8–2. Types of Cams.

The Rowbottom Machine Co., Waterbury, Conn.

of a cylindrical cam makes the action positive. *Box cam* is a general term for all positive-return cams that are not cylindrical. An example is Fig. 8–2 *b* which is also called a *face cam.*

There are many other special types having minor applications and the nomenclature is not well standardized.

8–2. Methods for Finding Pitch Curves and Profiles for Radial Cams with Various Types of Follower.—The *pitch curve* of a cam is the trace on the cam of the reference point of the follower. The *reference point* is

the point chosen to represent the motion of the follower. It is point 1 in Figs. 8–3, 8–4, and 8–5. The pitch curve is the cam profile only in case of point or line contact as in Fig. 8–3. The *base circle* of a radial cam is the circle with center on the cam axis that is tangent to the innermost part of the cam profile.

The V-edge follower, Fig. 8–3, is to be driven by a radial cam such that, starting in its lowest position, when the cam is in the phase shown, the follower will rise vertically from 1 to 2 while the cam turns counterclockwise 30°, from 2 to 3 during the next 30° of cam motion, and from 3 to 4 during the third 30°. It is required to find the profile of this part of the cam.

When the cam has turned through the first 30°, a point on radius Oc_2 must be in contact with the follower at point 2; hence Oc_2 must be made equal to $O2$. Similarly, radius $Oc_3 = O3$, and $Oc_4 = O4$. Then, for the cam profile, a smooth curve drawn through 1, c_2, c_3, and c_4 meets the specifications so far as given. The overall dimensions of the cam will depend on the size of the base circle. It is generally desirable to keep cams as small as practicable. For a specified travel of follower, however, this reduction is limited by two considerations. First, the smaller the cam, the smaller the length of periphery subtended by a given angle, and therefore, the steeper the cam, with corresponding increase of side pressure and friction. Indeed, a cam could be designed so steep that it would lock. The second consideration is that the cam shaft will have a certain minimum size for strength and stiffness, and the base circle must generally be larger than the shaft. The design for limiting pressure angle will be considered in § 8–6 and § 8–8.

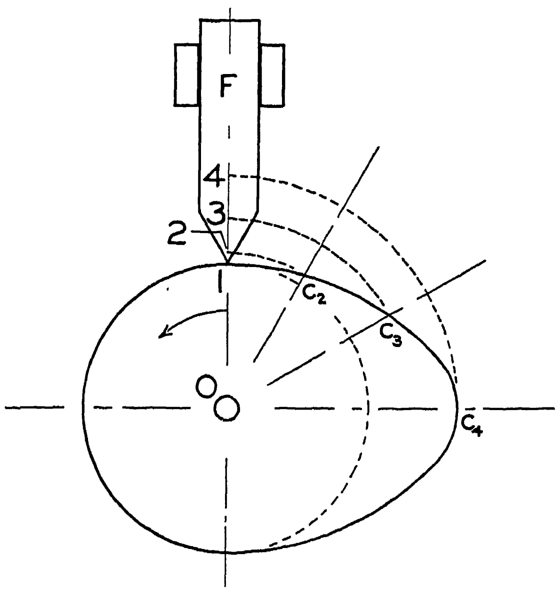


FIG. 8–3. V-Edge Follower Cam.

The method of obtaining the profile when a flat foot follower is used is shown in Fig. 8–4. Assuming specifications similar to those for the

previous case, the follower must rise from point 1 to 2, 3, and 4 while the cam turns clockwise through 30°, 60° and 90° respectively. Choosing the third point for illustration, an arc is drawn about O through 3, the new position of the reference point, to intersect the 60° radius Od_3 in d_3. Now d_3 is a point on the pitch curve, but the cam profile must make contact at some point c_3 depending on the rate of rise of the follower. The cam profile is obtained by drawing a smooth curve tangent to all such positions of the foot. In fact, a large number of such foot positions would generate the cam profile as an envelope.

The length of foot must be sufficient to give all contacts with the cam on its flat surface and never on its corner. The necessary minimum length will be found near where the follower is rising or falling at the highest velocity with respect to the angular speed of the cam. In this example the critical phase is close to 60°, and the required length of foot, about 5/8 in. It is given by a few trial tangents.

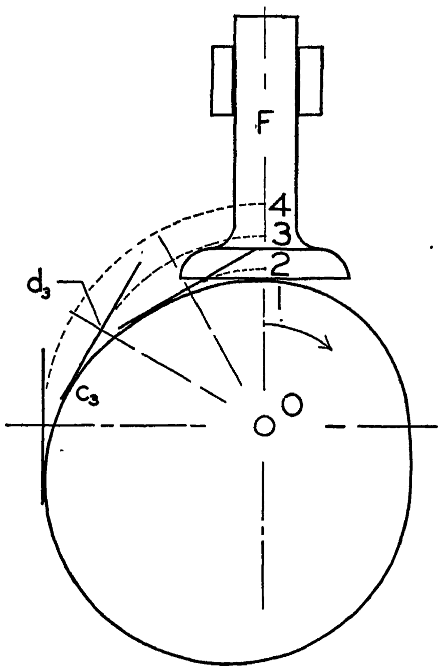


FIG. 8-4. Mushroom Follower Cam.

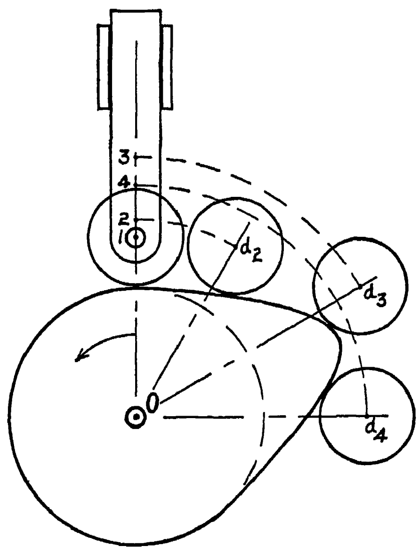


FIG. 8-5. Cam with Roller Follower.

It will be realized that accurate determination of any cam profile requires accurate thin-line pencil work. If the active surface of the follower is inclined or curved the above method is still applicable.

The case of a follower making contact through a roller requires a slightly different solution, the principle of which is illustrated in Fig. 8–5. The center of the roller rather than the contact point must be used as the reference point to measure the position of the follower. Suppose that the center of the roller is specified to be at points 1, 2, 3, and 4 when the cam has turned through 0°, 30°, 60°, and 90° respectively, counterclockwise. An arc is struck from 2, about O, to intersect the 30° radius in d_2. The cam profile must be tangent to the roller in this position. Note that the contact point is not on the radius Od_2, but considerably to the right of it. Hence the distance from O, on the radius Od_2, to the cam surface is not $O2$ minus the radius of the roller, and can not be so obtained. A fair number of roller positions will make it easy to draw the required tangent cam profile with accuracy.

8–3. **Desirable Follower Motions—Harmonic Cams.**—In very slow speed machinery, if it is desired to move a part from one position to another some time during the cycle, and if there are no conditions limiting this motion to a small part of the cycle, no great care is necessary in arriving at a satisfactory cam profile. Avoiding distinct bumps, almost any outline, with the required maximum rise, will serve. However, with cams to be operated at medium or high speed, the most careful theoretical design is essential. Any guesswork as to accelerations and velocities will almost certainly lead to trouble.

Cam followers are often designed to have harmonic motion. Such cams are easy to design and have fairly good characteristics. In fact, the idea is quite prevalent that it is the best possible shape, but that is incorrect. In practice, harmonic cams are generally called **crank cams.**

Harmonic motion is commonly defined as the motion of a mass accelerated toward a central position by a force directly proportional to the distance of the mass from that central position. The Scotch yoke will be recalled as a mechanism by which harmonic motion can be produced from uniform rotation. *Harmonic motion* is also the motion of the projection on the diameter of a circle, of a point moving at uniform velocity on the circumference, Fig. 8–6. This latter concept is a useful one in cam design, and it is also convenient to use one end of the travel as origin rather than the central position, i.e., point 1 or 7 rather than 4.

Suppose it is desired to design for a specified travel of follower by harmonic motion. The travel is laid off to scale as the distance 1 to 7, Fig. 8–6. A semi-circle is described on the travel as diameter and the semi-circumference is divided into a number of *equal* parts, depending on the number of points it is desired to plot. In this case six divisions are used and projecting b_2, b_3, etc., on the diameter yields the plotting points, 2, 3, etc., on the travel.

Now suppose it is desired that this rise take place during 120° of cam rotation. There would be a plotting point for each 20°, and, assuming that the cam rotates at uniform angular velocity, harmonic motion of the follower would result. The return of the follower might be also harmonic or a different specification might govern.

There are two distinct angular displacements involved here. 180° of motion of the radius arm in the harmonic circle takes place while the cam moves through 120°. Confusion will be avoided by calling the former *harmonic degrees* and the latter *cam degrees*. The relation between the two depends entirely on the cam specifications.

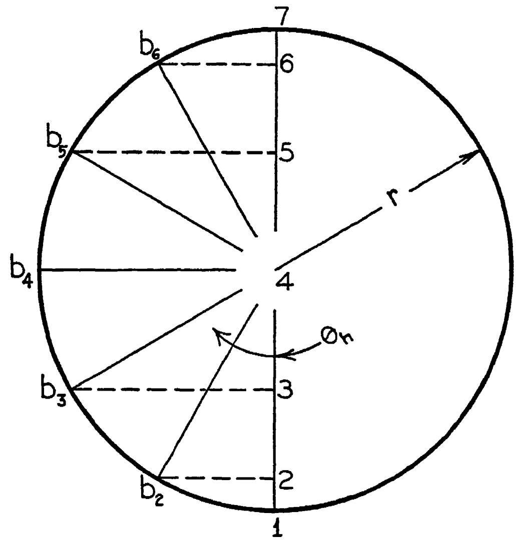


FIG. 8–6. Harmonic-Motion Circle.

The distance s of the follower from its lowest position 1 can be expressed in terms of the angle θ_h and the radius r of the harmonic circle, Fig. 8–6. Note that r must equal half the harmonic travel from rest to rest.

$$s = r - r\cos\theta_h \tag{1}$$

If V is the linear velocity of the follower and ω_h the angular velocity of the harmonic radius which must be constant,

$$V = \frac{ds}{dt} = \frac{d}{dt}(r - r\cos\theta_h) = r\sin\theta_h\frac{d\theta_h}{dt}$$
$$= \omega_h r\sin\theta_h \tag{2}$$

The acceleration,

$$a = \frac{dV}{dt} = \omega_h^2 r\cos\theta_h \tag{3}$$

If r is in in., and ω_h in rad per sec, V will be in in. per sec, and a in in. per sec^2.

Applying these relations to the above case where a complete harmonic rise of the follower takes place during 120° of cam rotation, we shall assume that the follower rises 6 in. and that the cam has a constant

speed of 90 rpm, or 120° in 2/9 sec. In the same time the harmonic arm revolves 180° or π rad in 2/9 sec, from which $\omega_h = 9\pi/2$. r = half the rise or 3 in.

The maximum velocity is at mid-travel, point 4, where $\theta_h = 90°$.

$$V_m = \omega_h r \sin \theta_h = \frac{9\pi}{2} \times 3 \times 1 = 42.4 \text{ in. per sec}$$

Maximum acceleration occurs at points 1 and 7.

$$a_m = \omega_h{}^2 r \cos \theta_h = 600 \text{ in. per sec}^2$$

8–4. **Desirable Follower Motions—Parabolic Cams.**—A cam designed to drive its follower with uniform acceleration and uniform deceleration or retardation is called parabolic. It may also be called a *constant acceleration* cam. When properly designed, no other cam will produce a given motion from rest to rest in given time with so small a maximum acceleration. In this respect it is the ideal cam. It is called parabolic because the travel of the follower, plotted against time on coordinate axes, yields parabolas; although the pitch curve and profile of the cam are generally not parabolas.

If a follower is given constant acceleration a from rest, the velocity V that it acquires in time t is at. Its average velocity is half this value and the distance that it travels is $s = \frac{1}{2}at^2$. This is the familiar expression for the travel of a body falling freely from rest under the action of gravity. Applied to the parabolic cam, a can have any practical value provided it is constant.

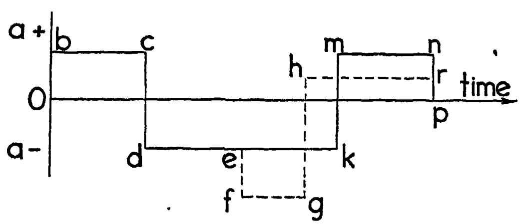


FIG. 8–7. The Ideal Acceleration Program.

Fig. 8–7 shows acceleration plotted on a time base. It can represent the program for a cycle in which a follower rises vertically from rest in a straight line, to rest at the top of its travel, and is returned to rest in its first position. There is, first, acceleration upward, *bc*, followed by acceleration downward, *de* (negative because the accelerating force is negative), while the follower is still *rising* to rest at the top. Negative acceleration *ek* brings the follower to its maximum downward velocity, and *mn* is positive to reduce this velocity to zero at the end of the cycle.

For any cam whatever, these four steps in the acceleration program are essential. If, in each step, the acceleration is uniform, it is a parabolic cam. If all acceleration ordinates are numerically equal (full-lined diagram) it is the **ideal symmetrical parabolic cam.**

Any departure from the ideal program, such as *fghr*, will cause at some time a higher acceleration and consequently a higher accelerating force. The requirements of machine performance generally do dictate such departures, but the closest approach to the symmetrical parabolic cam allowed by the specifications will generally result in the smoothest operating characteristics.

TABLE 8–1

CONSTANT ACCELERATION

Time Intervals from Rest	Total Travel $s = \frac{1}{2}at^2$	Increments of Travel
1	$\frac{1}{2}a \times 1$	$\frac{1}{2}a \times 1$
2	$\frac{1}{2}a \times 4$	$\frac{1}{2}a \times 3$
3	$\frac{1}{2}a \times 9$	$\frac{1}{2}a \times 5$
4	$\frac{1}{2}a \times 16$	$\frac{1}{2}a \times 7$
5	$\frac{1}{2}a \times 25$	$\frac{1}{2}a \times 9$

The third column of Table 8–1 shows that under constant acceleration from rest, the distances travelled in successive equal periods of time vary as the succession of the odd numbers. This arises from a remarkable property of numbers, namely that the difference between the squares of the successive numbers gives the odd numbers in succession. The relation is convenient in laying out parabolic cams.

For example, a follower is to rise 4 in. as its forward stroke, the first 2 in. under constant acceleration, the second 2 in. under constant deceleration. It is desired to plot the rise for six equal periods of time. The travels for the six periods will bear the relation, 1, 3, 5, 5, 3, 1. In the first period the travel is 1/18 of 4 in., in the second, 3/18 of 4 in., etc., since $1 + 3 + 5 + 5 + 3 + 1 = 18$.

8–5. **Characteristic Curves.**—Curves of displacement, velocity, and acceleration are helpful in studying the suitability of a proposed cam, especially if it is to be used at high speed. Sometimes it is good procedure to plot these curves first and design the cam to fit a desirable program of acceleration. To illustrate the method, a design involving both parabolic and harmonic movements will be developed.

A certain cam follower is designed for a total rise of $8\frac{1}{4}$ in. It first rises $2\frac{1}{4}$ in. under constant acceleration while the cam turns through 60°. The follower continues to rise at the attained velocity for the next 40° of cam rotation, and is brought to rest at the top of its upward stroke by a constant deceleration in the next 80°. After a dwell for 60°, the follower makes its return stroke under harmonic motion in the remaining 120° of cam rotation. The cam rotates once every 18 sec at uniform angular

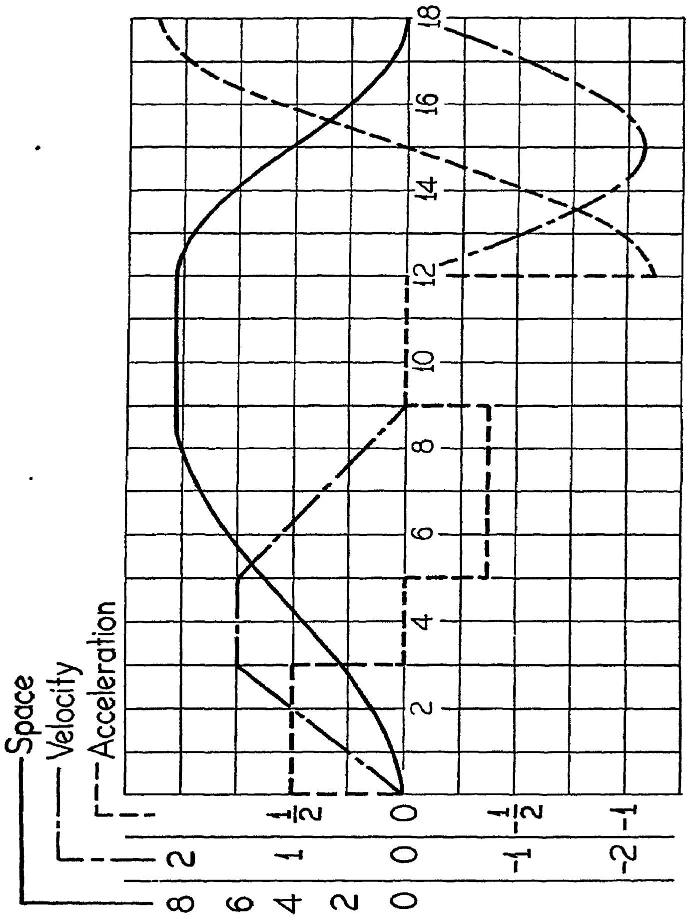


FIG. 8–8. Diagram of Cam Characteristics.

velocity. It is desired to obtain, on a time base, the characteristic curves of space or travel, velocity, and acceleration.

Since the cam revolves at constant speed, the time base can be laid off either in seconds or degrees. In Fig. 8–8, seconds are used, one second being equivalent to 20 degrees. The velocity has a linear increase for 3 sec, at the end of which time the velocity will be twice the average for the period, or $2 \times 2\frac{1}{4}/3 = 1\frac{1}{2}$ in. per sec. It remains at this value for 2 sec and then decreases uniformly to 0 in the next 4 sec. In this period the acceleration is downward and *therefore negative,* although the follower is still going upward.

The acceleration during the first three seconds is the gain in velocity divided by the time or $1\frac{1}{2}/3 = \frac{1}{2}$ in. per sec². It can also be obtained from the relation $s = \frac{1}{2}at^2$. In the next period with no change in velocity, the acceleration is zero, and in the third period the acceleration is $-1\frac{1}{2}/4 = -\frac{3}{8}$ in. per sec².

The travel curve is a parabola sloping upward for the first period, its equation being $s = \frac{1}{2}at^2$. It goes on straight, at the attained slope, for the second period and then becomes another parabola for the third period, ending tangent to the horizontal dwell line representing the highest point in the travel which is $8\frac{1}{4}$ in. above the initial position. The rise during the third period is $ut + \frac{1}{2}at^2$ where u is the initial velocity for the period, and a is negative. This completes the parabolic part of the cam covering 9 sec or 180°. For the next 3 sec, or 60°, there is a dwell, when the follower is stationary and the active cam profile is circular.

At the end of the 12th sec the follower starts downward under harmonic motion which is completed in 120 cam degrees or in 6 sec. The corresponding harmonic degrees are, of course, 180, so $\omega_h = \pi/6$. The maximum velocity occurs in the middle of this period with $\theta_h = 90°$. From (2) with $r = 8\frac{1}{4}/2$, maximum velocity

$$V_m = \omega_h r \sin \theta_h = \frac{\pi}{6} \times 4\frac{1}{8} \times 1 = 2.16 \text{ in. per sec}$$

In similar manner all necessary points for plotting the three curves can be obtained.

$$a_m = \omega_h{}^2 r \cos \theta_h = \frac{\pi^2}{36} \times 4\frac{1}{8} \times 1 = 1.13 \text{ in. per sec}^2$$

The displacement at the end of the 14th sec, for example, can best be computed by noting that the follower is 4/6 of 180 harmonic degrees from its position at the end of the cycle.

$$\begin{aligned} s_{14} = r - r \cos \theta_h &= 4\tfrac{1}{8}(1 - \cos 120) \\ &= 4\tfrac{1}{8}(1 + 0.5) = 6\tfrac{3}{16} \text{ in.} \end{aligned}$$

A construction similar to Fig. 8–6 affords a simple graphical solution for the values of s.

It should be noted that if the reference point of the follower does not travel in a straight path, the displacements must be measured *along the path* and are not then straight-line distances from the initial position.

8–6. **Pressure Angles—Parabolic Cams.**—The *pressure angle* of a cam is the angle between the direction of travel of the reference point of the follower, and the normal to the pitch curve at the reference point. It is a measure of the side pressure on the follower and its bearings. Large pressure angles cause rough running and may cause excessive wear of the cam, the roller, and the supporting bearings. Good bearings and hard wearing surfaces allow larger pressure angles to be used. In all important cam design the maximum pressure angle is a controlling specification. As will presently be seen, it dictates the size of cam necessary to meet given requirements of follower motion. Pressure angles up to 30° are generally satisfactory. Those above 35° require especially good bearings and contact surfaces.

The **pitch point** of a cam is the point on the pitch curve (defined in § 8–2) where the pressure angle is maximum.

The *pitch circle* of a radial cam is the circle with center on the cam axis, that intersects the pitch curve at the pitch point where the pressure angle is maximum.

In Fig. 8–9, q is the varying radius of the pitch curve of the cam in contact with the reference point of the follower. The velocity of this point on the cam for any phase is $q\omega$, where ω is the constant angular velocity of the cam. Assuming that the follower is under constant acceleration a upward from rest, its velocity is at. For a maximum value of the pressure angle ϕ, $\dfrac{d\phi}{dt} = 0$.

$$\tan\phi = \frac{at}{q\omega} \tag{4}$$

$$\frac{d}{dt}\tan\phi = \frac{d}{dt}\left(\frac{at}{q\omega}\right)$$

$$\sec^2\phi\,\frac{d\phi}{dt} = \frac{a}{q\omega} - \frac{at}{q^2\omega}\,\frac{dq}{dt} \tag{5}$$

If q_0 is the value of q when the acceleration begins,

$$q = q_0 + \tfrac{1}{2}at^2$$

$$\frac{dq}{dt} = at \tag{6}$$

From (5),

$$\frac{d\phi}{dt} = \cos^2 \phi \left(\frac{a}{q\omega} - \frac{a^2t^2}{q^2\omega} \right) = 0 \tag{7}$$

Dividing by $\frac{a}{q\omega} \cos^2 \phi$ which is not zero,

$$1 - \frac{at^2}{q} = 0 \tag{8}$$

or

$$q_m = at_m^2 = 2s_m \tag{9}$$

where q_m is the value of q at the pitch point at which ϕ is maximum. q_m equals twice the travel s_m of the follower from rest, provided the period of acceleration persists through time t_m. Substituting (9) in (4) for conditions at the pitch point,

$$\tan \phi_m = \frac{1}{\omega t_m} = \frac{1}{\theta_m} \tag{10}$$

where θ_m is the angle in radians through which the cam turns while the follower rises from rest to where the pressure angle is maximum. Equations (9) and (10) are only valid in a period of constant acceleration from rest, long enough to produce this kind of maximum pressure angle.

What is the situation if the initial constant acceleration period is shorter than the t_m of (9)? Assuming the usual program, the acceleration will be either zero or negative for the ensuing period. With $V-F$ constant at the attained value at, and q increasing, (4) shows that ϕ must decrease. Of course ϕ will decrease if a is negative. The answer, therefore, is that ϕ_m is the value attained at the end of the period of constant acceleration upward, if not before.

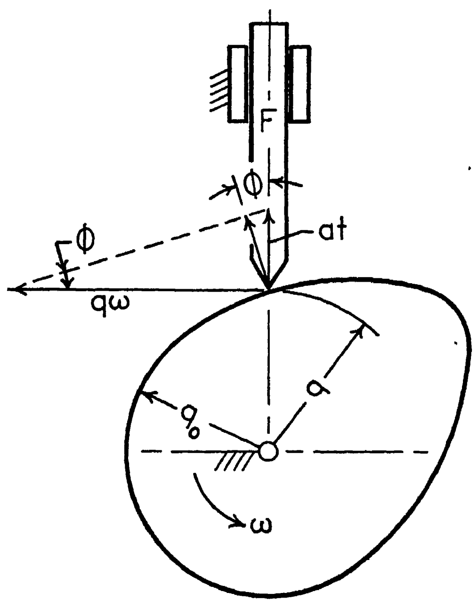


FIG. 8–9. Pressure Angles.

8–7. Parabolic Cam Design.—A symmetrical parabolic radial cam is required to run at 60 rpm, driving its follower through a one-inch roller in a straight vertical travel of one inch up and one inch down while the

cam turns 180°. The rest of the cycle is a dwell with the follower at its lowest position. The maximum pressure angle is specified as 30°.

From (10) or from the curve, Fig. 8–10, θ_m appears to be 99°. However, for a symmetrical parabolic cam to complete its action in 180°, the first acceleration period can be only 45°; therefore θ_m is 45°, and (9) and (10) do not apply.

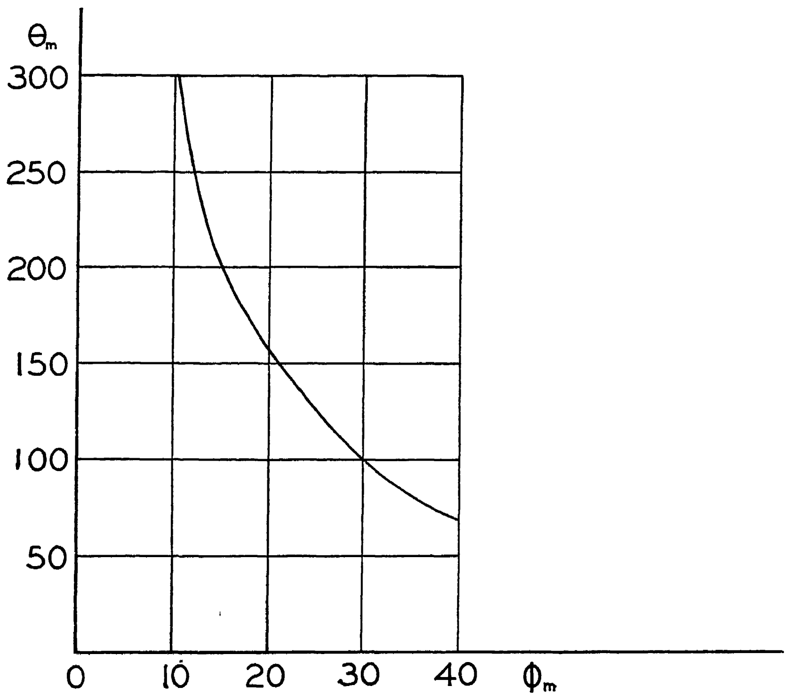


FIG. 8–10. Relation between Maximum Pressure Angles and θ_m on the Parabolic Cam for an Unlimited Acceleration Period.

The average speed of the follower in each of the four 45° periods is the same; therefore the distances travelled are equal. At the end of the first 45° period, $s = \frac{1}{2}$, $t = \frac{1}{8}$, so the average $V = s/t = 4$. The end velocity is twice the average or 8 in. per sec. This velocity is attained in 1/8 sec or $a = V/t = 64$. At 60 rpm, $\omega = 2\pi$.

With these as the values at the pitch point, (4) gives

$$q_m = \frac{at_m}{\omega \tan \phi_m} = \frac{64 \times \frac{1}{8}}{2\pi \times 0.577} = 2.21 \text{ in.}$$

Proceeding with the cam layout, Fig. 8–11, q_m is plotted from the cam shaft center P, along the line of travel of the follower, locating the pitch

point T. This is also the radius, $\theta = 45°$. Using 15° intervals of cam travel will give six points on the pitch curve for the 90° during which the

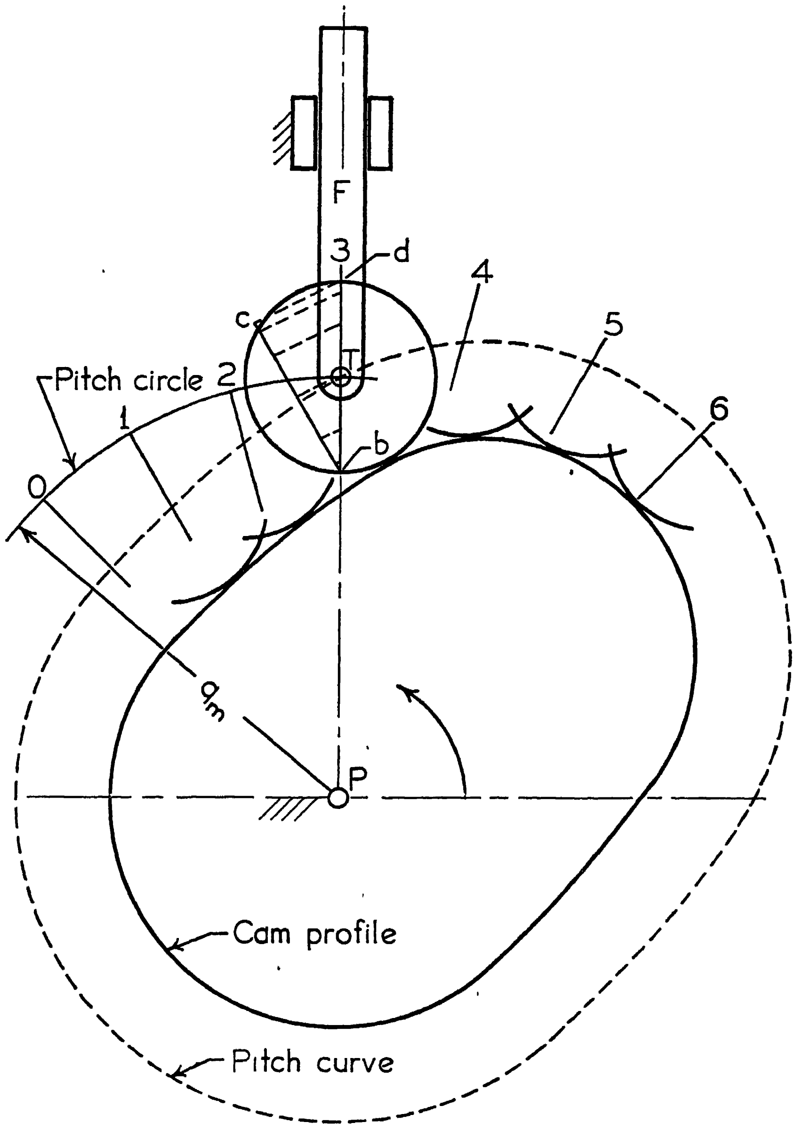


Fig. 8–11. Parabolic Cam. Scale, 1 in. = 1 in.

follower rises. For counterclockwise rotation, the rise will begin when radius PO is at PT, and radii 0, 1, 2, 3, etc., will be successively in action.

The follower must rise in six corresponding increments in the ratio, 1, 3, 5, 5, 3, 1. These are laid off to any scale on a line bc, drawn at any convenient angle with bd, and projected on the required travel bd. Next, by the procedure followed with Fig. 8–5, both the pitch curve and the cam profile are drawn. The cam is, of course, symmetrical about the line $P6$. A check of ϕ_m can be made by drawing a normal to the pitch curve at T.

8–8. **Pressure Angles—Harmonic Cams.**—If, in Fig. 8–9, V–F is changed to $\omega_h r \sin \theta_h$ instead of at, the case of the harmonic cam will be represented. Following the method of § 8–6, q_0 is the radius of the pitch curve in contact with the reference point when the follower begins its upward acceleration.

$$\tan \phi = \frac{\omega_h r \sin \theta_h}{q\omega} = \frac{kr \sin \theta_h}{q} \tag{11}$$

where $\omega_h/\omega = k$, a constant for each case.

$$\frac{d}{dt} \tan \phi = \frac{d}{dt} \frac{kr \sin \theta_h}{q}$$

$$\sec^2 \phi \frac{d\phi}{dt} = \frac{\omega_h kr \cos \theta_h}{q} - \frac{kr \sin \theta_h}{q^2} \frac{dq}{dt} \tag{12}$$

$$q = q_0 + r - r \cos \theta_h \tag{13}$$

$$\frac{dq}{dt} = \omega_h r \sin \theta_h \tag{14}$$

Substituting (14) in (12)

$$\frac{d\phi}{dt} = \cos^2 \phi \left[\frac{\omega_h kr \cos \theta_h}{q} - \frac{\omega_h kr^2 \sin^2 \theta_h}{q^2} \right] \tag{15}$$

Equating (15) to zero gives the relation for the maximum value of ϕ, called ϕ_m. Dividing by $\cos^2 \phi \times \omega_h kr/q$, which is not zero,

$$\frac{q_m}{r} = \frac{\sin^2 \theta_h}{\cos \theta_h} = \frac{\sin^2 \theta_h}{\sqrt{1 - \sin^2 \theta_h}} \tag{16}$$

Substituting for $\sin^2 \theta_h$ from (11),

$$k^4 \frac{r^2}{q_m{}^2} = k^2 \tan^2 \phi_m + \tan^4 \phi_m \tag{17}$$

giving q_m, the pitch radius for a specified ϕ_m, r, and k. The corresponding value of θ_h is given by (11),

$$\sin \theta_{hm} = \frac{q_m}{kr} \tan \phi_m \tag{18}$$

and θ_m by the relation,

$$\theta_m = \frac{\omega}{\omega_h} \theta_{hm} \tag{19}$$

8–9. **Harmonic Cam Design.**—The same specifications as used for the parabolic design, § 8–7, will be used here as far as possible, thus making comparison easy. A complete harmonic travel of one inch up in 90° with the return in the next 90°, followed by 180° of dwell, is required. As before, $\omega = 2\pi$. But, $\omega_h = 4\pi$, so $k = \omega_h/\omega = 2$. The extreme pressure

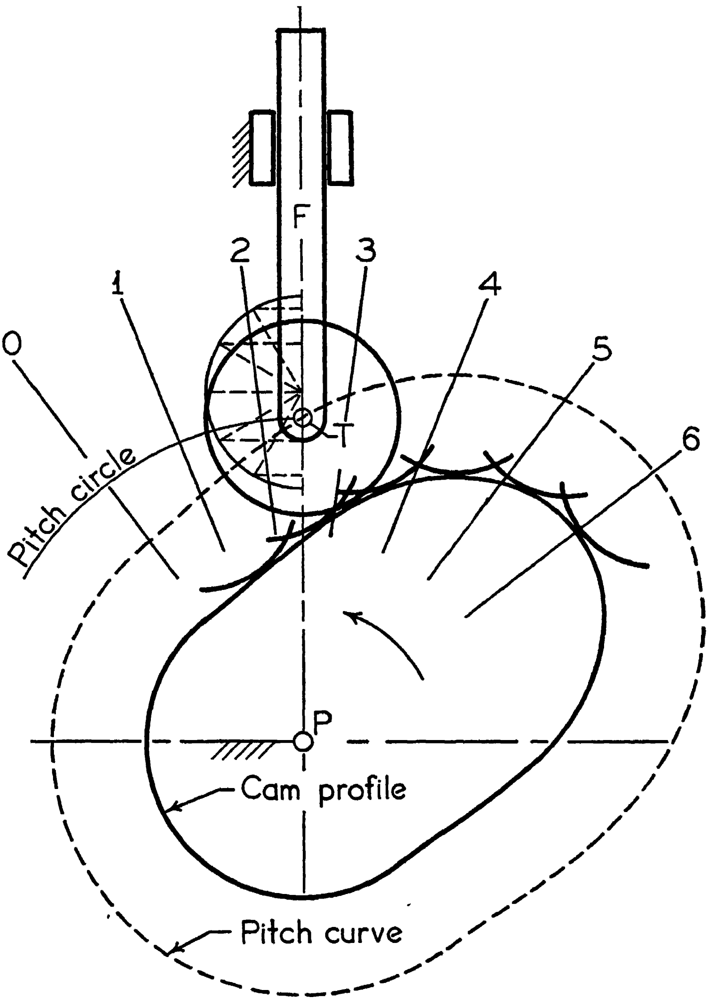


FIG. 8–12. Harmonic Cam. Scale, 1 in. = 1 in.

angle allowed is still $30° = \phi_m$. From (17),

$$16 \frac{r^2}{q_m^2} = 4 \times \frac{1}{3} + \frac{1}{9} = \frac{13}{9}$$

r, the radius of the harmonic circle, being 1/2, $q_m = 1.67$ in., which is plotted as PT, Fig. 8–12. From (18),

$$\sin \theta_{hm} = \frac{1.67}{2 \times \frac{1}{2}} \times \frac{1}{\sqrt{3}} = 0.962$$

giving $\theta_{hm} = 74°$ and $\theta_m = 37°$. Measuring 37° in the direction of cam rotation locates PO, the radius in action when the rise begins. The same 15° spacing is used as before, requiring six harmonic increments for the rise in 90° of cam rotation.

It is convenient to find q_0 from (13).

$$\begin{aligned} q_0 &= q_m - r + r \cos \theta_{hm} \\ &= 1.67 - 0.5 + 0.5 \cos 74° = 1.30 \text{ in.} \end{aligned}$$

This locates the lower end of the diameter of the harmonic circle, and the remaining construction is similar to that of Figs. 8–5 and 8–6.

Comparisons are now in order. The harmonic cam is considerably smaller than the parabolic, an important advantage. The major diameters are 2.60 and 3.42 in. respectively. The maximum acceleration for the harmonic cam is $r\omega_h^2 = \frac{1}{2}(4\pi)^2 = 78.8$, compared with 64 for the parabolic cam. This is the major merit of the latter, having increasing significance as the speed of operation increases.

8–10. **Cams with Other Characteristics.**—The **constant-velocity** or straight-line cam is seldom made because of its absurd acceleration characteristics. To reduce the acceleration bumps at the beginning and end of the travel, fairing curves of uncertain characteristics are often inserted in practice. Such profiles must serve where designing ability is not available, and they may be quite satisfactory provided the operating speed is low. When the two ends of the constant-velocity portion are faired in with suitable parabolas, the result is called a **combination cam**, a satisfactory profile for many applications.

Cams are sometimes made with **elliptical** pitch curves. They give fairly good starting and stopping characteristics to the follower travel, but have an acceleration peak considerably higher than that of the comparable harmonic cam. They can be easily machined for a follower with point contact or very small roller, in an elliptical chuck such as described in Chapter III. However, with rollers of practical size, the true elliptical profile may give severe bumps of acceleration, limiting satisfactory operation to low speeds.

If a portion of a pitch curve is made a **logarithmic spiral**, the cam has a constant pressure angle over this arc. Such a spiral, if used for any considerable portion of the cam, would give poor acceleration characteristics and has little practical merit.

The **mushroom follower** cam has a constant pressure angle which is independent of its velocity and acceleration, provided the motion of the follower is translation. If the follower rotates the pressure angle is variable.

8–11. **Unsymmetrical Cams.**—It is often good design to use different characteristics for the falling portion of the follower cycle compared to those used for the rising portion. It is safe to use a larger pressure angle on the falling than on the rising side, provided reverse operation is not required. This is accomplished automatically by the use of the offset follower, § 8–12.

If springs are used to return followers, the cam must compress the spring on the rise, Fig. 8–1, as well as accelerate the masses. By lengthening the period of rise, the maximum cam pressure can be reduced at the expense of providing a stronger spring for the increased acceleration necessary on the return stroke. Conversely, it may be more important to open a certain valve quickly than to close it quickly. These are only a few of the considerations that call for the design of nonsymmetrical cams.

8–12. **Followers on Offset and Irregular Paths.**—As previously mentioned, the pressure angle can be reduced on the rising side of the cam at the expense of increasing it on the falling side, by offsetting the line of travel of the follower as in Fig. 8–13. The technique for obtaining the pitch curve and profile is as follows.

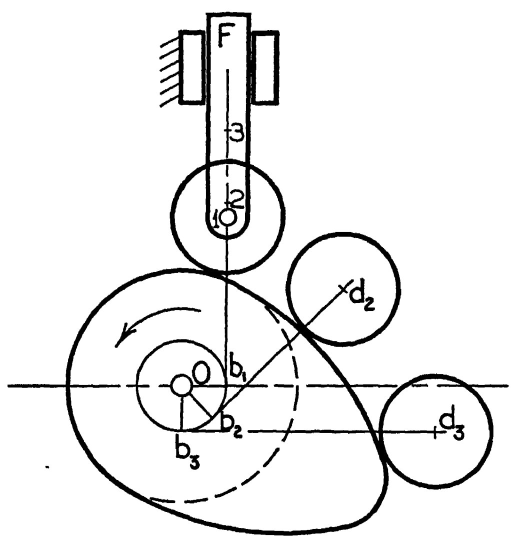


Fig. 8–13. Offset Follower.

For an offset Ob_1, draw the offset circle of radius Ob_1. Plotting each 45° of cam travel for illustration, lay off 45° increments from Ob_1 opposite to the sense of cam rotation. Normal to Ob_2 lay off $b_2d_2 = b_12$ giving point d_2 on the pitch curve. The profile is obtained from the pitch curve in the usual way.

Offsetting of the mushroom follower in the plane of the paper, Fig. 8–1, does not affect the pressure angle although it does affect the turning moment on the guides, and is sometimes utilized to reduce this turning

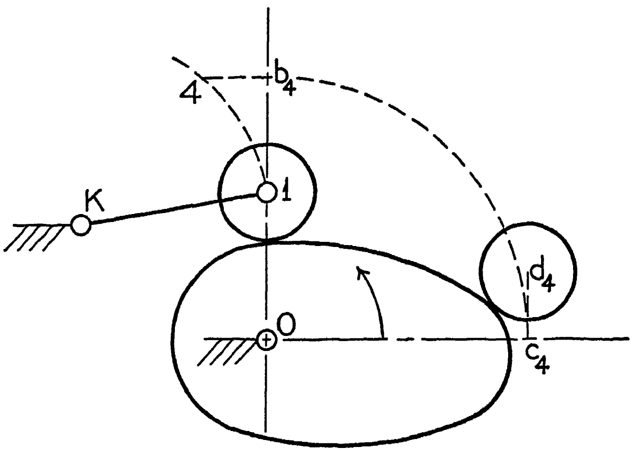


FIG. 8–14. Follower on Irregular Path.

moment on the rising stroke. There is another kind of offset, normal to the paper in Fig. 8–1, that is useful. This causes the follower to rotate in its guides through a small angle each cycle, insuring even distribution of wear over the foot, also better bearing action in the guides.

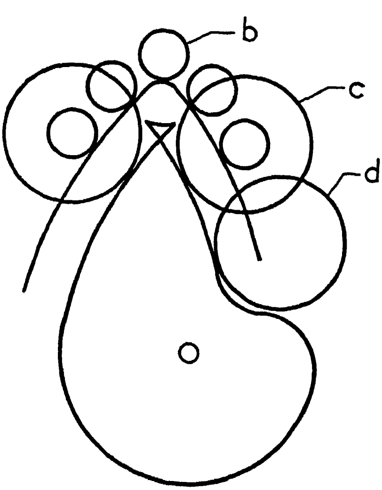


FIG. 8–15.

In Fig. 8–14, a roller follower is constrained to follow a curved path. Radius $O1$ through the reference point at the beginning of its rise will be used as reference radius. Suppose the reference point rises from 1 to 4 while the cam turns 90°. From 4 we draw a normal to the reference radius giving point b_4 which is transferred to the 90° cam radius as c_4. Erecting a normal c_4d_4 ($= b_44$) locates d_4 as the corresponding point on the pitch curve. This *normal offset technique* is not the only method possible but lends itself to accuracy.

The above method can be used when the reference point of the follower follows other than a straight path regardless of shape. For example, the

follower may be guided on any irregular path by pins or rollers traversing grooves in the fixed link.

8–13. **Some Limitations on Roller Followers.**—There are two limitations on the use of large rollers. One appears at *d*, Fig. 8–15, where the roller is too large for the concave part of the profile, and double contact results. The other difficulty is illustrated at the top. While a roller of size *b* would give a possible profile, size *c* gives an impossible looping profile, although the pitch curve is the same in each case. Proper attention to desirable pressure angles will make this difficulty less probable, especially with rollers of moderate size.

There is also a limitation on the small side. If rollers are too small, proper bearings cannot be housed in them. Also their load capacity is small due to the nature of their contact.

8–14. **Cylindrical Cams.**—Cylindrical cams, sometimes called barrel or drum cams, are made by fastening guide strips on the outside of a cylinder, Fig. 8–19, or by machining grooves in a cylinder, Fig. 8–16. The followers are regularly driven by rollers and move in direction parallel to the axis of the cylinder or approximately so.

If the roller were itself a true cylinder, it could not have pure rolling contact with the side of the groove, because the top and bottom of the groove have speeds proportional to their radial distances from the axis of the cylinder. If the roller is the frustrum of a cone with vertex at the axis of the cylinder, Fig. 8–16, it will have pure rolling while travelling in a circular groove, and substantially pure rolling in a helical groove unless the lead is excessive. The cone-shaped roller tends to rise from the groove due to the driving pressure, but it is easy to make and is the best practical solution.

The relative velocity of follower to cam is determined by using the pitch cylinder of the cam, the radius of which is q in Fig. 8–16. Cylindrical cam problems are regularly represented and studied on the developed surface of this pitch cylinder. The **pitch curve** is the trace of the axis of the roller (reference point) on the pitch cylinder.

The pressure angle in the plane tangent to the pitch cylinder, which is the plane of action, is given by the relation

$$\tan \phi = \frac{V^{-}F}{q\omega} \qquad (20)$$

ω being the angular velocity of the cam. Of course ϕ will be smaller at the large end of the roller and greater at the small end and still greater in a plane normal to the surfaces in contact, but (20) gives the mean value in the planes of action. ϕ will be maximum when $V^{-}F$ is maximum since q and ω are constant.

Design of Cylindrical Cam.—A cam having the general arrangement shown in Fig. 8–16 is to run at 240 rpm. For the rise of the follower, which is to be 10 in. in 150° of cam rotation, a combination profile is specified. Of the 150°, the first 45° is to be constant acceleration, the middle 60° constant velocity, and the last 45° constant deceleration. Following a dwell of 30°, the follower is returned in 150° by a symmetrical parabolic motion, and a 30° dwell completes the cycle. The maximum pressure angle on the rise is not to exceed 25°.

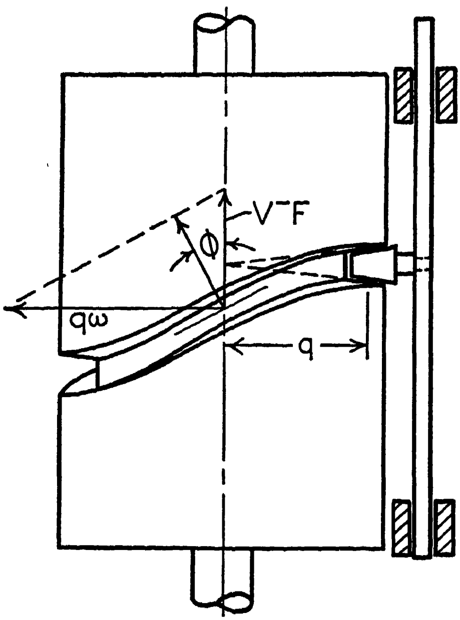


FIG. 8–16. Cylindrical Cam.

A plot of the characteristic curves is first made on a base of 360° of cam rotation which also represents 4 sec, the time of one revolution, and to another scale, the circumference of the pitch cylinder. The velocity curve, Fig. 8–17, should be the object of initial attack. The mean speed of the follower during the two 45° periods of parabolic action is half the maximum for the period of the rise. At this maximum speed the follower would rise 10 in. in $\frac{45}{2} + 60 + \frac{45}{2} = 105°$ or in 1.17 sec, fixing the velocity as 8.57 in. per sec.

For the return period, the maximum V^-F is twice the average or $2 \times \frac{-10}{1.67} = -12$ in. per sec, where 1.67 is the time in which the cam turns 150°. Assuming that even if the cam is operated in reverse it will not be under load, the pressure angle specification can be satisfied by placing $V^-F = 8.57$ in (20).

$$q = \frac{8.57}{\frac{2\pi}{4}\tan 25°} = 11.7 \text{ in.}$$

A cam with pitch diameter of about $23\frac{1}{2}$ in. is required.

Now proceeding with the curves of Fig. 8–17, a velocity of 8.57 in. per sec is attained in the first 45° which is $\frac{45}{360} \times 4 = \frac{1}{2}$ sec, or the ac-

celeration is + 17.14 in. per sec². It drops to zero for the constant velocity period and becomes − 17.14 for the 45° following. In the next

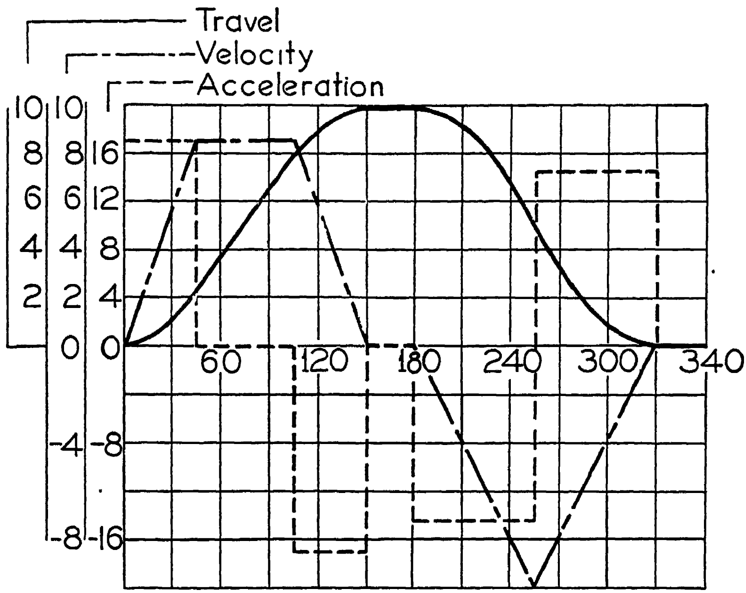


FIG. 8–17. Characteristic Curves for Cylindrical Cam.

75° or 5/6 sec the attained V–F is − 12 and the constant acceleration − 14.4. This becomes positive for the next 75° to bring the follower smoothly to rest.

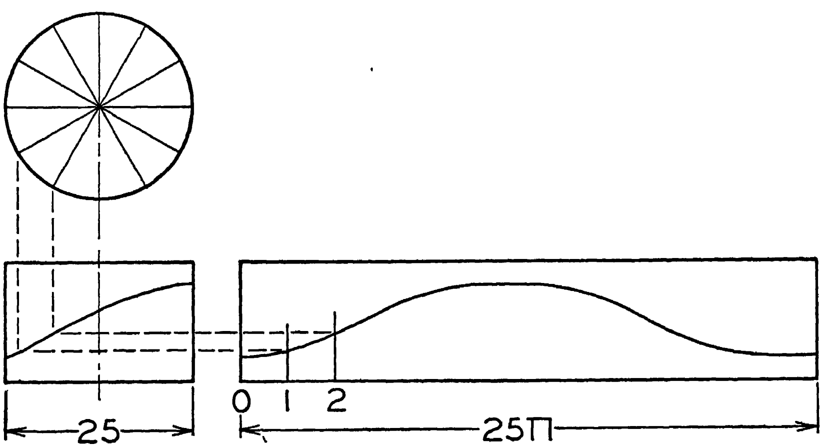


FIG. 8–18. Projection of Center Line of Groove on Cylindrical Cam.

The travel or space curve remains. In the first 45° or 1/2 sec the average V-F was 8.57/2 so the travel is 2.14 in. If it is desired to plot every 15°, there will be three increments of travel in the ratio, 1 : 3 : 5. The first increment is $\frac{1}{9} \times 2.14 = 0.238$, the second $\frac{3}{9} \times 2.14$ and so on.

It is convenient to reverse this curve for the 45° period of deceleration, plotting backward from the 150° ordinate. The middle 60° is of course a straight line, and is accurately tangent to the parabolas. The falling part of the curve consists of two tangent parabolas as shown computed in similar manner.

The modern method of producing cams, to be explained later, does not depend on the drawing of the groove on the cylinder but is based on

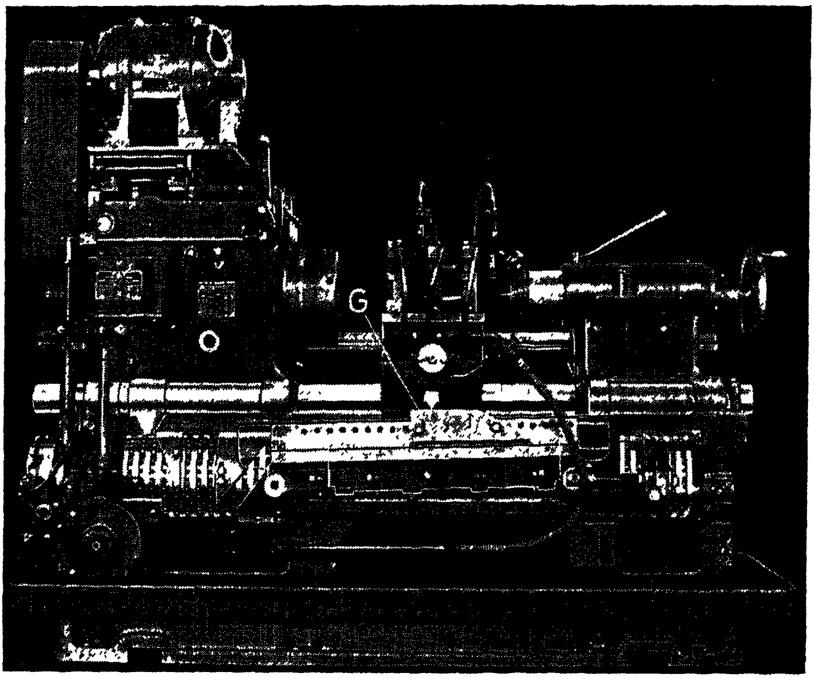


Fig. 8–19. The Fay Automatic Lathe.
The Jones and Lamson Machine Co., Springfield, Vt

a profile drawing of the former cam, § 8–20. It is often necessary in design, however, to know in advance where the groove will lie on the surface of the cylinder. The method is shown in Fig. 8–18.

Assuming that the length of the active part of the conical roller measured along its axis is $1\frac{1}{2}$ in., the outside diameter of the cam cylinder must be 25 in. (the pitch diameter being $23\frac{1}{2}$). The developed surface

is 25π in. and the travel of the follower must be plotted to the same space scale which in Fig. 8–18 is 1 in. $= \frac{25\pi}{3}$ in. The projection on the cylinder requires an end view as shown.

This solution affords a comparison between the combination program of motion and the symmetrical parabolic. If the latter were used also for the rising portion of this cam, the required pitch diameter as computed would be 32.8 in. instead of 23.4 for the same pressure angle specification. However, the maximum acceleration would be 14.4 instead of 17.14, resulting in a smoother drive with lower bearing loads.

An example of the extensive use of cylindrical cams is found on the Fay automatic lathe shown in Fig. 8–19. A separate view of the principal cams, Fig. 8–20, shows that the drums have a great many tapped holes so that guide straps, serving as active cam surfaces, can be fastened

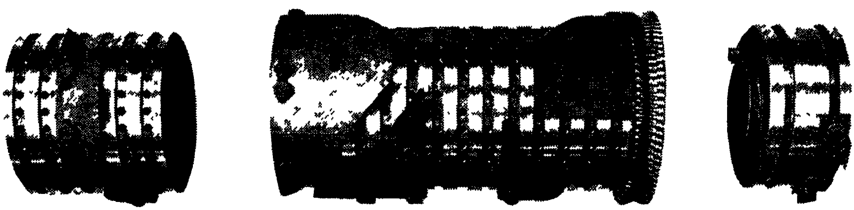


FIG. 8–20. Cylindrical Cam Construction.

in a wide variety of locations. In this way required motions (feeds and cuts) can be given to the several tools for machining a variety of parts.

The carriage, Fig. 8–19, which generally carries several tools, is clamped to the center bar, and both receive axial movement from the large cam at the left. The end of the carriage toward the reader rests on the front former *G*. This contact is through a pivoted shoe behind the indicator, Fig. 8–19, and is in shadow. The former, actuated by the cam at the right, controls the tilting of the carriage about the center bar, which action feeds the tools towards the axis of the work. When the machine is set up for a job, and a piece is inserted between the centers, there is no manual work necessary except to move the starting lever, until the operation on the piece is completed.

Space does not permit further description of this highly developed machine. The Jones and Lamson Machine Co. publish a well illustrated book entitled "The Fay Automatic Lathe."

8–15. **The Swash Plate.**—A plane disk mounted in fixed position on a revolving shaft so that its plane makes an angle other than 90° with the shaft axis constitutes a swash plate. A follower, bearing on the plate and constrained to move axially, will have harmonic motion if it has point

contact as indicated in Fig. 8–22. It will have approximately harmonic motion if a small roller is used, Fig. 8–21, or if a ball-hinged sliding foot bears on the plate. With the latter arrangement this mechanism has been used for small reciprocating pumps and compressors. It can be given the positive-return characteristic by the use of a U link on the foot of the follower to afford bearing on both sides of the disk. It has been a favorite of amateur designers of multi-cylinder engines.

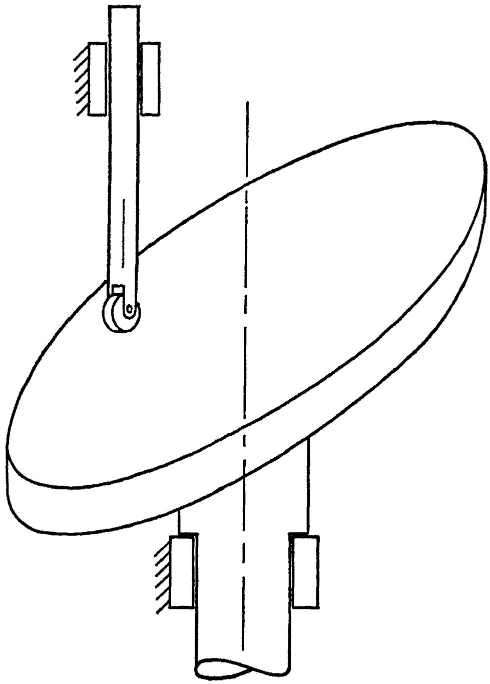


Fig. 8–21.

Its weakness lies in the difficulties encountered in developing bearings between plate and follower that will carry real loads with durability and reasonably good mechanical efficiency. Rollers, mounted as in Fig. 8–21, have only point contact with the plate, and the action is improved if the face of the roller is curved. However, the swash plate is a useful cam, where loads are light.

Fig. 8–22 will be used for analysis. As the shaft *of* revolves, the contact point of the follower *h* is displaced vertically above a horizontal plane represented by the circle *bdc*. The trace of *h* on the swash plate is the ellipse *bhk*. While the shaft turns through θ from *ob* to *od*, the rise of the follower is *dh*.

$$dh = ae = (fo)\,\frac{ba}{bo} = (fo)\,\frac{bo - ao}{bo}$$

$$dh = (fo)\left(1 - \frac{ao}{do}\right)$$
$$= (fo)(1 - \cos\theta) \quad (21)$$

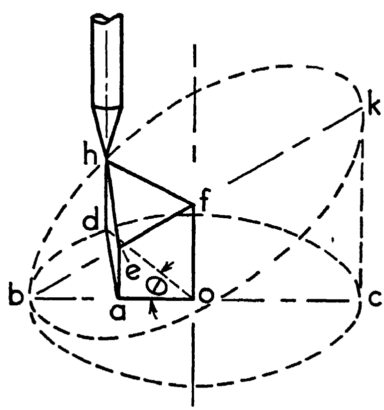


Fig. 8–22.

This is the same as equation (1) for harmonic motion, *fo* being half the harmonic travel.

8–16. **Some Special Cams.**—A toe-and-wiper cam is shown in Fig. 8–23. It can be used only for reciprocating action through a small angle. In this case it lifts an exhaust steam valve. Its main advantage is that there is little rubbing at the cam surfaces. In fact with centrode design, as in Chapter II, pure rolling could be easily attained. A small amount of rubbing, however, is not always objectionable as it tends to keep the surfaces smooth.

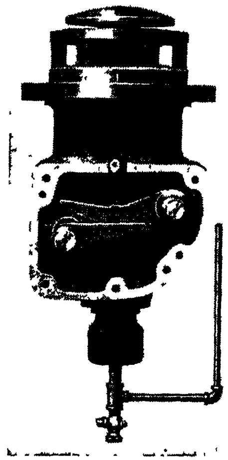


FIG. 8–23. Wiper Cam. *The Skinner Engine Co., Erie, Pa.*

The positive return cam of Fig. 8–24 is a box cam in appearance as well as by definition. It can only be freely designed for a half cycle, for example, side h to the right of the center line. Thereafter, the profile of side K is governed by the fact that the breadth between any pair of parallel tangents must have the constant value d. The characteristic curves for a rise in 180° will be reversed for the ensuing 180°. This limitation can be completely circumvented by using two cams, one of which bears only on the top of the box, while the other bears only on the lower surface.

If, in Fig. 8–24, point or line contacts are inserted in the follower at top and bottom on a line through the cam axis, a **constant diameter cam** becomes necessary. That is, all diameters through the cam axis must be equal.

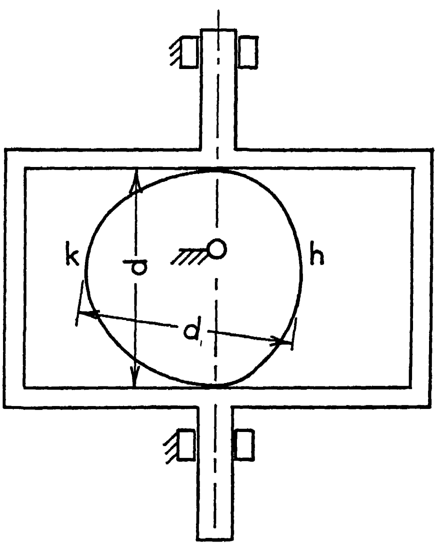


FIG. 8–24. Constant-Breadth Cam.

A further conversion to roller followers would require a cam shape neither exactly of constant diameter nor of constant breadth, but the limitation as to design would persist. The so-called *heart-shaped cam* formed of three equal circular arcs is a constant-breadth cam.

A good example of the positive return principle is the valve cam of Fig. 8–25 used on nine-cylinder radial aircraft engines. Two roll-

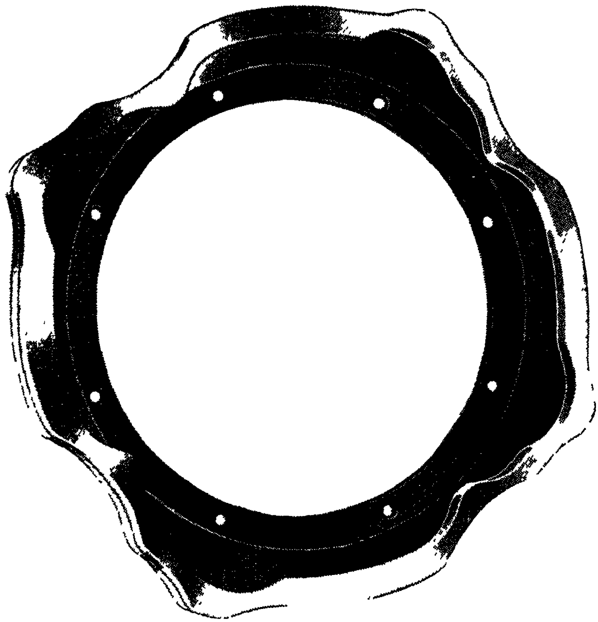


FIG. 8–25. Radial Aircraft Engine Cam.

ers with axis in the same radial plane are attached to each follower (push rod), one roller running on the outer cam surface, the other on the inner surface. Only one push rod is required per cylinder with this design. It is really a "push and pull" rod, operating both the inlet and exhaust valves, as can be observed from the contour of the cam. This cam operates at one-eighth engine speed and in the opposite sense. An alternative design uses two external cams.

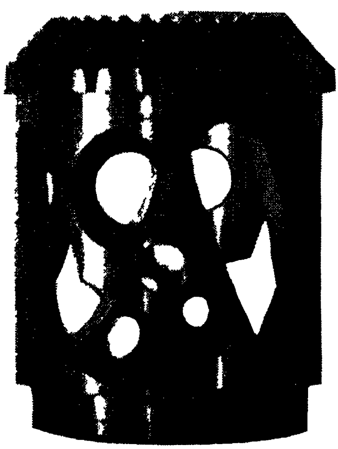


FIG. 8–26. Propeller Feathering Cam.

The Rowbottom Machine Co., Waterbury, Conn.

The unique cylindrical cam illustrated in Fig. 8–26 is housed axially in the hub of an aircraft propeller and used to control the inclination or feathering of the blades as required by changing speed of the plane. In this case, the cylindrical cam is really the follower, and is given rotation by rollers which engage the grooves shown in the cam. The rollers are attached to a link extending through the cam and given translation by a pis-

ton in an oil cylinder. By valve control, the pilot can admit high pressure oil to either side of the piston, thereby rotating the cam in either sense. The bevel teeth engage a bevel pinion on each blade, the blades being otherwise free to rotate in their bearings. In this manner all blades are feathered uniformly.

8–17. **Acceleration of Cam Followers.**—The equivalent-mechanism methods presented in Chapter VII for obtaining the accelerations of cam followers were selected for their relative ease of manipulation combined with control of accuracy. Practice with these methods is necessary for reasonable speed and dependability, and it is essential that the appropriate method be selected for each case. Table 8–2 is designed to assist in this selection. Not all combinations are included. Those omitted represent cases either quite simple or where cam action is impossible. An example of the latter is where both cam and follower have active surfaces that are plane.

TABLE 8–2

ACCELERATION METHODS FOR CAM FOLLOWERS

Cam			Follower		Method
No.	Profile	Motion	Profile	Motion	Equivalent mechanism
1	curved	rotation	curved	rotation	quadric
2	curved	rotation	curved	translation	slider-crank
3	curved	rotation	plane	rotation	slider-plane
4	curved	rotation	plane	translation	slider-plane
5	plane	rotation	curved	rotation	slider-plane
6	plane	rotation	curved	translation	slider-plane
7	curved	translation	curved	translation	double-slider
8	curved	translation	plane	rotation	slider-plane
9	curved	translation	plane	translation	slider-plane
10	plane	translation	curved	translation	slider-plane

8–18. **Acceleration by Graphical Differentiation.**—If a space or travel curve having ordinates s is plotted on a time base (t), it is possible by drawing tangents at various points to evaluate velocity V as ds/dt. Drawing a velocity curve through these points and again drawing tangents it is possible to determine ordinates for the acceleration curve as $a = dV/dt$.

This would be a more valuable method if it were possible to draw tangents to curves of changing curvature with accuracy. It is difficult to avoid considerable cumulative error in drawing the tangents and plotting the curves. The alternative graphical methods depend at worst on the determination of the radius of curvature at a point. Such curvatures on cams, if not specified, can be obtained with fair accuracy.

It is also possible to determine acceleration from a velocity curve plotted on a travel base, using the length of the subtangent as the basis of computation. Neither of these methods gives the accuracy demanded in engineering practice.

8–19. **Acceleration on an Automotive Valve Cam.**—It is desirable to open the valves of internal-combustion engines to their maximum lift in a small fraction of the cycle. On high-speed automotive and aircraft engines, the designer faces a difficult problem in endeavoring to keep accelerations low enough to avoid failure or serious wear, while at the same time providing the valve movements required for thermodynamic economy.

The cam of Fig. 8–27 is representative of truck engine practice. The lower 180° is circular. The purpose of the 30° ramp is gradually to take

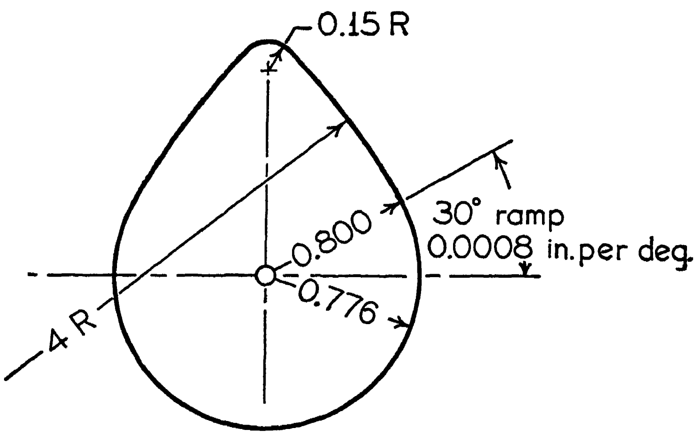


Fig. 8–27. Automotive Valve Cam for Mushroom Follower.

up the clearance, or a large part of it, so that the valve will begin to rise as the active arc of the cam comes under the follower. In the position shown, the cam is symmetrical about the vertical center line.

The acceleration curve is given in Fig. 8–28 on a degree base. Starting from the radius OM as zero, degrees are measured in sense opposite to the cam rotation as usual. The $37\frac{1}{2}°$ position is taken to illustrate the method of attack for contact on the arc of 4 in. radius. Contact is at P which is located accurately by laying off the angle $OTP = 7\frac{1}{2}°$. The equivalent-mechanism method requires a translation of the foot of the follower to T, the center of curvature of the cam. The slider, or point K, is on B.

$$K_{ON} = (KO)\omega_B{}^2 = 3.2\omega_B{}^2.$$

Assuming that $\alpha_B = 0$, this value can be plotted as $O''K''$, and using the

scale, 1 in. = $2\omega_B^2$ in. per sec², the vector is 1.6 in. long. $T_{KN} = 0$, and $O_{TN} = 0$ (relative rectilinear translation in each case). Therefore T_{KT} ($= K''T''$) and O_{TT} ($= T''O''$) must close the diagram, giving $T_O = 1.59$ in. which is the $37\frac{1}{2}°$ ordinate of the acceleration curve.

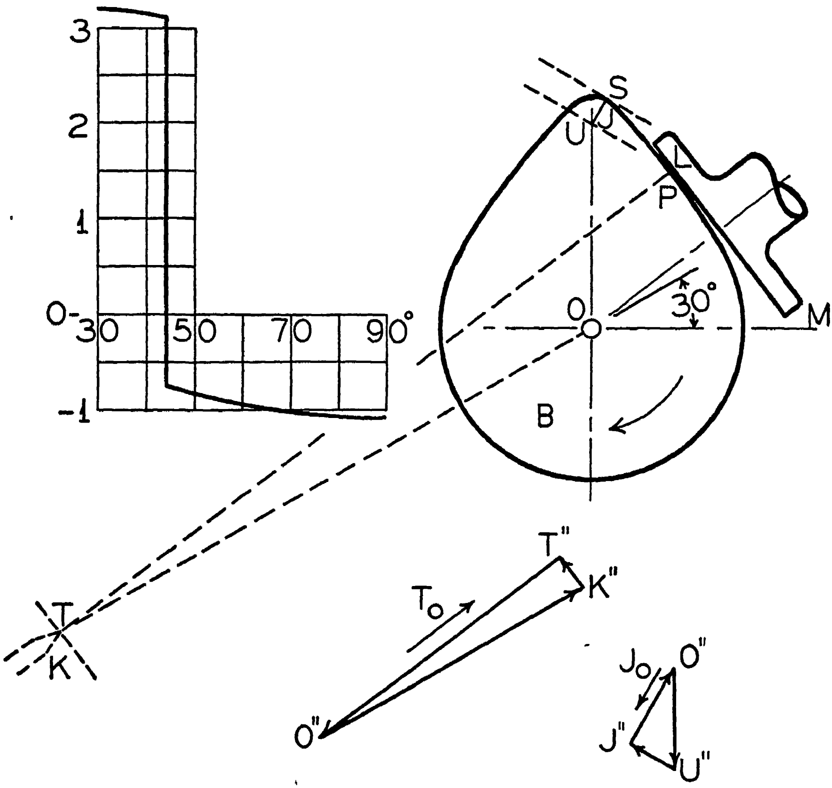


FIG. 8–28. Acceleration Values.
Space scale, 1 in. = 1 in. Acceleration scale, 1 in. = 2 ω_B^2 in. per sec².

Swinging the center line of the follower into the 60° position gives contact on the nose arc at S. The foot of the follower must be moved to the center of curvature J. The slider or driving-contact point of B is now U. The vector diagram $O''U''J''$ follows regularly and gives the vector $J_O = -0.48$.

The reversal in sense of the acceleration occurs at 44°, where the contact passes from the large to the small radius. Upward acceleration occupies only 14° of the 60° angle of rise. The peak of acceleration could be reduced more than 50% by using a symmetrical parabolic cam, but large valve opening would not be attained as quickly. Incidentally, the

symmetrical parabolic cam can be approximated here, by decreasing the 4 in. radius and increasing the nose radius.

It may have been noted that this solution was made without the use of a stated cam speed. It is often convenient to express accelerations in terms of the square of the speed of the constant-speed link. For an engine speed, in this case, of 3000 rpm, the cam speed would be 1500, and the peak of acceleration of the valve, 79,000 in. per sec^2, or about $200 \times g$.

8–20. **Production of Cams.**—The machining of cams on ordinary machine tools—lathes, milling and grinding machines—is slow and troublesome unless the profile is simple, as, for example, the tangent cam with circular nose. Special attachments are available for the standard tools, by the use of which speed and accuracy are improved. The profile milling machine is fairly well adapted to the production of irregular contours.

In recent years, however, highly specialized cam-cutting machines have been developed, an example of which is given in Fig. 8–29. The set-up shown is for a disk cam. The cutter head slides on the vertical column, its position being controlled by the former cam through a roller follower called a former roll.

Noting that the former roll has the same diameter as the cutter, it follows that the cam produced will be an exact reproduction of the former in profile, provided they are turned at the same speed. Unless the cam profile is quite simple—straight lines and circular arcs—the former is made from an accurate drawing of the cam. Usually a sheet-metal template is first cut and used to transfer the profile to the cast iron disk from which the former is made. By a judicious mixture of machine and hand work, using the template as a guide, it is possible to reproduce the most complicated profile.

With the set-up shown, the machine can be used for internal, box, or, in fact, any type of radial cam. Suitable speeds are available so that grinding wheels can be used in place of the milling cutters for finishing hardened cams or for smoother surfacing.

If it is desired to produce a cam for use with a mushroom follower, the procedure outlined above is still used. The profile of the cam is reproduced, but not the motion of the follower. In case a smaller cutter or a larger grinding wheel than any of the former rolls available needs to be used for finishing cuts, a different former is required. Starting with the cam profile drawing, it is necessary to plot a pitch curve for a roller of the diameter of the finishing cutter or grinding wheel. With centers along this pitch curve arcs are drawn the size of the former roll. The usual tangent curve gives the profile of the former required. Note that

the size of the follower roller to be used with the completed cam is of no concern in the production problem. For the roughing cuts, of course, the above relations among cutter, former roll, and former need not be rigidly held, but are necessary with finishing cuts.

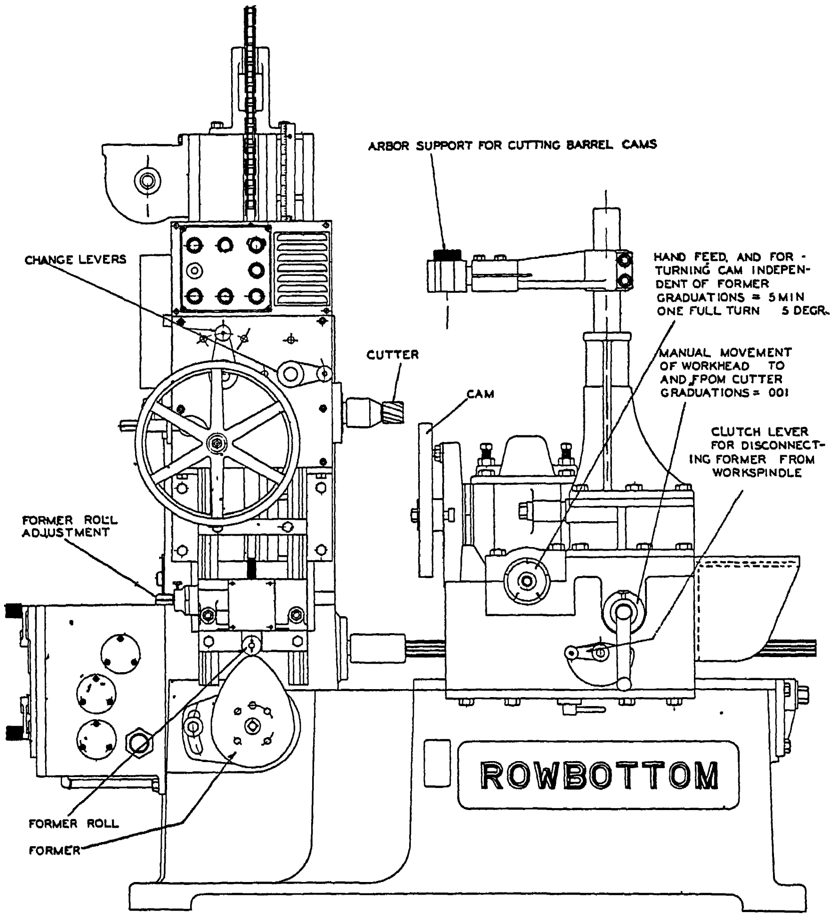


FIG. 8–29. Universal Cam Milling Machine.

The Rowbottom Machine Co., Waterbury, Conn.

For the production of cylindrical cams, vertical arbor supports are provided, together with feed-gear connections for turning the blank. The cam groove as designed is drawn on the developed pitch surface of the cam cylinder. Assuming that the groove is to be finished with a cutter of size equal to the roller of the follower, the pitch curve of the cylindrical cam must be plotted as for a radial cam. With centers along this new

pitch curve, arcs equal to that of the former roll are struck, and the tangent curve gives the required profile of the former.

QUESTIONS AND PROBLEMS

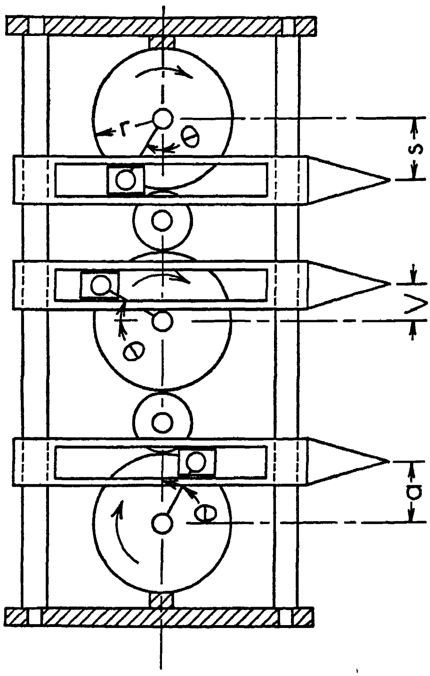
1. Classify cams according to
   (*a*) the characteristics of the follower,
   (*b*) the characteristics of the driver.
2. Make outline sketches of the following:
   (*a*) A box cam with a roller follower that travels on a curved path.
   (*b*) A disk cam with a mushroom follower that travels on a straight but offset path.
   (*c*) A positive-return cylindrical cam with roller follower.
3. The follower of a radial cam is to rise vertically 4 in. in 8 sec by harmonic motion.
   (*a*) Compute the rise for each second.
   (*b*) What will be the velocity and acceleration at the end of the third second?
4. The follower of a radial cam is to rise vertically 4 in. in 8 sec, the rise for the first 2 in. being under constant acceleration, and for the remaining 2 in. being under constant deceleration.
   (*a*) Compute the rise for each second.
   (*b*) What will be the velocity and acceleration at the end of the third second?
5. Which of the followers of problems (3) and (4) have the least maximum acceleration and by what per cent? Ans. (4) by 16.5%.
6. A follower moves in a straight line as follows,—
   (*a*) upward 1 in. in 4 sec under constant acceleration,
   (*b*) upward at the attained velocity for 2 sec,
   (*c*) upward under constant deceleration to rest in 8 sec,
   (*d*) dwell for 2 sec,
   (*e*) returns in 8 sec, 4 sec being constant acceleration and 4 sec being constant deceleration. On a time base plot curves of velocity, acceleration, and travel.
7. A follower rises in a straight path 2 in. in 6 sec under harmonic motion to rest, and returns in 6 sec by a program that will give the least possible maximum acceleration. On a time base plot curves of velocity, acceleration, and travel.
8. A disk cam with $1\frac{1}{2}$ in. diameter roller follower, having a maximum pressure angle of 25°, is required to cause its follower to move in a straight line through the cam axis. The program of motion is symmetrical parabolic with rise and fall in 180° of cam rotation, the total rise being 2 in., there being 180° of dwell with the follower

in lowest position, and the cam speed being 120 rpm. Construct the pitch curve and profile.

9. A radial cam is required to move its follower through a one-inch roller in straight radial travel, the total rise being $1\frac{1}{2}$ in. The program of motion is a symmetrical harmonic rise and return in 120° of cam rotation, the other 240° being dwell with the follower in lowest position. The cam speed is 600 rpm, and the maximum pressure angle 30°. Design the cam profile.

10. On a degree base construct curves of velocity, acceleration, and travel for the follower of Prob. 9.

11. In this figure three Scotch yokes are shown connected by gearing in such manner as to produce kinematically the first and second derivatives of the cosine function, equation (1), which expresses harmonic displacement s. Find the scales to which the lower two pointers indicate the velocity and acceleration respectively.



12. A face cam, Fig. 8–2 (*b*), revolves at 120 rpm, driving its follower through a 2-in. roller. The follower is to rise on a straight radial travel of 4 in. in 160° of cam rotation by symmetrical parabolic motion, the maximum pressure angle being 35°. After a dwell at the top for 20°, the follower falls by symmetrical parabolic motion in 100° after which there is a dwell for 80°. There is no pressure angle limitation for the falling period. Design the cam to the extent of determining the inner and outer profiles.

13. Draw the three characteristic curves for the cam of Prob. 12. What is the maximum pressure angle for the return period?

14. A Diesel-engine fuel pump is to be operated by a disk cam with mushroom follower. The cam speed is 300 rpm. The pump plunger is to have a straight line stroke of 1/2 in. The rise and fall is to be symmetrical and parabolic in 180° of cam rotation. Design the cam on a base circle of $1\frac{1}{2}$ in. Give the diameter of the mushroom, allowing it to project 1/4 in. beyond all contact points.

15. (*a*) Indicate on your drawing of the cam of Prob. 14 the direction of offset of the follower that would reduce guide-bearing pressure on the upward stroke, for an assumed sense of rotation of the cam.

    (*b*) Indicate the kind of offset that would distribute the wear.

16. A cylindrical cam two feet in pitch diameter rotates uniformly once in 36 sec and drives its follower in a path parallel to the cylinder axis through a 2-in. roller as follows:

    (*a*) It is accelerated by harmonic motion for $4\frac{1}{2}$ in. in 60° of driver rotation. (Use only the accelerating half of the harmonic cycle.)

    (*b*) It continues at the attained velocity for 45°.

    (*c*) It is decelerated to rest at the top by harmonic motion in 60° of cam rotation.

    (*d*) After a dwell of 15° it is returned by a reversal of the above program and a dwell of 15° completes the cycle.

    (1) What is the stroke of the follower? Ans. 14.3 in.

    (2) Draw curves of velocity, acceleration, and space on a degree base.

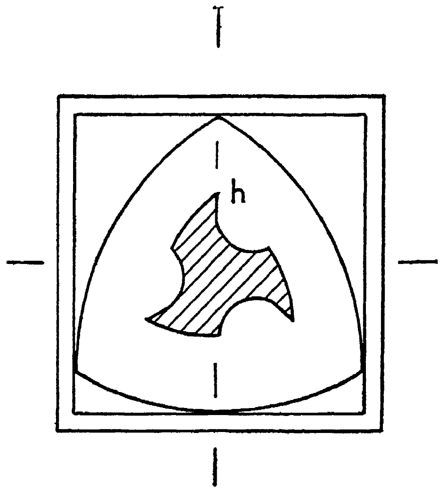
    (3) For a roller of 2-in. active radial length, plot the center line of the groove on the developed surface of the cylinder and project it on the plan view.

    (4) Find the maximum pressure angle. Ans. 29.3°.

17. A cylindrical cam revolving at uniform speed is required to move its follower axially 4 in. and return, in 120° of cam rotation. For a symmetrical parabolic motion and a maximum pressure angle of 30°, find the cam pitch diameter. (Note that the answer is independent of speed. Degrees can be used for time.)

18. Design a swash-plate cam inclined at 30° to its axis, to give its follower an axial travel of 6 in. Extend the plate 1 in. radially beyond the center line of the two-in. roller of the follower. Will the motion be harmonic?

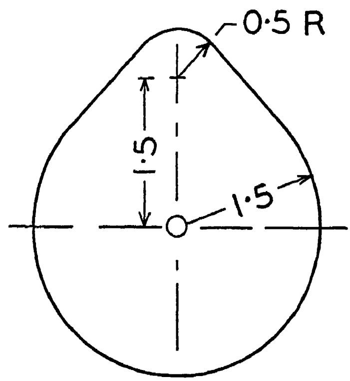
19. Draw this heart-shaped cam and its square stationary guide about three times the size shown and locate point h one-quarter of the cam diameter from the top. Plot the path of h for a cycle. If the cam and box were hardened steel, how could it be set up and driven to do useful drilling?



20. In the case of the aircraft-engine valve cam of Fig. 8–25, consider the crankshaft to rotate clockwise and the cam counterclockwise. On the outside of a 2-in. circle, lay off the angular positions of a bank of nine cylinders and number them consecutively. Cut a 2-in. paper disk and on it mark four cam lobes spaced at 90° to represent the lobes that operate the inlet valves. Rotate the disk in the circle, and write the firing order of the cylinders (same as the order in which inlet valves open). Is torque applied to the engine shaft uniformly? What would be the performance of eight cylinders in a radial bank?

21. A mushroom-follower cam, similar to that of Fig. 8–27, has a radius at the beginning of the lift (end of the ramp) of one inch. The main radius of the cam is the same, 4 in., but the nose radius is 0.3 in. Plot the acceleration and velocity curves.

22. This is a tangent cam with which curved followers only can be used. Find its curve of acceleration and its maximum pressure angle for a one-inch roller follower having straight radial travel. It is advisable to use figures two or three times full size for quantitative work.



CHAPTER IX

SPUR GEARING

9–1. Dimensions and Pitches of Gear Teeth.—Gearing is the most convenient and most generally used means of transmitting mechanical energy from one rotating body to another where the parts to be so connected are not too far apart. Gears, of a sort, were used in some of the most primitive machines. Throughout the history of machine development, gearing and bearings have vied for first place in importance.

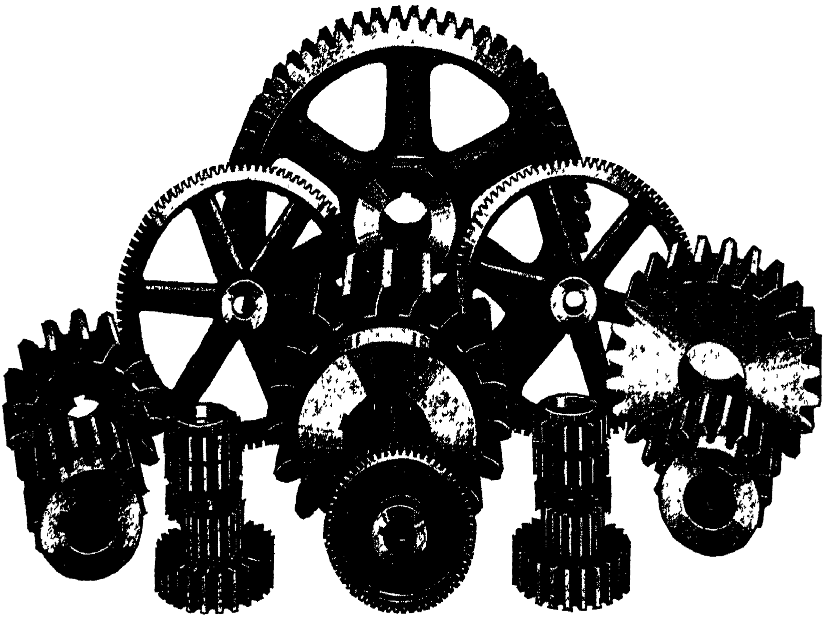


FIG. 9–1. Spur Gears with Cut Teeth.
Foote Bros. Gear & Machine Co., Chicago, Ill.

Some nomenclature is given in Fig. 9–2 that will apply, in general, to all systems of spur-gear teeth. A *spur gear* is one having teeth parallel to the axis of the gear.

The *addendum circle* is the circle bounding the outer ends of the teeth.

The *pitch circle* of a gear is the circle that rolls on the corresponding circle of the mating gear without sliding. If two plain friction cylinders, having diameters equal respectively to the two pitch circles of a pair of mating spur gears, were rolled together without sliding, they would have the same relative angular velocity as the gears.

The *pitch point* is the point of tangency of two mating pitch circles and is, of course, on the line of centers.

The *meshing-depth circle* or *working-depth circle* is the boundary up to which the teeth of the mating gear come. Due to the rolling action, the tooth contact does not extend as far as the meshing-depth circle.

The *dedendum circle,* sometimes called the *root circle,* is the inner boundary of the intertooth space.

The *pitch surface* is the surface containing all possible pitch circles of a gear. For a spur gear the pitch surface is a cylinder.

The *face* and *flank* of the *tooth* are the areas on the side of the tooth, above and below the pitch surface respectively. This should not be confused with the *face of the gear* which is, for a spur gear, the length of the teeth.

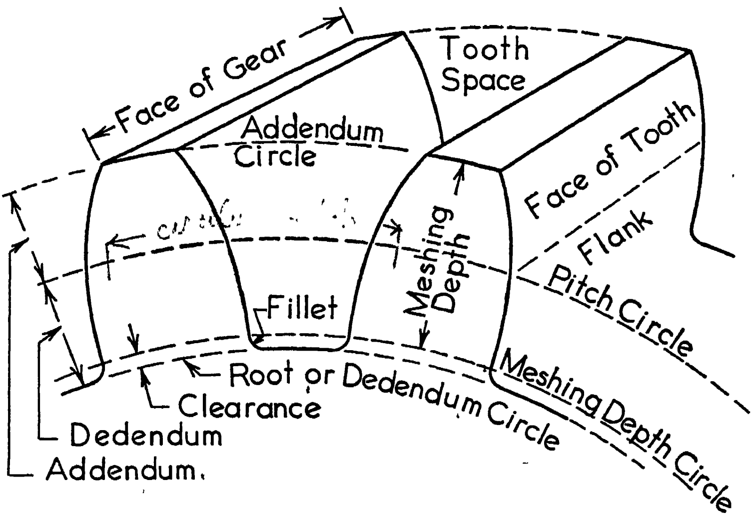


FIG. 9–2. Nomenclature of Gear Teeth.

The *addendum* is the radial distance from the pitch circle to the addendum circle. The radial distance from the pitch circle to the mating-depth circle is commonly equal to the addendum as indicated in Fig. 9–2, but this is not always the case.

The *clearance* is the radial distance from the mating-depth circle to the dedendum circle.

The *dedendum* is the radial distance from the pitch circle to the dedendum circle.

The *circular pitch* of a gear is the distance from any point of a tooth, on the pitch circle, to the corresponding point on an adjacent tooth, measured along the pitch circle.

The *diametral pitch* of a gear is the number of teeth per inch of pitch diameter, the pitch diameter, of course, being the diameter of the pitch circle.

Both the above mentioned pitches are measures of tooth size: the circular pitch, a direct measure; the diametral, an inverse measure. The circular pitch is the length of an arc, and this arc is the portion of the pitch circle occupied by a tooth and a space. Normally, the tooth

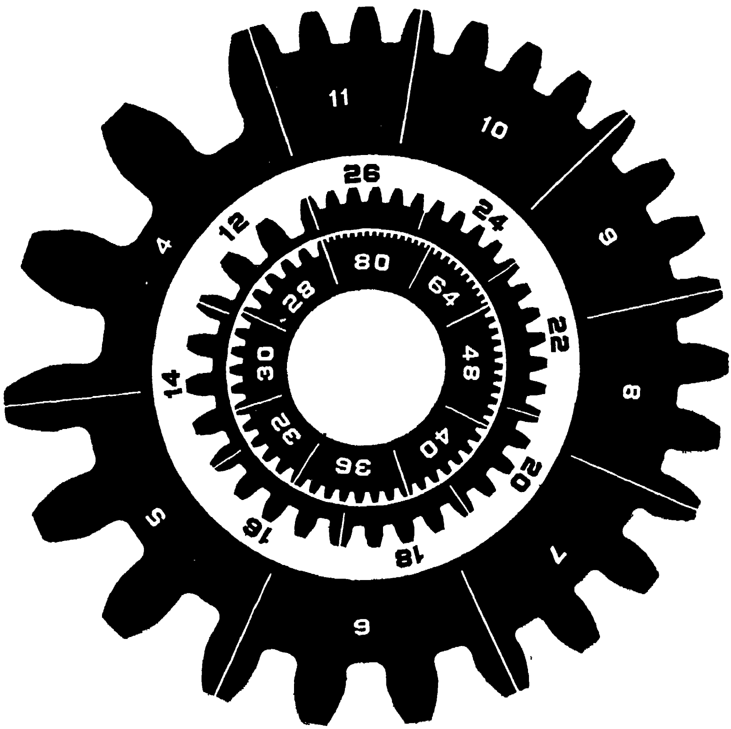


Fig. 9–3. Diametral Pitches.

Barber-Coleman Co., Rockford, Ill.

occupies the same length of pitch-circle arc as the space, but, if less, there is looseness between the meshing teeth called *backlash*. Diametral pitch is a ratio. Obviously, the larger the diametral pitch, the more teeth a gear of given size must have, and the smaller its teeth must be.

A *pinion* is the smaller of a pair of gears, the larger being called the *gear*.

Module is the pitch diameter in inches (or millimeters) divided by the number of teeth.

Fig. 9–3 shows the actual size of the diametral pitches of full-depth involute teeth from 4 to 80.

It is frequently necessary to find the diametral pitch from the circular and vice versa.

Let p represent the circular pitch,
P, the diametral pitch,
N, the number of teeth,
D, the pitch diameter in inches.

From the definitions,

$$p = \frac{\pi D}{N}$$

and

$$P = \frac{N}{D}$$

Evaluating D in each equation and cancelling N gives

$$P = \frac{\pi}{p} \quad \text{and} \quad p = \frac{\pi}{P} \tag{1}$$

9–2. **The Fundamental Proposition.**—The primary requirement for satisfactory gear operation is that, for all possible tooth contact, the relative angular velocity ratio of the two gears shall be constant. If the driving gear is running at constant speed, and is compelling the driven gear to turn at a certain speed at one instant, and, an instant later, with changed tooth contact, at a different rotative speed, the result will be an angular acceleration imposed on the driven gear, producing acceleration forces on the teeth. These forces resulting from such acceleration are in addition to the regular transmitting forces, and are the principal cause of noise in worn or poorly made teeth. In machine design, the maximum of these accelerating forces is called the "dynamic increment." It is often greater than the power thrust.

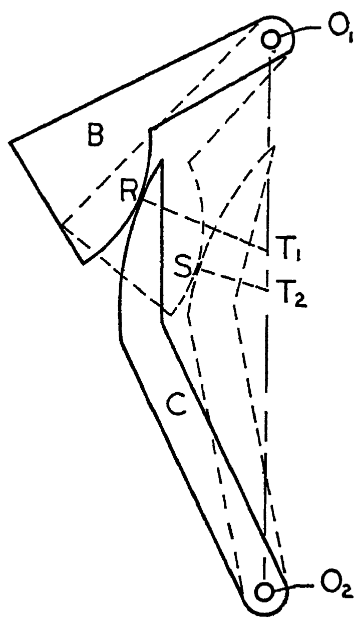


Fig. 9–4.

The nature of tooth outlines that will transmit at constant angular velocity ratio will be investigated with the aid of Fig. 9–4 which shows two simple contact surfaces that might be the sides of two teeth, one on gear 1 turning about axis O_1, the other on gear 2 turning on axis O_2. These surfaces represent the general case of direct

contact surfaces. The only limitation placed on their shape is that they be convex to each other so that only one contact at a time is possible.

In the full-line position, R is the contact point, and RT_1 the normal to the surfaces at the contact point. According to § 2–10, T_1 is the centro bc. T_1 also qualifies as the pitch point. It has the same velocity as a point on both B and C relative to all third bodies; consequently,

$$(O_1T_1)\omega_B = V^{-}T_1 = (O_2T_1)\omega_C \tag{2}$$

or

$$\frac{\omega_B}{\omega_C} = \frac{O_2T_1}{O_1T_1} \tag{3}$$

An instant later, when B and C have moved to the broken line positions, and the contact point is at S, by the same reasoning the pitch point will be at T_2, and

$$\frac{\omega_B}{\omega_C} = \frac{O_2T_2}{O_1T_2} \tag{4}$$

Manifestly, to keep ω_B/ω_C constant, the pitch point (centro) T must remain fixed for all possible contacts. Furthermore, if several pairs of teeth are simultaneously in contact, the conclusion applies to all.

The law controlling the shape of satisfactory gear teeth is that **all profiles must be such that the normals to the surfaces at all possible contact points must pass through a fixed point on the line of centers.** This point is the pitch point. Curves which make contact in such manner as to fulfil this law are said to be **conjugate** to each other.

There is no limit to the number of distinct profiles that will give theoretically correct conjugate action. Suppose that a metal gear tooth is made of any shape whatever, except that its surface is convex at all possible points of contact. If this tooth is fastened on a cylinder and made to mesh with another cylinder of plastic material, the two cylinders being rotated at constant relative angular velocity, the metal tooth will form, in the plastic cylinder, a perfect conjugate tooth surface. Involute curves, however, have so many advantages, both theoretical and practical, over other possible profiles for gear teeth, that they are almost exclusively used.

9–3. **Involutes as Tooth Profiles.**—In Fig. 9–5, a cylinder is represented in end view by the circle of radius Ot. At the distant end of the cylinder, a flange extends to c. On this cylinder is wrapped a cord dtb, having a pencil point attached at b. Starting with the pencil point at a, and keeping the cord taut as it is unwound, the pencil point will describe the involute ab, on a flange attached to the cylinder. The curve can also be described by rolling a straight rod on the cylinder without sliding, and

having the pencil point attached to the rod. The loci of the relative motions of all points on the rod with respect to the cylinder will be involutes or segments of involutes.

Consider now that the cylinder is rotated counterclockwise, the cord being reeled in until b is in position t. Point a will also be in position t. If the cord is now pulled in a straight line from t to b, the cylinder will be rotated clockwise and the same involute ab will be described, because the relative motion between cord and cylinder is the same as before.

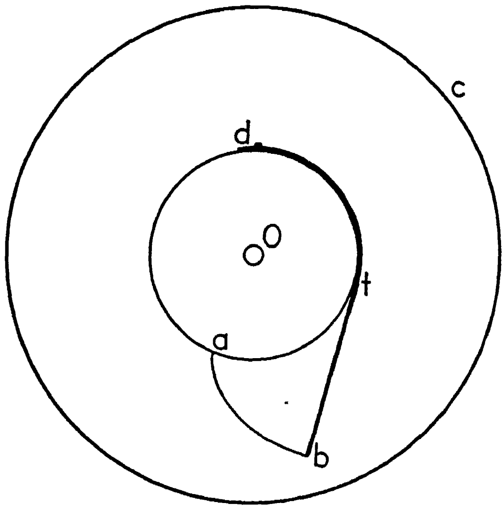


FIG. 9–5. Generation of the Involute.

The involute has one very distinctive property. The circle representing the cylinder from which the cord unwinds is called the **base circle,** and the involute at any point has a direction normal to the tangent running from the point to the base circle. At a, the involute is normal to the base circle, and its curvature is infinite. Advancing from a, each successive portion of the curve has a longer tangent and reduced curvature. The rate at which the curvature changes depends on the size of the base circle. All involutes described on the same size of base circle are identical, and on base circles of different size, are different.

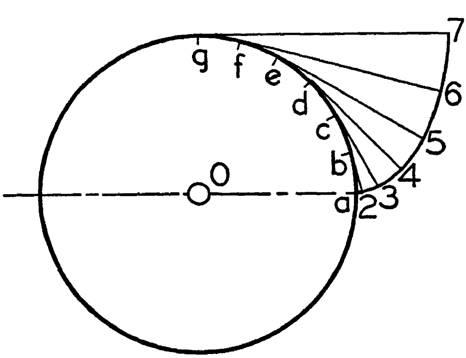


FIG. 9–6. Methods for Drawing the Involute.

A practical method for drawing the involute is illustrated in Fig. 9–6. Select as much arc of the base circle to be used as will give about the length of involute required, and divide this arc into as many equal parts as it is desired to have plotting points on the curve. From each division point on the arc draw tangents, as $b2$, $c3$, $d4$, etc. With dividers, step

off the length of arc ab, and, on the tangent, step off the same distance $b2$ giving the point 2 on the involute. As another example, the length $d4$ on the tangent must equal the arc ad.

Greatest accuracy does not result from using extremely small divider steps. A little practice will enable one to step on the outer edge of the arc sufficiently to offset the difference in length between arc and chord. Do not press the divider points into the paper. In drawing the curve through the plotted points, remember that the involute has a direction normal to its base circle at its initial point, and has the sharpest curvature at the same point. It is sometimes convenient to compute the equal arc lengths from the angles they subtend.

Another method which is not theoretically exact but which can generally be executed with satisfactory accuracy, is Rankine's approximation for rectifying an arc. Taking the case of point 4 on the tangent $d4$, produce the chord ad to h (not shown), making $dh = \frac{1}{2}\,ad$. With center h and radius ha swing an arc to intersect $d4$ at 4. Then $d4$ is the length of the arc da very closely.

Involute curves will now be examined to determine whether they fulfil the fundamental requirement for the outlines of gear teeth, as developed in § 9–2. In Fig. 9–7, two gear blanks, 1 and 2, are mounted to rotate on fixed centers O_1 and O_2 respectively, and on these blanks the base circles of radius O_1a and O_2h are drawn. A cord is wound clockwise on the base circle of gear blank 1, is stretched tightly between a and h, and is wound counterclockwise on the base circle of gear blank 2. Now imagine that the two bodies are rotated in the sense of the arrows, the cord being always taut. A pencil point, attached to the cord, initially at a, will travel down the common tangent ah of the two base circles as the cord unwinds from base circle 1, and winds on base circle 2. In traveling from a to R, the pencil will describe on body 1 the involute bR, and on body 2 another involute, a portion of which is dR. These two curves are described simultaneously by the one pencil. Imagine the flanges, extending from the two bodies, to be spaced axially with sufficient clearance so that the cord can travel between them. Then the pencil, being double pointed, can describe simultaneously an involute on each body.

The direction of involute bR at R is normal to the tangent to its base circle Ra, and the direction of involute dR at R is normal to the tangent to its base circle Rh. Hence if bR is made the profile of a tooth on body 1, and dR the profile of a tooth on body 2, the normal to the surfaces in contact at R will be ha. Since the curves are described simultaneously by a pencil point traveling on the straight line aR, the locus of the contact point is aR. Further, since R is a general point, the normal to the

surfaces at any possible contact point will be along the common tangent ah and will cut the line of centers in the fixed point T. The resulting angular velocity, of ratio ω_1/ω_2, will be the constant O_2T/O_1T, thus proving that involute curves meet the principal requirement for satisfactory tooth profiles. In other words they are conjugate.

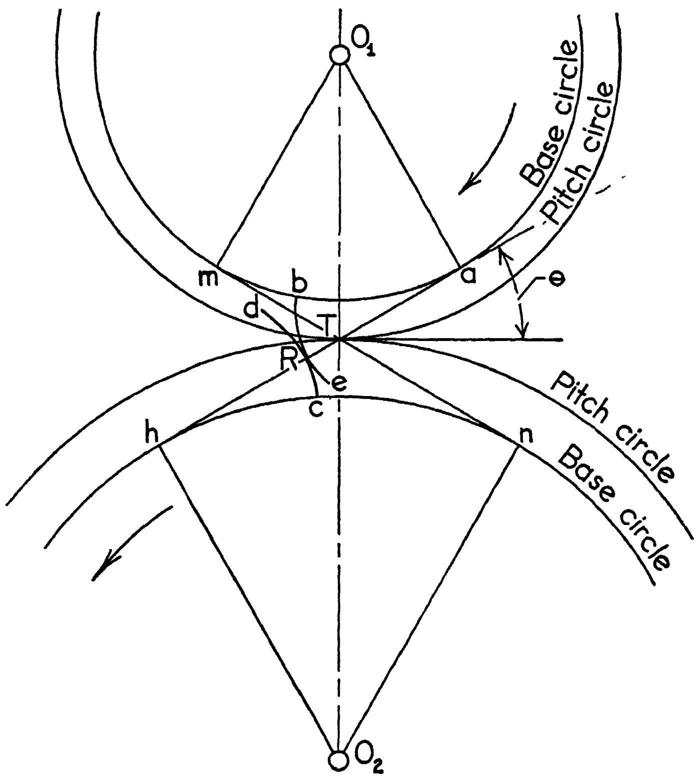


Fig. 9–7. The Proof Diagram for Involutes.

As the pencil point moves from R toward h, the involute bR will be continued toward e, and dR will be completed at c when the tracing point has moved to h. The curves for the opposite sides of the two teeth will be described by a pencil point moving along the common tangent mn.

The **pressure angle,** also known as the angle of obliquity, is the angle between the line of action and a normal to the line of centers. It is the angle θ in Fig. 9–7. The *line of action ha* is the locus of the contact point. It is normal to all possible surfaces of contact between the teeth at the instant those surfaces are in contact. It is, therefore, the line of

action of all transmitting forces between the teeth for ideal frictionless tooth action.

If the center distance of involute gears is changed, they still transmit at constant relative angular velocity but the pressure angle is changed. Suppose O_2 is lowered a small distance. Its involute *cd* is carried down together with its base circle. Its base circle cannot change in size as long as its involute *cd* (its tooth profile) remains unchanged. The line of action *ha* has a new position and inclination θ, but must still be the common tangent to the base circles. T moves downward, and both involutes (teeth) have changed contact points.

There is a perfect analogy between involute-gear action and transmission by means of a crossed belt running on the base circles as pulleys. The belt would lie on the lines of action *ah* and *mn*. If the centers O_1 and O_2 are moved farther apart, a longer belt would be required but all parts of the belt must have the same linear velocity; therefore the two base circles have the same linear velocity, and $\frac{\omega_1}{\omega_2} = \frac{O_2h}{O_1a}$ = a constant. Thus the pitch point moves on the line of centers, and the pitch circles change in size though not in ratio, but *the base circles are fixed on the gears as long as the involutes are fixed.*

These facts also appear from the geometry of Fig. 9–7. The triangles O_1aT and O_2hT are similar, having right angles at a and h, and equal angles at T, and remain so as O_2 and T move.

$$\frac{\omega_1}{\omega_2} = \frac{O_2T}{O_1T} = \frac{O_2h}{O_1a} = \text{a constant}$$

This property of involute gears has important practical significance. Even slight wear in bearings changes center distances measurably. Backlash would of course be increased, but perfect gear-tooth action would still be possible.

The **layout** of a pair of involute spur gears requires first the location of the gear centers and pitch point from the specified pitch and number of teeth in gear and pinion. Through the pitch point T, Fig. 9–7, the line of action *ha* is drawn making the specified angle of action θ with a normal to the line of centers. The two base circles must be tangent to the line of action and this determines their size and location.

On the base circle of gear 1, an involute such as *be*, Fig. 9–7, is constructed at any convenient place around the circumference. The involute is next traced on a piece of transparent paper which must extend over the center O_1, and have that center marked on it. By rotating the tracing paper about O_1, one side of all teeth of gear 1 can be indicated by prick points and drawn. By turning the tracing paper over, the traced involute

gives the opposite sides of the teeth of gear 1. The above process is repeated for the teeth of gear 2, using an involute constructed on its base circle.

The teeth must be carefully spaced by measuring arcs on the pitch circles equal to the circular pitch. Angles equal to $360°/N$ may well be used. Note that the circular pitch is an arc on the pitch circle, *not on the base circle.* In completing the profiles, one will be guided by the standards given in § 9–4. Barring other specifications, the flanks of the teeth between base circle and fillet will be radial straight lines.

A convenient dimension on involute gears is known as **base pitch.** It is circular pitch measured along the base circle. It is also called *normal pitch* because it is the distance between successive contact points along the line of action.

Backlash is looseness or play of meshing teeth. It is generally measured by feelers at the pitch point, in which case it is a dimension along the line of action.

9–4. **Standard Proportions for Gear Teeth.**—In the earlier years of gear manufacture, each company specializing in the production of gears and gear cutting tools developed its own standards. This resulted in lack of interchangeability, difficulty in making repairs, the necessity of carrying large replacement stocks, and general waste. Absence of a reasonable amount of standardization generally results in waste. The American Gear Manufacturers' Association (A.G.M.A.) sponsored a program which resulted in the adoption by the American Standards Association (A.S.A.) of the standards of Table 9–1.

Each of the four types has a special field of application. The merits and limitations of each will appear as our study proceeds. A comparison on the basis of strength can be made with the aid of Fig. 9–8. For the same pitch and number of teeth in the gear, the $14\frac{1}{2}°$ full-depth form is the weakest of the three. A change of the pressure angle to 20° gives the tooth much more sturdiness at the base where the bending moment imposed on it as a cantilever beam is greatest.

The stub tooth has a broad base and shorter length of cantilever. The latter results in a lower bending moment for the same applied tooth load. The stub tooth is best as to strength, but there are several other comparisons which will be developed later. The composite tooth is treated in § 9–8. Its strength is practically the same as that of the $14\frac{1}{2}°$ full-depth involute.

Cycloidal gearing has not been included, because, while the oldest and at one time most used system, it is now almost obsolete except for large cast-iron gears of coarse pitch where the teeth are cast to shape and not machine cut. The outlines of the teeth are cycloidal curves

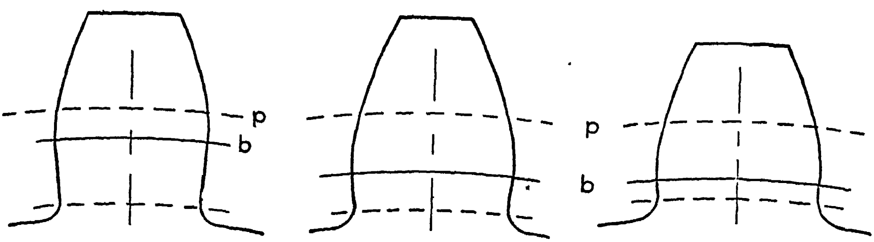
TABLE 9–1

A.S.A. Standards for Spur Gear Teeth

	For Formed Cutter Method	For Production by Generating Methods		
	$14\frac{1}{2}°$ Composite	$14\frac{1}{2}°$ Full-Depth Involute	20° Full-Depth Involute	20° Stub Involute
Addendum	$\frac{1}{P}$	$\frac{1}{P}$	$\frac{1}{P}$	$\frac{0.8}{P}$
Minimum Dedendum	$\frac{1.157}{P}$	$\frac{1.157}{P}$	$\frac{1.157}{P}$	$\frac{1}{P}$
Minimum Clearance	$\frac{0.157}{P}$	$\frac{0.157}{P}$	$\frac{0.157}{P}$	$\frac{0.2}{P}$
Radius of Fillet	$1\frac{1}{3}\times$clearance	$1\frac{1}{3}\times$clearance	$1\frac{1}{3}\times$clearance	$\frac{0.3}{P}$

P = diametral pitch

which give theoretically constant angular velocity ratio, but each side of the tooth has a double curve, making it more difficult to cut accurately. Further, a change in the shaft center distance of cycloidal gears spoils the correctness of the action, while, in this respect, involute teeth are perfect except for backlash.



(*a*) $14\frac{1}{2}°$ Full Depth (*b*) 20° Full Depth (*c*) 20° Stub

Fig. 9–8. Standard Involute Tooth Profiles.

These are drawn using plotted involutes, § 9–3. For comparison, all are for 20-tooth gears of 2 diametral pitch. The exact contour of the tooth below its base circle will depend on the shape of the cutter tooth and the method of generation.

Example: A pair of $14\frac{1}{2}°$ full-depth 4-pitch involute gears has 32 and 48 teeth. Find the principal dimensions.

Using subscript 1 for the smaller gear and 2 for the larger, the pitch diameter $D_1 = 32/4 = 8$ in., $D_2 = 48/4 = 12$ in. The center distance is $(D_1 + D_2)/2 = 10$ in. The addendum of each gear is $1/P = \frac{1}{4}$ in., Table 9–1, the minimum dedendum 0.2893 in., the clearance 0.0393 in., the radius

of fillet 0.0524 in. The diameter of the base circle of the smaller gear, $D_{b1} = D_1 \cos\theta = 8 \times 0.9681 = 7.745$ in., and $D_{b2} = 11.617$ in. The circular pitch $p = \pi/4 = 0.7854$ in. and, if there is no backlash, this is equally divided between circular thickness of tooth and circular thickness of space. The base pitch is $p \cos\theta = 0.7854 \times 0.9681 = 0.76035$ in.

Gear cutters, including hobs, for all forms of teeth are more commonly listed and stocked in the following preferred diametral pitches:

from 1 to 3 varying by quarters,
from 3 to 6 varying by halves,
from 6 to 12 in whole numbers,
from 12 to 24 in even numbers.

Teeth coarser than one diametral pitch should be specified in circular pitch. The above pitch numbers have not been standardized, but are indicative of common practice.

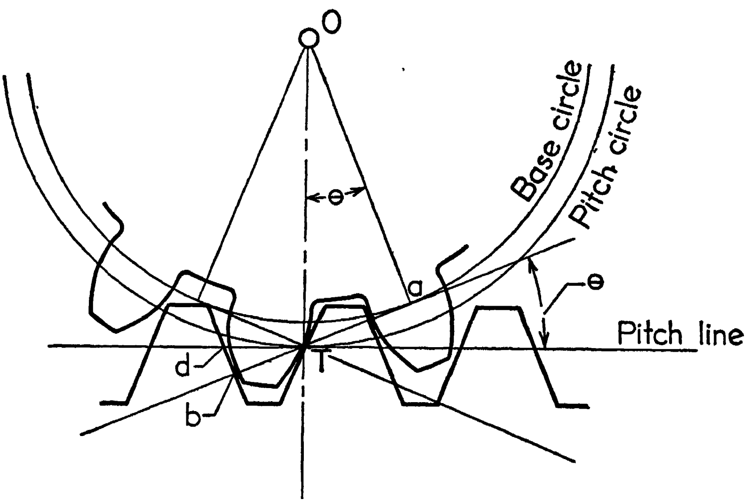


FIG. 9–9. Rack and Pinion.

A special "fine pitch" tooth form [1] is used for instrument and clock work. The form is 20° involute with addendum $1.350/P$, working depth $2.700/P$, clearance $0.3269/P$. The diametral pitches range from 24 to 120.

9–5. **The Rack and Pinion.**—A rack, theoretically, is a segment of a gear of infinite pitch diameter, so the pitch circle of a rack is a straight line. In Fig. 9–9 a rack is shown in mesh with its pinion. Using the cord-and-pencil-point device of § 9–3, we imagine the cord wrapped on the

[1] Buckingham, Manual of Gear Design, Section Two, p. 156.

base circle of the pinion, and the pencil point traveling down the tangent aT. Since the pressure angle cannot vary, the cord may be imagined to extend down this line to infinity. Now each portion of the involute on the rack has infinite radius and is normal to the tangent aTb, as it is drawn, and since the rack has no rotation, a straight-sided rack tooth results. As the side of the rack tooth is normal to the line of action (parallel to Oa), it is inclined at the pressure angle θ to its own center line.

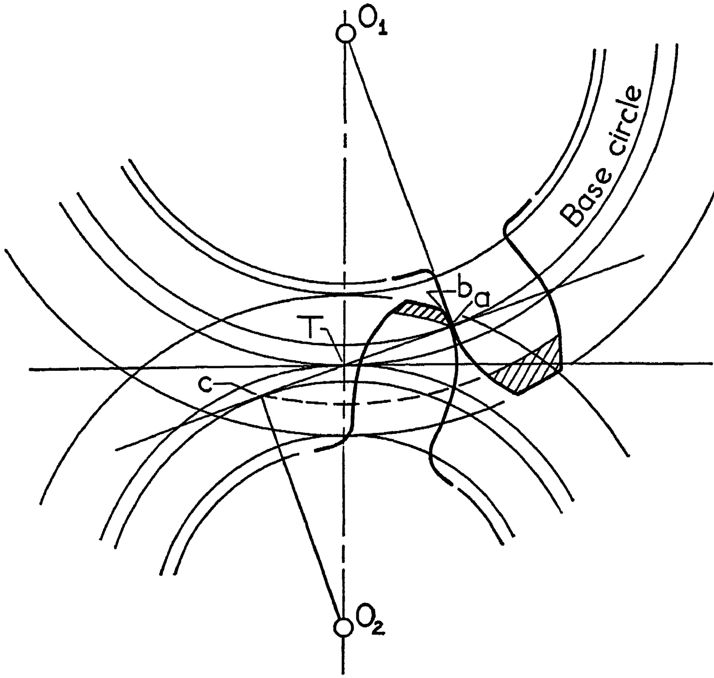


FIG. 9–10. Tooth Interference.

From its nature, the rack is limited to reciprocating action, but it is easy to manufacture, and can be made and measured with precision. As will be shown later, it is used as a cutter to generate other cutters and gears having curved involutes.

9–6. Interference and Undercutting.—Interference on involute gears results from contact where the surfaces are not both involute in profile. Suppose that the contact point of the two teeth shown in Fig. 9–10 travels up the line of action from T. Between T and a there is action between involutes, a being the point of tangency of the line of action with the base circle of gear 1. If action occurs beyond a, as it will in this case, there will be interference, and a is called the *interference point*. Although

the profile of the lower tooth is involute to its corner b, there can be no involute above a on the upper tooth. If the flank is formed as shown, there will be contact off the line ac, and the lower tooth will gouge the flank of the upper tooth. Interference is often defined as contact that is not on the involute line of action.

All this presupposes that the profiles of the teeth between base circles and fillets are radial straight lines (a contour called the radial flank), or curves that will give the teeth an even wider base. By making the flank sufficiently concave, interference can be avoided. This is called **undercutting.** It weakens the tooth close to the section subjected to the maximum bending moment, and is therefore highly objectionable.

Another solution is to remove metal at the tip of the tooth from a to b. This is called *tip relieving*, and is frequently done, but in certain cases it requires special cutters. Shortening the addendum would have the desired effect. One good solution however is to select, where possible, standard gears that do not project beyond the interference points a and c. Such selection is treated in § 9–18. Other methods of eliminating interference and of putting practical limitations on undercutting can be understood better after a survey of the methods of gear cutting.

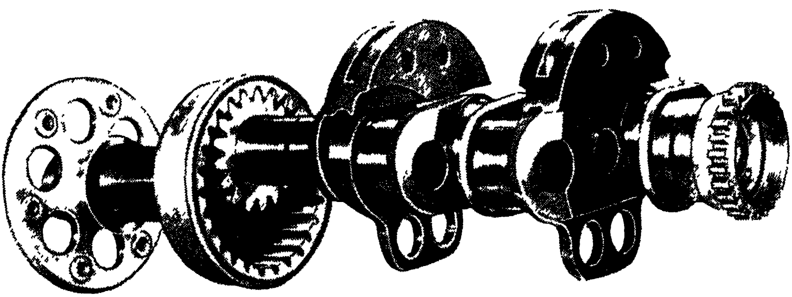


FIG. 9–11. Engine to Propeller Reduction by Annular Gearing. (Two counterweights removed.)

Lycoming Division of Aviation Corp., Williamsport, Pa.

There exists another type of interference, namely where the outer corner of a tooth strikes the radius at the bottom of the mating tooth space. In the rare cases when it occurs, it is necessary either to relieve the tips of the offending teeth or to cut a sharper fillet by extending the corners of the teeth of the cutter used on the mating gear.

9–7. **Annular Gears.**—If one gear of a pair of spur gears is enlarged until its pitch radius is infinite, a rack results. If the change in curvature of the pitch circle is continued, both gear centers appear on the same side of the pitch point, and the result is an annular or internal gear and pinion.

The results are rather remarkable. The addendum of the annular tooth is now toward its gear center, and the tooth appears to be turned inside out, as in fact it is. An application is illustrated in Fig. 9–11. In aircraft, desirable propeller speeds are generally less than desirable engine speeds, so direct drive is a compromise. The annular gear reducer is a compact drive which, in this application, allows both engine and propeller to operate at optimum speeds.

The layout necessary to derive tooth profiles is similar to that for external gears. The line of action *db*, Fig. 9–12, is drawn through the

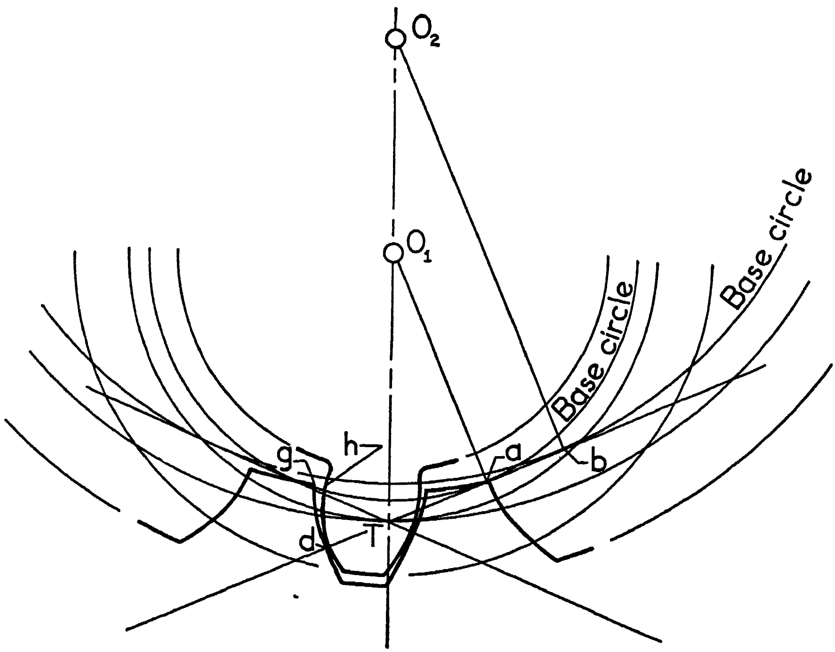


Fig. 9–12. Annular Gear Teeth.

pitch point T, at the required pressure angle with a normal to the line of centers O_2O_1. On this line of action, normals O_2b and O_1a are dropped from the gear centers giving the respective base-circle radii of the two gears. Now imagine two cords, one wound on each base circle and both stretched toward d. They will lie together from a to d and beyond. The peripheral speeds of the two base circles should be the same, and they will be, since the triangles O_1aT and O_2bT are similar. Therefore, the two cords will travel at the same speed down the common tangent ad, while the two gear blanks revolve clockwise at such angular velocities that their pitch circles do not slide. Now, if a pencil point is attached to both cords

at *a*, it will trace, as it travels to *d*, the involutes *gd* on the annular gear 2, and *hd* on 1. As these curves roll and slide on each other, the line of action will always pass through *T*, so constant angular velocity ratio results.

It may be noted that the annular tooth does not project inward quite up to its own base circle. Suppose the pencil point is traveling from *d* toward *a*. At *a*, it has completed the involute on the tooth of gear 1, and the remainder of that tooth is radial inside its own base circle. The pencil point might continue from *a* to *b* on one cord and complete the involute on the annular tooth to its base circle, but there can be no in-

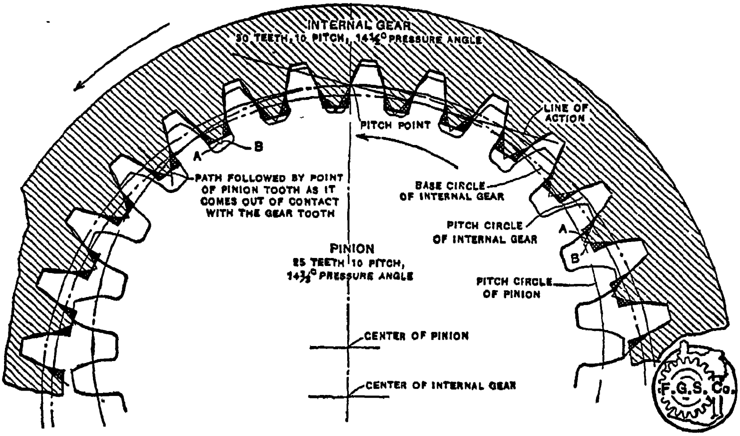


FIG. 9–13. Secondary Interference of Annular Gears.

The Fellows Gear Shaper Co., Springfield, Vt.

volute on the mating tooth to mesh with this portion. Further, if this portion is not omitted, interference will result on the radial flank of the pinion tooth.

There is a secondary interference possible that is peculiar to annular gears. This is most marked when the pinion is so large as to approach the size of the annular gear. An extreme case will illuminate the difficulty. Assume that the pinion has one tooth fewer than the gear. Any tooth of the pinion will not withdraw completely out of the intertooth space in any phase, and still this tooth must move back one space every revolution.

Fig. 9–13 indicates the extent of this secondary interference when a 30-tooth annular gear meshes with a 25-tooth pinion. The advice of the Fellows Gear Shaper Co. on this point is, "As a general rule, the smallest permissible *difference* between the number of teeth in the pinion and internal gear, to give proper tooth action without considerable

modification of tooth shape, is 7 teeth for 20° stub-tooth form, and 12 teeth for full-length $14\frac{1}{2}°$ involute form."

Hobs (§ 9–11) or rack cutters cannot be used to cut the internal gear, so the only generating method possible is the gear shaper (§ 9–9). Another difficulty peculiar to annular gears arises unless the cutter is fairly small. When the cutter is being fed to depth initially, it may trim the corners of a few teeth on the gear, cutting away useful contact surfaces. The same condition may produce assembling difficulty. If the pinion is above a certain size relative to the gear, the only possible way to assemble the pair is to slide them together axially. These facts indicate that internal gears require more designing than external gears.

The 20° pressure angle gives more latitude in avoiding these difficulties, but special values of addendum and dedendum are required. Fig. 9–12 shows that the difference between pitch radius and internal radius of the gear tooth must be considerably less than the standard addenda of Table 9–1 to avoid interference. The following dimensions [1] have been found satisfactory for the 20° pressure angle where the pinion has 16 or more teeth.

Addendum of pinion tooth	$\dfrac{1.25}{P}$
Radial length of gear tooth inside pitch circle	$\dfrac{0.60}{P}$
Circular thickness of pinion tooth	$\dfrac{1.7528}{P}$
Circular thickness of gear tooth	$\dfrac{1.3888}{P}$

The use of unequal tooth thickness along the pitch circle is explained in § 9–12.

In spite of the difficulties, annular gears have decided merit. The form of the gear tooth could hardly be better designed for strength if that were the only requirement. There is less sliding, that is, a nearer approach to true rolling, than with comparable external gears. A long contact line is possible. Well designed and accurately cut annular gears have smooth performance and long life. The computation of their contact ratio will be covered in § 9–16.

9–8. $14\frac{1}{2}°$ Composite System and Cutting Methods.—The basic rack of this system has tooth profiles that are straight lines (true involutes) for only the middle three-eighths of the working depth. Above this the tips are relieved by a blending circular arc of radius $3.750/P$, and the flanks are filled in with an arc of like curvature. The system was origi-

[1] Buckingham, Manual of Gear Design, Section Two, p. 108.

nally a combination of the involute and cycloidal systems, hence the name. The tip relieving avoids interference which would otherwise be serious with low numbers of teeth. The length of contact of each pair of teeth is reduced however.

Gears of this system are regularly cut with formed milling cutters, Fig. 9–14 (*a*). The finishing cutter is ground to the shape of the intertooth space. The setup is as shown in Fig. 9–14 (*b*) which indicates that helical gears designed to connect non-parallel shafts, as well as spur gears, can be so cut. The milling machine and indexing head are usual machine shop equipment, and the cutters are not expensive. Unless quantity production is considered, this is an inexpensive method compared to using highly specialized gear cutting machines. However, a high degree of precision can seldom be achieved with the formed cutter method, so it is limited to the production of relatively slow-speed gears.

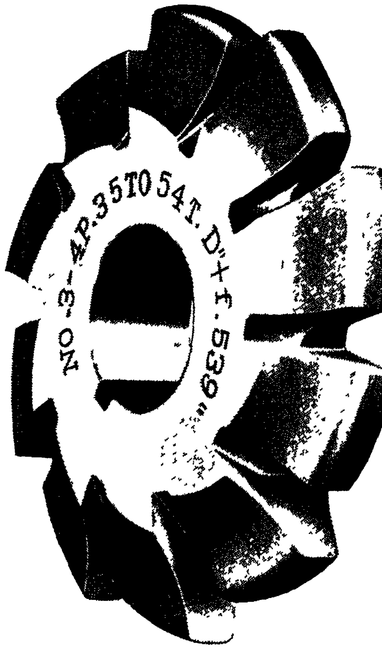


FIG. 9–14 (*a*). Formed Cutter.
Brown and Sharpe Mfg. Co., Providence, R. I.

The cutters required to machine gears of any number of teeth of a *single* pitch are listed in Table 9–2. For the same pitch, the shape of the intertooth space changes with the number of teeth. The rack has nearly straight sided teeth with center lines parallel. The twelve-tooth pinion has sharply curved teeth with center lines of adjacent teeth making an angle of 30°. Between these extremes, the shape of the intertooth space changes with every change in number of teeth, the changes being more marked at the small number end, as indicated by Table 9–2. Due to the tip relieving, there is no interference in these gears if cut by the proper number of cutter or by properly shaped hobs, as long as the tooth numbers are twelve or higher. Fewer than twelve teeth are not used because of the weak undercut shape.

It is plain that this system can, at best, give only an approximation of accurate profiles. For example, the No. 4 cutter is used for 26 to 34 teeth inclusive. It can have correct shape for only one of these tooth numbers. The "extra" cutters give a closer approximation. By ad-

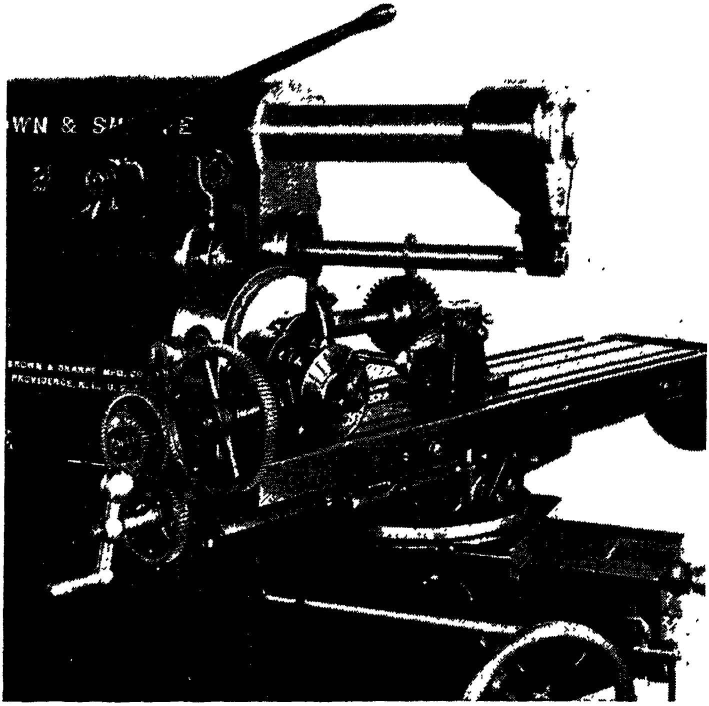


FIG. 9–14 (b). Cutting Gear Teeth by the Formed Milling Cutter Method.
Brown and Sharpe Mfg. Co.

TABLE 9–2
FORMED CUTTERS FOR THE COMPOSITE SYSTEM

Regular		Extra	
Cutter No.	Teeth in Gear	Cutter No.	Teeth in Gear
1	135 to rack	1.5	80–134
2	55–134	2.5	42– 54
3	35– 54	3.5	30– 34
4	26– 34	4.5	23– 25
5	21– 25	5.5	19– 20
6	17– 20	6.5	15– 16
7	14– 16	7.5	13
8	12– 13		

justing the depth to which the cutter is fed into the gear and pinion,[1] a further approximation of the theoretically correct profile is possible.

9–9. The Generating Principle and the Gear Shaper.—The generating principle is the basis of the most accurate methods for machining gear

[1] Buckingham, Earle, Manual of Gear Design, Section Two, p. 47.

teeth. These methods are also most economical, provided production is in such quantity as to justify the expensive special machines required.

If a gear blank were made of some plastic material such as putty, and rolled with a properly cut involute metal gear at proper meshing depth and with pitch circles in pure rolling contact, teeth would be molded on the plastic gear. Neglecting, for the moment, practical difficulties, the molded gear would have accurate involute teeth regardless of the number of teeth in metal or molded gear, and even clearance could be molded in, if the teeth of the metal gear were made that much longer. Now, if to this generating principle is added the shaping process, an accurate gear cutting method results. The above metal gear is now made the cutter, as shown in Fig. 9–16, and is traversed axially across the face of the blank to remove the metal. The centers of the cutter and blank are first fed towards each other until the proper depth is attained; then the rotation of both cutter and blank begins and one complete rotation of the blank completes the gear. The shape of the chips removed in successive cuts is shown in Fig. 9–15 (*a*), and at (*b*) the relative position of cutter and blank is shown.

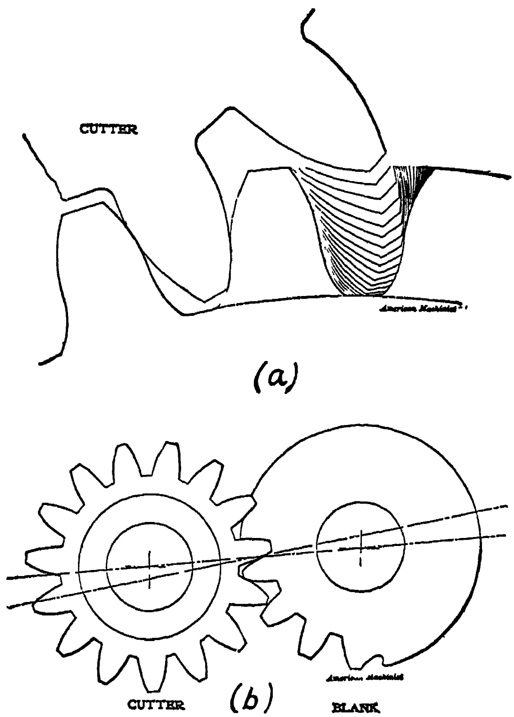


FIG. 9–15. The Generating Principle.

The Fellows gear shaper, Fig. 9–16, performs the several feeding operations automatically. The gear being cut is shown at the right of the cutter which is traversed vertically. The vertical cutter spindle is reciprocated by the horizontal rocking beam fulcrumed back of the large nut. In this cut, the guard of the driving mechanism has been swung back to show the crank and connecting rod which drive the rear end of the rocking beam. Both are adjustable in length. The length of cutter stroke is varied to suit gears of different face width by adjusting the

length of the driving crank, and by changing the length of the connecting rod the stroke of the cutter is positioned with respect to the gear blank. Fig. 9–17 is a detail view of the cutter. Its teeth have cutting angles and clearance like a regular shaper tool.

The cutting operation is started with the gear stationary and the cutter being fed in as it reciprocates, until the proper depth of tooth is

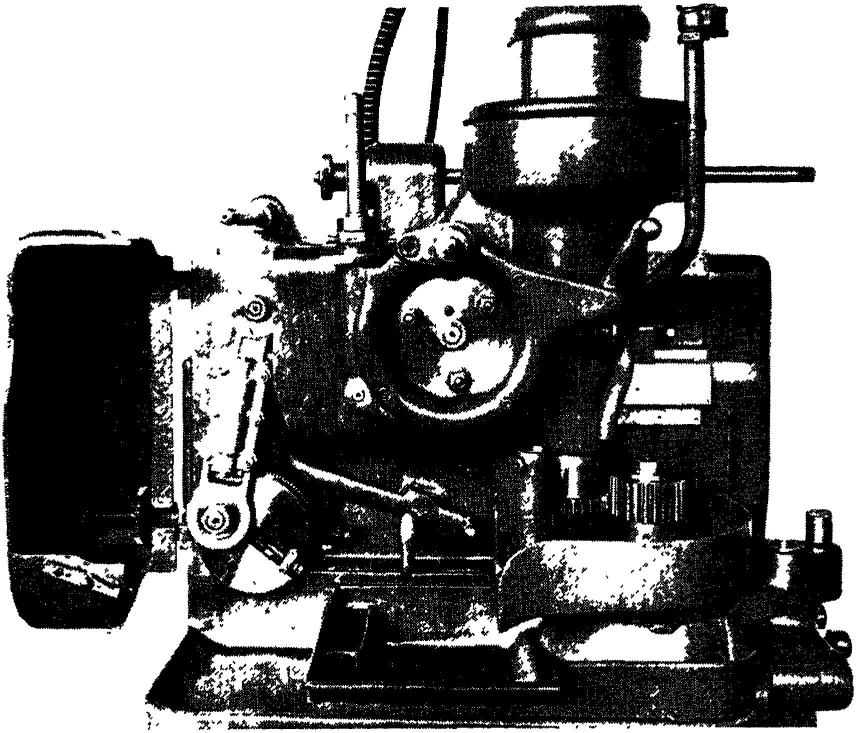


FIG. 9–16. High Speed Gear Shaper.
The Fellows Gear Shaper Co., Springfield, Vt.

attained. Then gear blank and cutter are slowly rotated, both under the control of feed gears, while the cutting proceeds. The entire process is automatic.

An outstanding advantage of the generating process is that one cutter only is required for all gears of the same pitch, pressure angle, and form, and this single cutter will generate theoretically correct tooth profiles on all, regardless of the number of teeth they may have.

9–10. **Rack Cutters—Maag Gears.**—As previously mentioned, the rack cutter can be ground and measured accurately with less difficulty

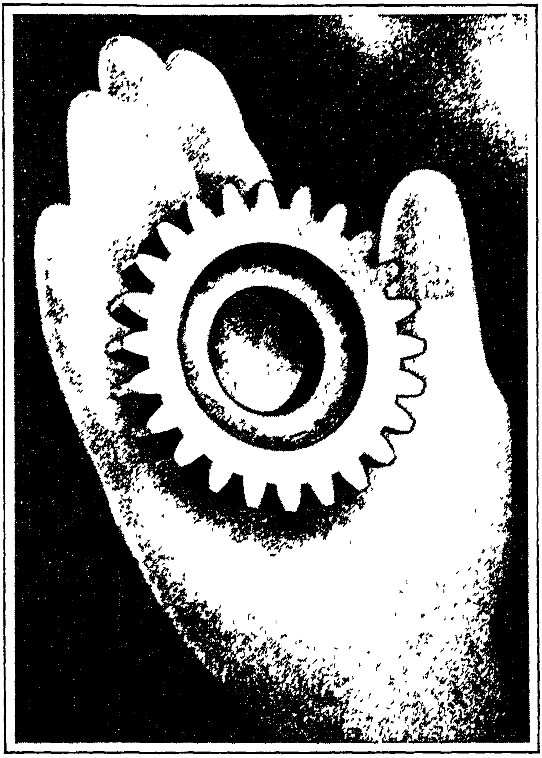


FIG. 9–17. Fellows Gear Cutter.

than is encountered with the cutter having curved teeth. Fig. 9–18 shows the generating action. This must be accompanied by reciprocating shaper motion. Fig. 9–19 shows the set-up for cutting a large gear on a machine specially designed for the rack-cutter method. One disadvantage is the large amount of indexing necessary. For quite small gears of fine pitch, rack cutters are made with a few more teeth than are in the gear, so that a gear can be completed without indexing.

Maag gears, developed in Switzerland and manufactured now by the General Machinery Corp., are based on the use of the rack cutter, the use of nonstandard pressure angles to avoid interference and improve strength, and the use of unequal addenda. The latter will be treated in § 9–12. Of course, all these devices are now used by other manufacturers.

An example of the effect on the profile of an eight tooth pinion, of changing the pressure angle from $14\frac{1}{2}°$ to $27°$, combined with unequal addenda, is illustrated in Fig. 9–20. On the left is the standard $14\frac{1}{2}°$ profile. As few as five teeth in the pinion can be used without under-

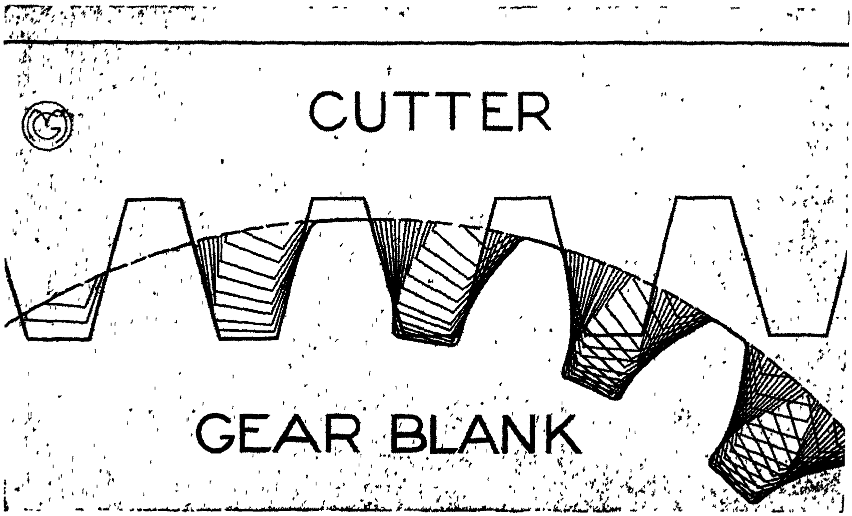


Fig. 9–18. Action of Rack Cutter.

General Machinery Corp., Hamilton, Ohio.

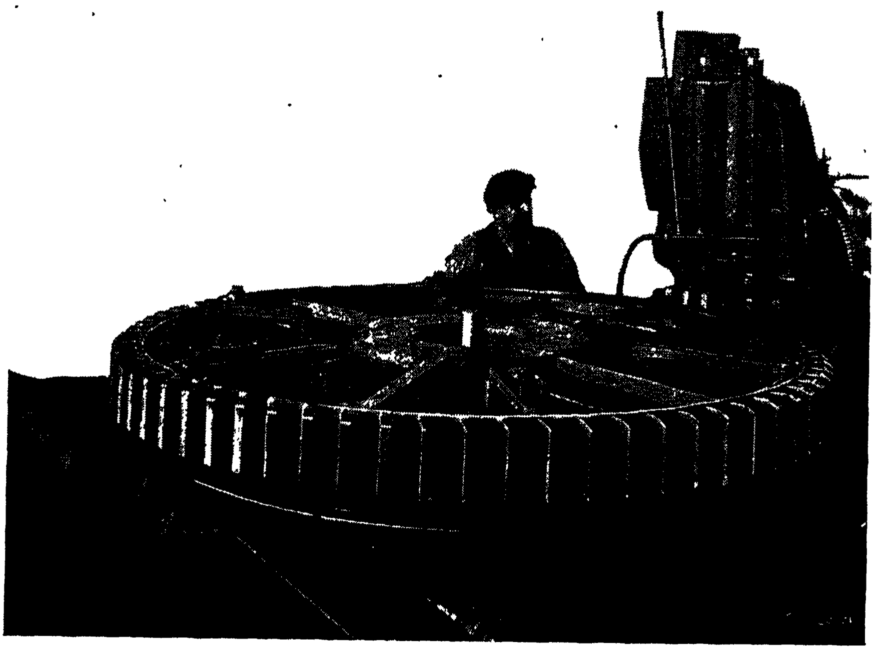


Fig. 9–19. Shaping Maag Gears.

General Machinery Corp., Hamilton, Ohio.

cutting by departing from standards in this manner. Of course the necessarily high pressure angles increase the bearing loads somewhat.

9–11. **The Gear Hob—a Generating Cutter.**—In manufacturing a hob, it is first given the form of a straight or cylindrical worm, then

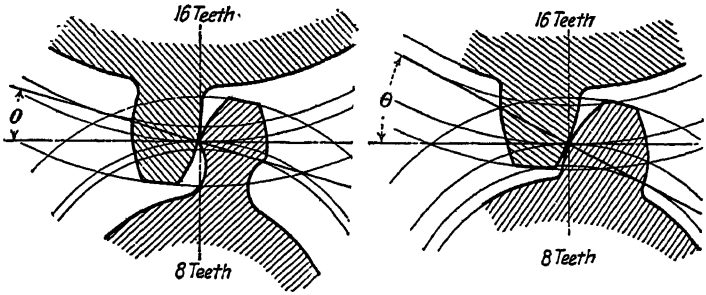


FIG. 9–20. Effect of Increase in Pressure Angle.

From a pamphlet by the Niles Tool Works Division of The General Machinery Corp.

gashed axially to produce the teeth in the shape illustrated in Fig. 9–21. A hob will generate a gear with which it would mesh properly as a worm. If the axis of the hob lies in the plane of revolution of the blank, it will generate a worm wheel, Fig. 10–26. If the hob axis is properly inclined

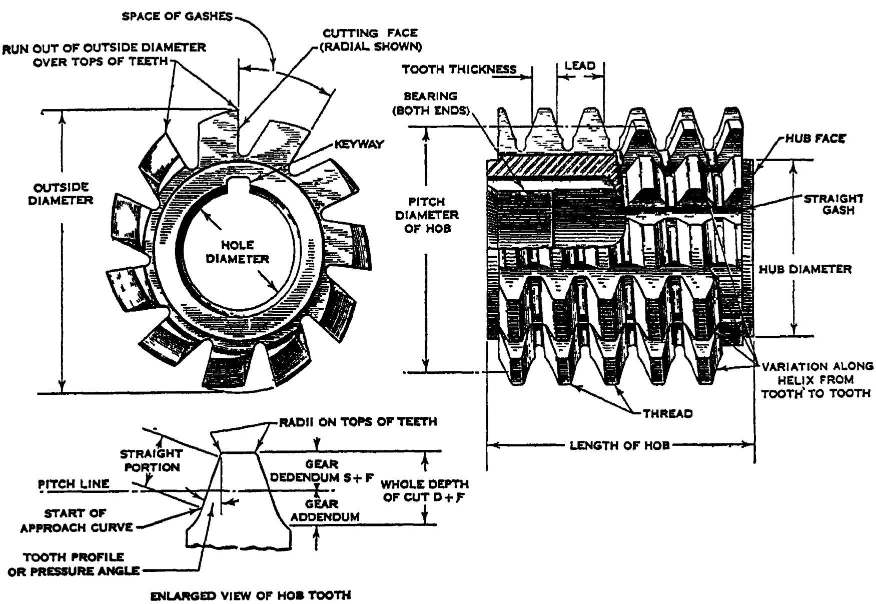


FIG. 9–21. Single Thread Right Hand Hob.

Barber-Coleman Co., Rockford, Ill.

from this position by 90° minus its own helix angle, it will generate spur gears as in Fig. 9–22.

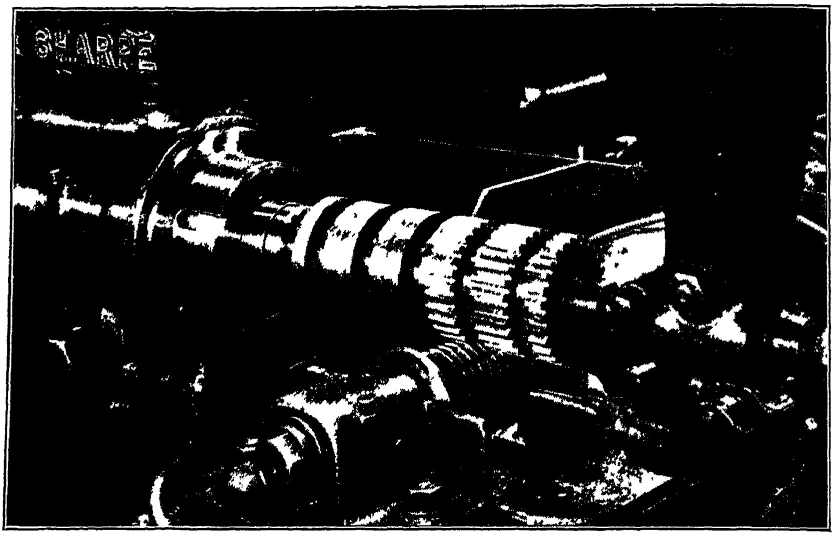


FIG. 9–22. Hobbing Six Spur Gears with One Setup.
Brown and Sharpe Mfg. Co., Providence, R. I.

An axial plane passed through a straight involute worm, Fig. 9–21, gives a section through the threads which has the form of the involute rack. The sides of the teeth have straight-line profile. The hob, there-

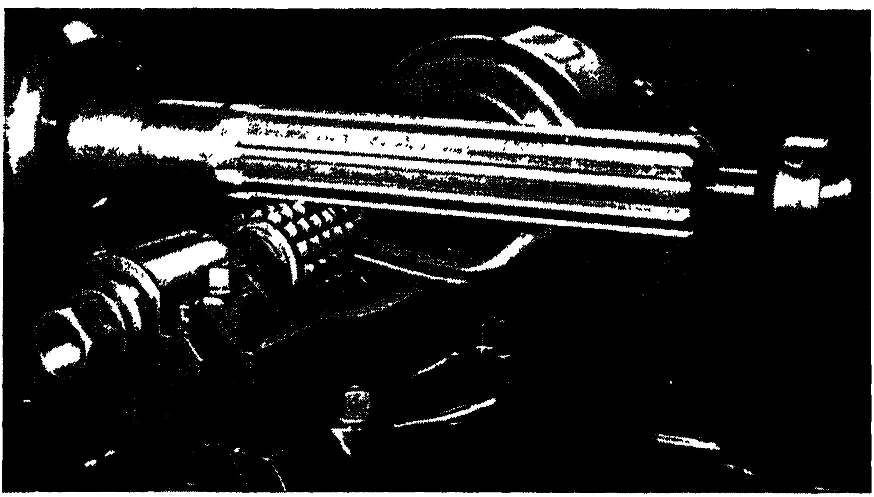


FIG. 9–23. Hobbing Splines.
Barber-Coleman Co.

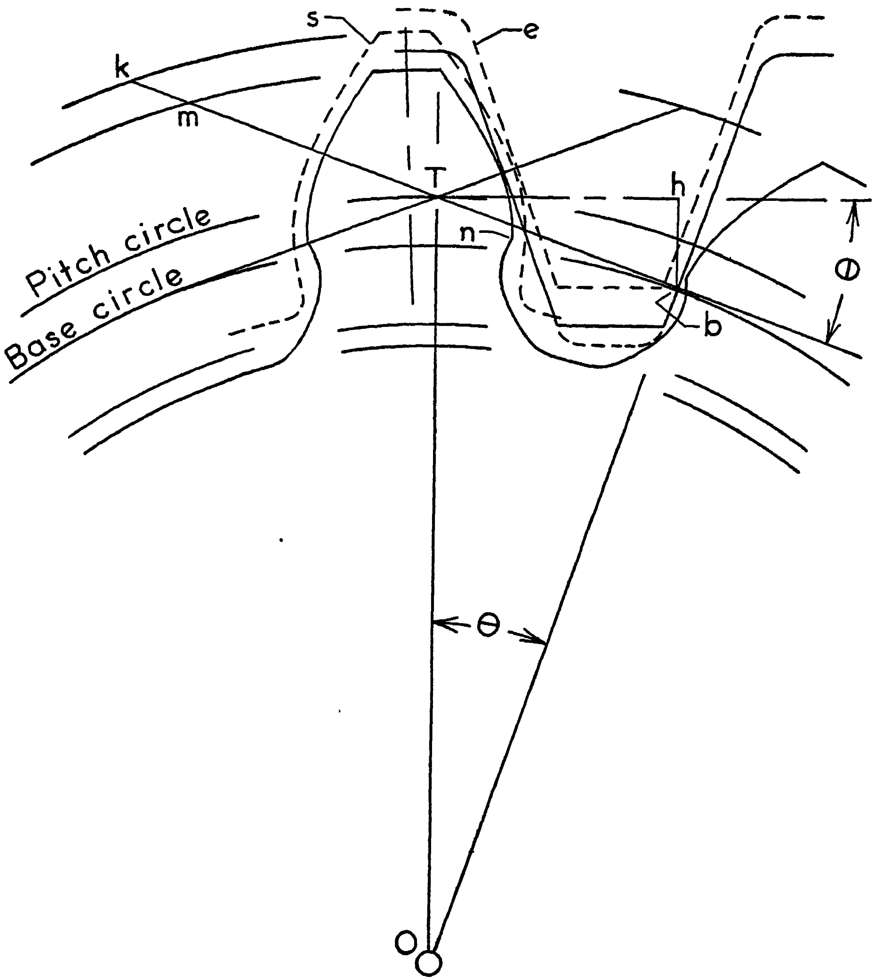


FIG. 9–24. Some Gear and Rack Relations.
(12 teeth, 20° full depth.)

fore, has exactly the same generating characteristics as the rack cutter. It has the advantage over the rack cutter that its action is continuous, thus eliminating indexing during the production of a complete gear.

The hob is used also for the economical production of multiple splines on shafting, Fig. 9–23. The splined hub and mating shaft really constitute a gear clutch. Hobbing has a wide range of application in gear production. For ordinary commercial gears it is speedy and economical. It is also used in the production of large spur and herringbone gears for

such high speed applications as turbine speed reducers and ship drives. The precision required of such gears necessitates constant temperature rooms and continuous operation even though days may be required to cut one gear.

9–12. **Unequal-Addendum Gears.**—Fig. 9–24 represents in the full-lined view a standard 20° full-depth 12-tooth pinion as it would be cut by a standard rack-shaped cutter (rack or hob). The extension of the addendum of the rack tooth necessary to cut the clearance is shown in broken line. There would be no interference between the gear and a rack of the solid-line profile, because the generating action has removed all the interfering metal, but the undercutting is severe.

All undercutting is caused by that portion of the cutter tooth below the interference point *b*, and this includes the undercutting above the base circle of the pinion (up to *n*). Suppose now that the cutter is given the position of the broken-line rack *e*, while the pinion blank is increased in diameter so that teeth having profile *s* are cut. Next place this enlarged pinion in mesh with a rack of profile and position *e*. The following gains result:

(1) Undercutting is eliminated.

(2) The pinion tooth is thicker at the pitch circle and below it. [(1) and (2) combine to strengthen the pinion tooth decidedly.]

(3) Useful contact is increased by the distance *mk* at one end, plus a small amount at the other end.

The rack tooth is weaker to resist forces along the unchanged line of action, but it is still probably stronger than the new pinion tooth. A decided gain in both the strength of the combination and in the length of useful contact has been achieved, and only standard cutters have been used.

Consider next that the rack is replaced by a gear of many teeth to mesh with pinion *s*. Its addendum must be *hb*. Its outside diameter must be smaller than standard by the amount that the pinion has been made larger than standard. Its teeth must be thinner than standard at the pitch circle by the amount that the pinion teeth have been made thicker. Unequal addendum gears can be cut with either generating shaper or rack cutter of standard form, and there will be no change in the pressure angle of the combination. Unless the gears are of unequal tooth numbers, however, there can be no gain from using unequal addenda.

What is the possibility of the large gear undercutting or interfering with the pinion *s*? It will not, because the involute addendum of the large gear tooth must lie entirely within the contour *e*, in all meshing positions.

If a rack does not undercut a pinion, the pinion will not be undercut by any gear of the same addendum as the rack operating on the same line of action.

The A.G.M.A. Handbook gives tables of dimensions for both $14\frac{1}{2}°$ and 20° full-depth unequal-addendum gears. Table 9–3 is quoted as an

TABLE 9–3

DIMENSIONS FOR UNEQUAL ADDENDUM GEARS

Pinion Teeth	20° pressure angle, one diametral pitch		
	X	Circular Thickness of Pinion Tooth	Circular Thickness of Gear Tooth
8	1.0642	1.9581	1.1835
9	0.9472	1.9156	1.2260
10	0.8302	1.8730	1.2686
11	0.7132	1.8304	1.3112
12	0.5963	1.7878	1.3538
13	0.4793	1.7453	1.3963
14	0.3623	1.7027	1.4389
15	0.2453	1.6601	1.4815
16	0.1284	1.6175	1.5241
17	0.0114	1.5749	1.5667

X is the amount by which the diameter of the pinion is to be increased and the diameter of the gear decreased from standard.

For other pitches divide all values by the diametral pitch.

example. Considering the 12-tooth pinion of Fig. 9–24 to be of one diametral pitch and letting r_1 and N_1 represent the pitch radius and number of teeth of the pinion respectively,

$$\frac{Tb}{r_1} = \sin\theta \quad \text{and} \quad hb = r_1 \sin^2\theta$$

$$hb = \frac{N_1}{2P}\sin^2 20° = \frac{12}{2 \times 1} \times 0.342^2$$
$$= 0.70185 \text{ in.}$$

The rack or large gear will be cut back the standard addendum one, minus hb. The outside diameter of the pinion must therefore be increased in the amount

$$2(1 - 0.70185) = 0.59628 \text{ in.}$$

The original tooth thickness measured along the pitch circle, called the *circular thickness*, was $\pi/2$ for a diametral pitch of one. The circular thickness of the pinion tooth will be increased by

$$2(1 - 0.70185) \tan 20° = 0.2170$$

giving a circular thickness of 1.7878. The mating rack or gear will have its tooth circular thickness reduced by 0.2170 in. Compare these results with the tabular values.

The explanations so far given have been based on the assumption that rack-type cutters would be used. Identical results can be obtained with generating shaper cutters. The kinematics of that process will be further developed in the next article.

9–13. **Variable-Center-Distance Systems.**—From the elevation so far attained in the study of gearing, it is desirable that we take another look at the fundamentals of the involute system. In Fig. 9–2 some involute teeth were shown. What is their pressure angle? We do not know. They have none until brought into mesh with other teeth to establish a pressure line. In Fig. 9–8 (*a*), the tooth is said to have a $14\frac{1}{2}°$ profile. That is allowable only because it is a standard tooth designed to be brought into standard relation with another standard tooth. When we depart from these standards, as in the case of unequal-addendum and variable-center-distance systems, it is necessary to examine the action from a thoroughly fundamental standpoint.

The purpose of departing from standards is to obtain strong profiles and a long line of action, to avoid interference, undercutting, and excessive pressure angles, and to accomplish these things with the greatest possible economy of tools.

In all treatment of the variable-center-distance topic it is to be assumed that the numbers of teeth in the two gears considered always remain unchanged. An increase in center distance, as from bd to bf in Fig. 9–25, will cure the obviously bad case of interference. It will also increase the pitch radii and therefore the circular pitch. The original pitch is called the *nominal pitch* of the gears on the new center distance. Speed ratios are *always* dictated by tooth numbers; therefore, the ratio of pitch radii must be unchanged. This locates the new pitch point e, such that $bc/dc = be/fe = N_b/N_d$.

The line of action must always pass through the pitch point and be tangent to the two base circles, giving the familiar pair of similar triangles. The relations $be/fe = bm/fn = N_b/N_d$ dictate the *relative* size of the base circles, also of the pitch circles. In words, *the base circles are proportional to the pitch circles in size, and both are proportional to the numbers of teeth.* Further consideration leads to two distinctive cases:

Case I. Standard generating shaper cutters are to be used. The cutter selected would be of the nominal pitch. It would generate the standard teeth shown in full line, to operate on the standard line of action hk, if the gears were to operate on the standard center distance bd. It

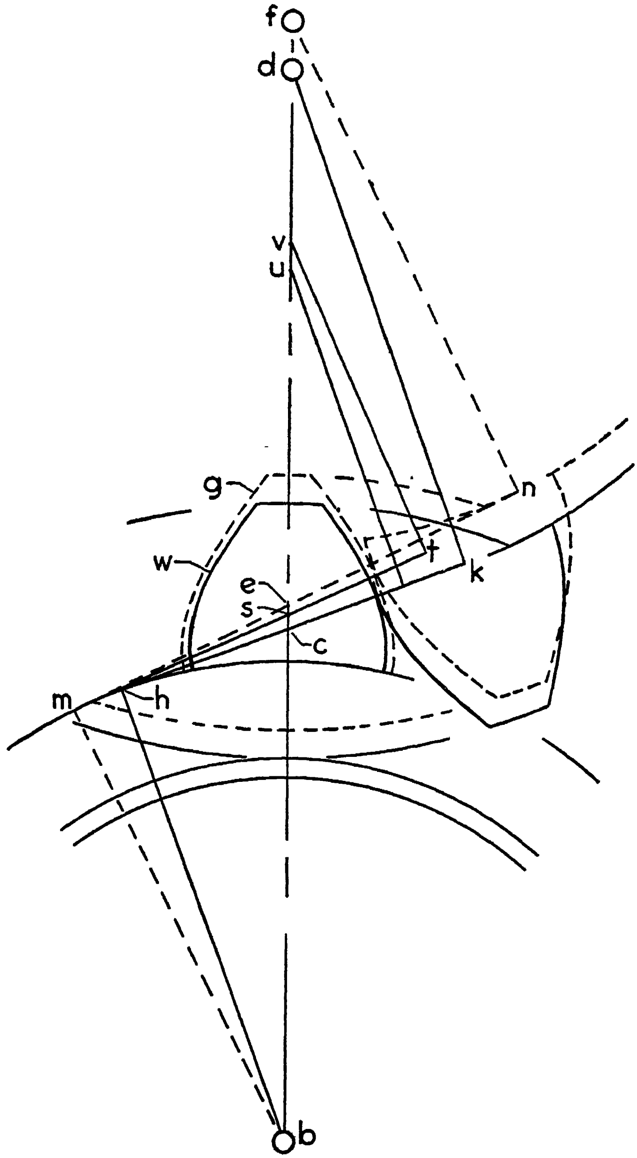


FIG. 9–25. Effect of Changing Center Distance.

will now be shown that this standard cutter can be used to generate the nonstandard gears to operate on center distance *bf*.

The standard teeth would be severely undercut, *b* more than *d*, because it is smaller. It is necessary then to give the smaller gear more than half

the correction, say 60% for illustration. The outside radius of gear blank b will be made larger than standard by 60% of df. The standard cutter will have its center moved up from u to v, where uv = 60% of df. Teeth of standard depth will now be cut on b, its outside diameter having been enlarged accordingly.

The base circle of the cutter will be tangent to some line of pressure st, and s will be the temporary pitch point for the cutting action. From the geometry of the figure, cs is 60% of ce, and the teeth on b will be generated with circular thickness increased by 60% of the total final increase in circular pitch when the pitch point is at e. This gives the profile g. The involutes are the same (except for length) as the standard full-line involutes of b because they have the same base circle of radius bh. The teeth therefore have the same base pitch as the standard teeth. This fact is the key to the use of the shaper cutter in generating both variable-center-distance gears and unequal-addendum gears. The same basic involutes can be generated at various pressure angles and used to transmit at still other pressure angles.

Gear f can be generated in similar manner and by the same standard cutter to have 40/60 as much increase in outside radius and circular tooth thickness as gear b. Now gears b and f will operate perfectly together, having teeth of profile shown in broken line. Their pressure line of operation will be mn. There will be no backlash, interference, or undercutting. The length of the line of action is considerably increased. They have the same tooth numbers and angular-velocity ratio as the standard gears. A single standard cutter does the work.

Case II. Standard rack-profile generating cutters are to be used. The theory of this operation is much simpler because the *cutting* pressure angle does not change when the cutter is moved out to generate teeth on a gear blank larger than standard. This is clearly shown in Fig. 9–24. As in Case I, all involutes generated on a gear belong to the standard base circle of that gear. When placed in mesh with another oversized gear of the same nominal pitch, similarly generated, operation will be on a pressure line established by the common tangent to the two standard base circles. While the methods differ, the end products in cases I and II are identical. The pressure angle is increased. The real circular pitch is increased on both gears.

To take full advantage of the possibilities of the involute curve in gear applications, it is necessary to use variable center distance, variable pressure angle, and variable and unequal addenda. There is nothing to prevent the use of all these variations in a single design. The real limita-

tions on design are economic, namely the use of standard cutters and the practical advantages of having gears interchangeable.[1]

9–14. **Interchangeable Gears.**—The requirements of an interchangeable set of involute gears such that any pair will mesh satisfactorily are as follows:

(1) the same pitch (diametral, circular, and base),

(2) the same circular tooth thickness,

(3) the same pressure angle with any pair of the set in mesh as with any other pair,

(4) that the addendum of any gear shall not exceed the dedendum minus the clearance of any other gear,

(5) that no pair shall interfere.

The requirements for proper operation of a single pair of gears differ from the requirements for a set only in respect to (2). Item (3) is not pertinent when only one pair is involved. That is sufficient to admit unequal-addendum and variable-center-distance gears as pairs, but to exclude them as sets.

9–15. **The Effect of Sharpening Cutters.**—It is necessary to have clearance angles on all cutting edges, so that the tool can bite into the work and remove the chip without scrubbing. This is often referred to as *relief*. These clearances are illustrated somewhat in Figs. 9–14 (*a*), 9–17 and 9–21. When the cutting edges become dull they are sharpened by grinding the faces of the teeth. What effect does this have on performance?

In the case of the rack cutter, the teeth are so made that there is no change in the cutting profile caused by sharpening. Of course, a hob will have its outside and pitch diameters decreased, but the consequent feeding in of the tool will not change its pressure angle or its generating action.

With the generating shaper cutter the desired uniformity of performance is obtained by forming its teeth so that its involutes on all sections normal to its cutting direction have the *same base circle*. Fig. 9–25 will serve as illustration. Consider contour g cut off at the top so that the top lands of g and w are equal. Then g might represent the profile of the cutter tooth when new, w when much used. In the latter case the cutter must be fed farther into the blank in order to cut an intertooth space of standard circular thickness. The cutter carries its base circle with it; hence the generating will be done at a smaller pressure angle. But the base circle of the blank is unchanged, and the cutter, whether new or old, will generate the same involutes on it and the same tooth profiles in all respects.

[1] For a complete table of dimensions of a variable-center-distance system based on the $14\frac{1}{2}°$ full-depth standard, see Buckingham, Manual of Gear Design, Section Two, p. 73.

9–16. **Contact Ratio.**—The locus of the contact point of a meshing pair of involute gears is a straight line, and the contact point travels along this line at the speed of the two base circles. Fig. 9–26 represents the general case for gears that do not undercut each other. For gear 1 driving gear 2, the length of contact is EF. If EF is laid on either base circle as an arc it will subtend the angle of action for that gear. This leads to some definitions.

The *angle of approach* is the angle through which a gear turns while the contact point moves from initial contact to the pitch point.

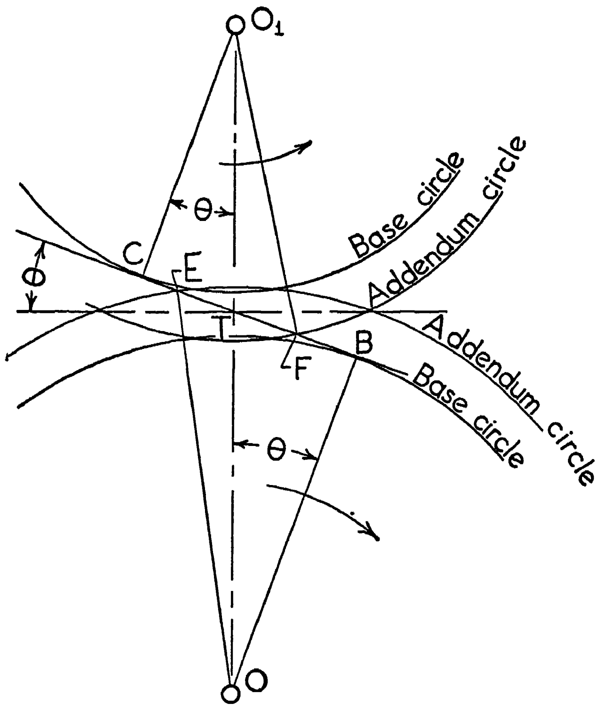


FIG. 9–26. Extent of Contact.
This represents 20° full-depth gears of 20 and 30 teeth.

The *angle of recess* is the angle through which a gear turns while the contact point travels from the pitch point to final contact.

The *angle of action* is the angle through which a gear turns while one of its teeth is in action and is the sum of the approach and recess angles.

The *pitch angle* is the angle on a gear that its circular pitch subtends. It is $360/N$ in degrees.

The **contact ratio** is the angle of action divided by the pitch angle.

The angles of action are unequal for meshing gears of unequal size while the contact ratios are the same, hence the convenience of the latter term.

The arc, on the base circle, that subtends the pitch angle is $p \cos \theta$, where p is the circular pitch, Fig. 9–26. Therefore,

$$\text{Contact ratio} = \frac{EF}{p \cos \theta} \tag{5}$$

$$EF = CF + EB - CB$$

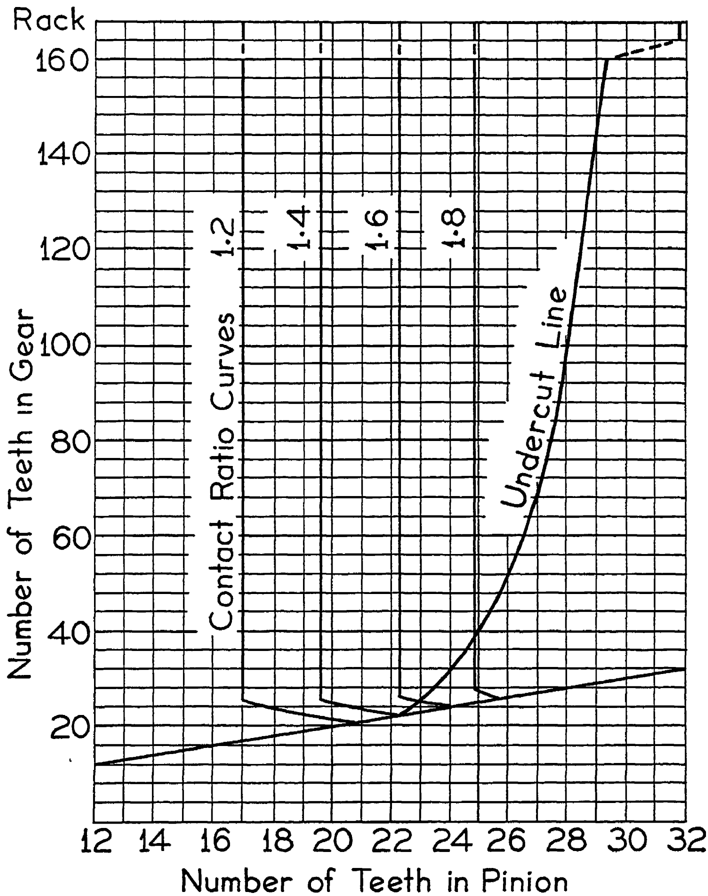


FIG. 9–27. Contact Ratios and Undercut Limits for $14\frac{1}{2}°$ Full-Depth Standard Gears.

Points to the right of the undercut line represent combinations that will not undercut each other.

Contact ratios are for gears cut by hob or rack cutters. For points to the left of the undercut line, contact ratios will be slightly higher if the gears are cut on Fellows generating shapers.

Using r_0 to represent outside radius, r the pitch-circle radius, and r_b the base-circle radius,

$$EF = \sqrt{r^2_{01} - r^2_{b1}} + \sqrt{r^2_{02} - r^2_{b2}} - (r_1 + r_2) \sin \theta \tag{6}$$

From (5) and (6),

$$\text{Contact ratio} = \frac{\sqrt{r^2_{01} - r^2_{b1}} + \sqrt{r^2_{02} - r^2_{b2}} - (r_1 + r_2) \sin \theta}{p \cos \theta} \tag{7}$$

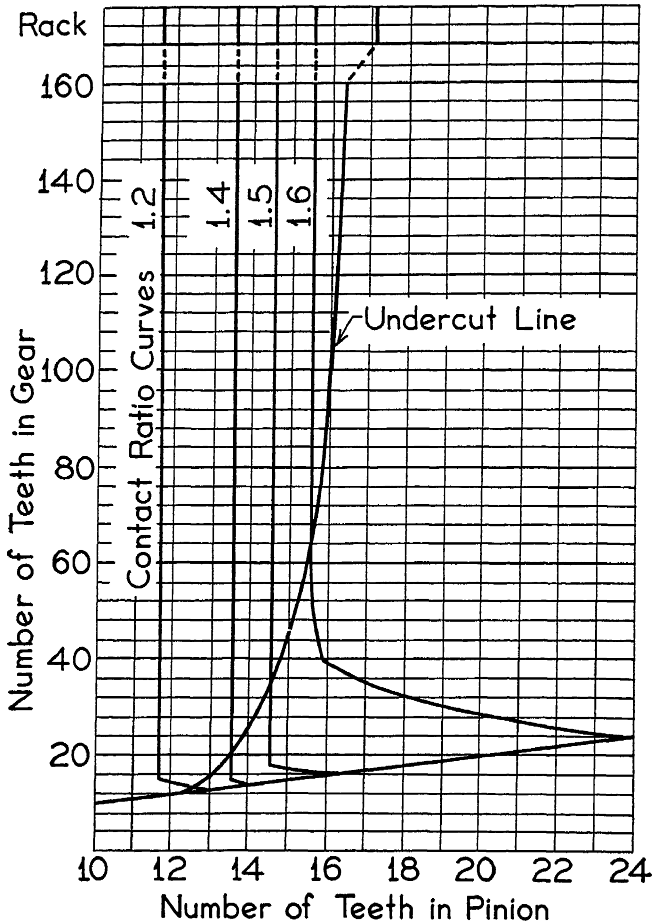


FIG. 9–28. Contact Ratios and Undercut Limits for 20° Full-Depth Standard Gears.

Points to the right of the undercut line represent combinations that will not undercut each other.

Contact ratios are for gears cut by hob or rack cutters. For points to the left of the undercut line, contact ratios will be slightly higher if the gears are cut on Fellows generating shapers.

For satisfactory spur-gear performance, it has been found necessary to have a minimum contact ratio of 1.4 for gears operating at medium or high speeds. This means that two pairs of teeth are in contact 40% of the time, thus allowing the pair coming into contact to pick up the transmission smoothly as the outgoing pair unloads. For very slow speed gears, contact ratios down to one are permissible.

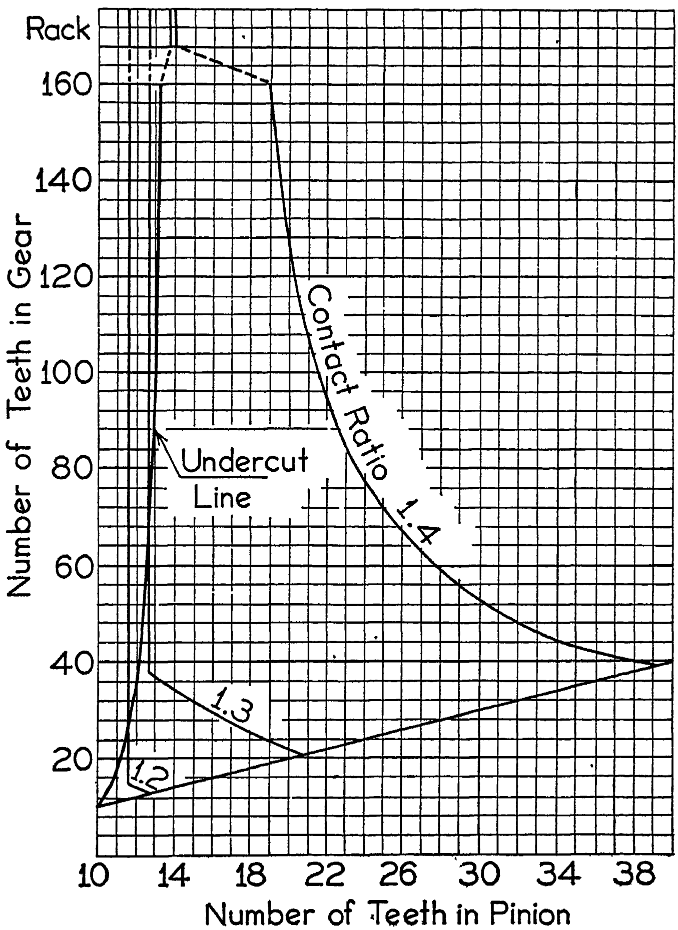


FIG. 9–29. Contact Ratios and Undercut Limits for 20° Stub Standard Gears.

Points to the right of the undercut line represent combinations that will not undercut each other.

Contact ratios are for gears cut by hob or rack cutters. For points to the left of the undercut line, contact ratios will be slightly higher if the gears are cut on Fellows generating shapers.

The maximum possible length of contact is the length of the line of action between interference points or CB in Fig. 9–26, and, for unequal gears, that is possible only with unequal addenda. When the addendum circles fall outside the interference points, the line of contact may actually be less than CB due to undercut above the base circle as shown at n in Fig. 9–24. The contact ratio curves of Figs. 9–27, 9–28, and 9–29 make it possible to select speedily standard gears of given speed ratios that will give desired contact ratios.

Example: The contact ratio is required for 8 diametral pitch, 20° stub gears of 32 and 48 teeth. The common addendum is $0.8/8 = 0.1$ in., $r = N/2P$, and $r_b = r \cos \theta$, giving

$$\begin{aligned} r_1 &= 2 & r_2 &= 3 \\ r_{01} &= 2.1 & r_{02} &= 3.1 \\ r_{b1} &= 1.88 & r_{b2} &= 2.82 \end{aligned}$$

From (7),

$$\text{Contact ratio} = \frac{\sqrt{4.41 - 3.534} + \sqrt{9.61 - 7.952} - 5 \times 0.342}{\frac{\pi}{8} \times 0.940} = 1.40$$

This result may also be read from Fig. 9–29. The contact ratio is, of course, independent of the pitch, but eliminating the pitch in (7) complicates the expression.

The **contact ratio for annular gears** might be deduced from (7), but a direct approach is more illuminating. In Fig. 9–30 the contact line for the general case is shown as ce (compare with Fig. 9–12). The only new symbol is r_{i2} which represents the internal or inside radius of the gear. The length of contact is

$$\begin{aligned} ec &= ea - ac \\ &= ea - bc + ab \\ ec &= \sqrt{r^2_{01} - r^2_{b1}} - \sqrt{r^2_{i2} - r^2_{b2}} + (r_2 - r_1) \sin \theta \end{aligned}$$

$$\text{Contact ratio} = \frac{\sqrt{r^2_{01} - r^2_{b1}} - \sqrt{r^2_{i2} - r^2_{b2}} + (r_2 - r_1) \sin \theta}{p \cos \theta} \qquad (8)$$

Example: Find the contact ratio of a 60-tooth annular gear meshing with a 20-tooth pinion. The pressure angle is 20°, and the tooth proportions are as recommended in § 9–7. Using $P = 1$ as a convenient value for computing the various radii required in (8), the contact ratio is

$$\frac{\sqrt{11.25^2 - 9.397^2} - \sqrt{29.40^2 - 28.19^2} + (30 - 10)0.342}{\pi \times 0.9397} = 1.584$$

By disregarding the recommended value for r_{i2} and allowing the gear tooth to extend in as far as the interference point a, the contact line might be given the maximum value ae, and the contact ratio would then be

$$\frac{\sqrt{11.25^2 - 9.397^2}}{\pi \times 0.9397} = 2.1$$

9–17. **Undercutting.**—If the addendum circle of a gear encloses the interference point of its mating gear, the mating gear will either have to be undercut or there will be interference. If the gears are generated in such manner that the interfering metal is removed from tooth flanks, the result will be undercutting, and that term will now be used to cover both possibilities. Of course any generating cutter will remove all metal that would interfere with a mating gear of the form and size of the cutter.

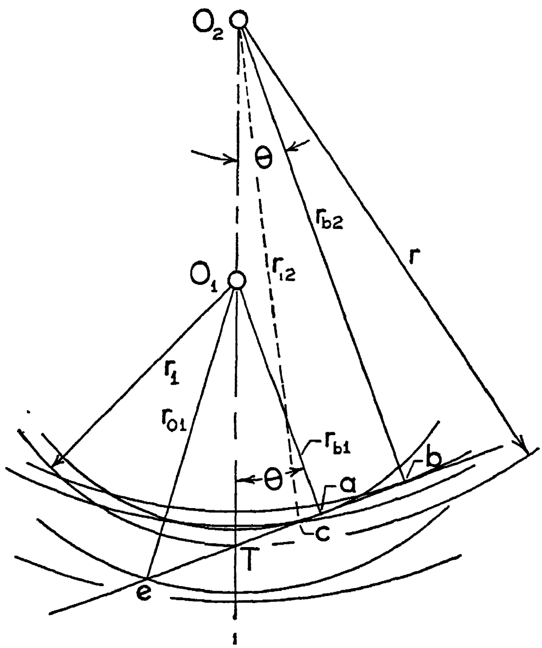


Fig. 9–30.

The situation when a rack and pinion are in mesh can be read from Fig. 9–24. The interference point is b, and the addendum of the rack must not exceed hb if undercutting is to be avoided. Calling this limiting addendum a, and the pitch radius of the pinion r,

$$a = hb = (Tb) \sin \theta = r \sin^2 \theta \tag{9}$$

Applying this to the 20° full-depth standard form,

$$a = \frac{1}{P} = \frac{2r}{N} = r \sin^2 20° \tag{10}$$

$$N = \frac{2}{0.342^2} = 17.1$$

No undercutting difficulty will be encountered with a 20° full-depth rack

and pinion if the pinion has 18 or more teeth. In fact, the undercutting would be quite negligible with 17 teeth.

The general case of two unequal gears is represented in Fig. 9–31. Subscript 2 is used for the larger gear. It can be seen by inspection that the interference points, C and B, place a lower limit on the addendum of

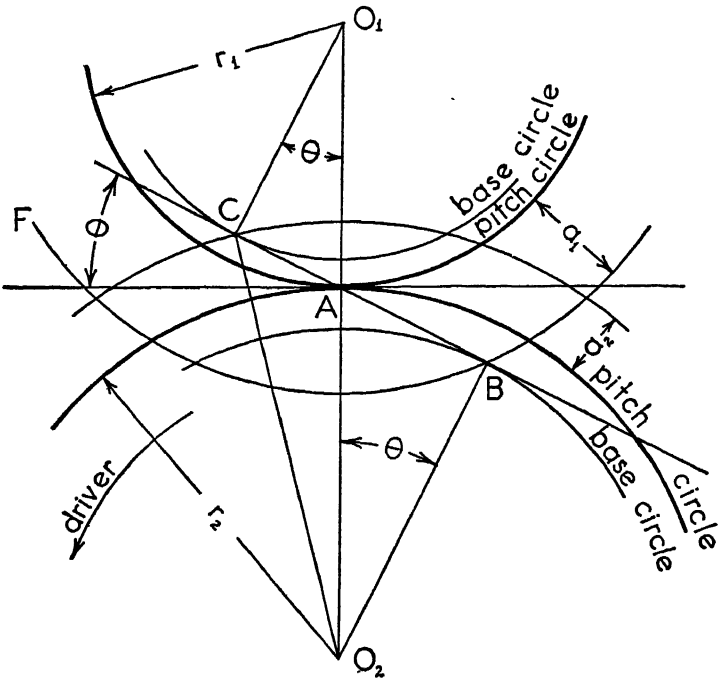


FIG. 9–31. Undercutting in Gears.

the larger gear. If the gears are standard, therefore, a_2 alone needs to be considered. If the addendum does not exceed a_2 as shown, there will be no undercutting.

$$a_2 = \sqrt{(r_1 \sin \theta + r_2 \sin \theta)^2 + r_2^2 \cos^2 \theta} - r_2 \tag{11}$$

This relation may be read from the three right triangles of Fig. 9–31 and reduces to

$$a_2 = \sqrt{(r_1^2 + 2r_1r_2) \sin^2 \theta + r_2^2} - r_2 \tag{12}$$

and further to

$$a_2 = r_2 \left\{ \sqrt{\left(\frac{r_1^2}{r_2^2} + 2\frac{r_1}{r_2}\right) \sin^2 \theta + 1} - 1 \right\} \tag{13}$$

Since $r = \dfrac{N}{2P}$ and $\dfrac{r_1}{r_2} = \dfrac{N_1}{N_2}$,

$$a_2 = \frac{N_2}{2P}\left\{\sqrt{\frac{N_1^2}{N_2^2} + 2\frac{N_1}{N_2}\sin^2\theta + 1} - 1\right\} \tag{14}$$

A convenient form for computation is

$$\frac{2a_2P}{N_2} + 1 = \sqrt{\left\{\frac{N_1^2}{N_2^2} + 2\frac{N_1}{N_2}\right\}\sin^2\theta + 1} \tag{15}$$

Example: What are the smallest 20° full-depth standard gears of a given pitch that will transmit in a speed ratio of 4 to 1 without undercutting?

For a_2, the standard addendum $\dfrac{1}{P}$ is substituted. This eliminates P from (15), confirming that the result is independent of pitch. $N_1/N_2 = \frac{1}{4}$.

$$\frac{2}{N_2} + 1 = \sqrt{(\tfrac{1}{16} + 2 \times \tfrac{1}{4})0.342^2 + 1}$$

$$N_2 = 61.8 \quad \text{and} \quad N_1 = 15.45$$

This illustrates the manner in which plotting points were obtained for the undercut curves of Figs. 9–27, 9–28, and 9–29. Satisfactory use of (15) is not easy without a very good table of square roots.

9–18. **Use of Contact-Ratio and Undercut Curves.**—The design of spur gears for practical application involves problems of kinematics, strength, and wear resistance. The kinematic problems with which we are here alone concerned occur in great variety. In most applications on which it is desired to use standard gears, the graphs of Figs. 9–27 to 9–29 afford a good starting point. In addition to answering the direct questions as to whether a given pair of gears will require undercutting and what their contact ratio will be, the following uses are typical.

(1) Suppose that high-speed spur gears are required to transmit in a speed ratio of 2 to 1 using the most compact arrangement possible. What are the fewest teeth that could be used in standard gears?

A contact ratio of 1.4 is the minimum desirable. Fig. 9–27 shows that 20 and 40 teeth of $14\frac{1}{2}$° full-depth form would have this contact ratio, but would be severely undercut. To avoid undercut completely also, 27 and 54 teeth would be necessary. This compares with 15 and 30 from Fig. 9–28, and 30 and 60 from Fig. 9–29. The 20° full-depth form, therefore, should be used. If the speed were low and conditions such that the contact ratio could be reduced to 1.2, it would be possible to use 20° stub gears of 12 and 24 teeth.

(2) It is desired to use 4-pitch gears on approximately $11\frac{1}{2}$ in. center distance to connect shafts to run at 1400 rpm and 600 rpm. What form and tooth numbers should be used?

The speed ratio is 7 to 3, so for every 7 teeth on the gear there must be 3 on the pinion. Hence the sum of the teeth must be divisible by 10. $N = P \times D$, where D is pitch diameter, and

$$N_1 + N_2 = P(D_1 + D_2)$$
$$10k = 4(2 \times 11.5)$$

where k must be a whole number. The nearest value for k is 9. Then $N_1 = 3 \times 9 = 27$ teeth and $N_2 = 7 \times 9 = 63$. All three forms are shown by the graphs to be satisfactory. The $14\frac{1}{2}°$ full-depth form is preferable due to its low pressure angle. The center distance would be $11\frac{1}{4}$ in.

(3) A 26-tooth, $14\frac{1}{2}°$, full-depth gear is required. How can the teeth be machined to avoid undercutting?

First consider hobbing. Fig. 9–27 shows that a standard hob would undercut this gear, in fact all such gears up to 32 teeth. A gear-shaper cutter with radial flanks, having 32 teeth, would cut the 26-tooth gear without undercutting it, since the 26–32 combination gives a point to the right of the undercut line.

(4) Suppose that in the previous case only $14\frac{1}{2}°$ hobbing equipment was available, what might the procedure be? The gear might be cut with standard addendum. It would be weakened by the undercutting but would be otherwise satisfactory. If the gear does not need to be interchangeable, and if it is to be meshed with a larger gear, avoid the undercutting by using the unequal addendum method, § 9–12. If the mating gear is not considerably larger, increase the center distance, § 9–13.

(5) A $14\frac{1}{2}°$, full-depth, 20-tooth gear is cut with a standard, generating-shaper cutter of 24 teeth, and meshed with a 60-tooth gear. How will the gears perform? There will be interference. The 60–20 combination gives a point much farther to the left of the undercut line, Fig. 9–27, than the 24–20 pair. Of course a special "full tipped" cutter can be purchased which will remove sufficient interference metal from the flanks of the pinion to avoid interference.

These few examples suggest the variety of information which can be read from the graphs with a little practice in their use. With the increase of design for quantity production, there has been an increasing use of nonstandard forms, including variation of addendum, center distance, and pressure angle. In such design, these graphs are useful in making the initial survey.

9–19. **Gear Finishing Processes—Grinding, Lapping, Shaving, Flame Hardening.**—There exists a quite definite relation between the accuracy and smoothness of the active tooth surfaces, and the speed at which the gears will run satisfactorily. Also, wear resistance is closely related to finish. Quite accurate profiles can be produced with cutters using the generating principle, better by grinding, or grinding followed by lapping, and recently shaving has been developed as an accurate finishing process. In addition, gears cannot be cut after they are hardened, and, if hardened, they may warp so that grinding is necessary to restore the required accuracy.

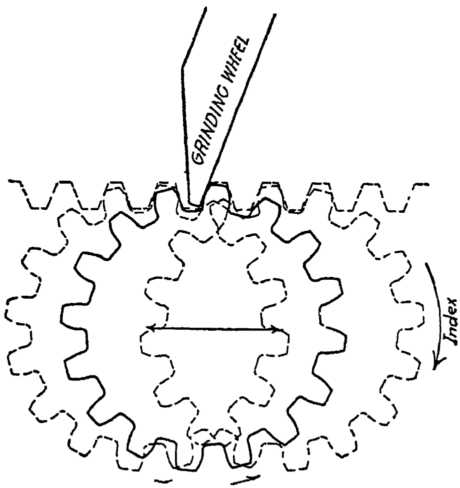


FIG. 9–32. Grinding Gear Teeth by the Generating Principle.

The generating method is regularly used in grinding, and the principle of the operation is illustrated in Fig. 9–32. The grinding wheel, trued to a plane surface on the cutting side, revolves in a fixed plane. The cutting side coincides with the straight side of the tooth of an imaginary rack on which the gear rolls. The gear is thus reciprocated until one side of a tooth is ground, then indexed for the next tooth. Fig. 10–23 shows how the gear motion is obtained on the Lees-Bradner grinder.

A **lap** is a metal surface, generally cast iron or brass, charged or impregnated with very fine particles of abrasive. Gears are lapped either by running them in mesh with another gear which has been charged as a lap, or by running the gears in pairs while a slurry of oil and abrasive is applied to the meshing surfaces. Some axial motion is also desirable. Only a small amount of metal is removed in lapping, but it comes from the minute high spots. All traces of abrasive must be removed before the gears are placed in service. Gears are often lapped without first being ground.

Shaving is a cutting process and is applied to the finishing of gears by using the generating principle combined with helical-gear action. The shaving cutter is shaped like a gear and has fine grooves cut on the face of its teeth. These grooves provide the cutting edges as shown in Fig. 9–33. The machine on which the operation is performed is illustrated

in Fig. 9–34. The helical action plus the meshing action results in the removal of fine chips in the shape of needle-like curls. The cross cutting of the tooth surfaces tends to remove any wash-board effect left by the shaping cutter. Of course, shaving is a cutting process and cannot be used on hardened gears.

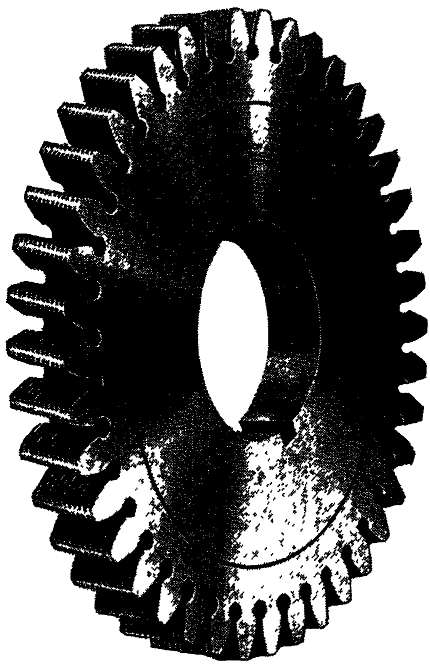


FIG. 9–33. Shaving Cutter.
National Broach and Machine Co., Detroit, Mich.

It is possible by the use of a cam control on the shaving machine to finish the teeth with a slight bulge or crown. The teeth are thicker at the center by from one to three thousandths of an inch. They are called *elliptoid teeth.* This shape has been found to compensate, in some measure, for shaft deflection and to prevent concentrated pressure at the ends of the teeth due to slight misalignment of the gears from any cause.

Reference has already been made to the inaccuracies that result from possible warpage when gears are hardened in the ordinary manner, which requires that the whole gear be raised to a high temperature. **Flame hardening** largely avoids this difficulty. One method, applied only to gears of fairly coarse pitch, hardens only one tooth at a time. Oxyacetylene flames are applied to both sides of the tooth, and the torches are traversed along the tooth by power feed at such rate as to heat the tooth locally to the required temperature. A quenching spray follows immediately behind the torches, and one tooth at a time is thus completely hardened. Another process, most successful on gears of finer pitch, is to revolve the gear slowly so that all the teeth are subjected to the heating effect of enveloping torch flames. The teeth are quickly heated to hardening temperature while the body of the gear is relatively cool. An automatically timed mechanism drops the gear into the quenching tank.

9–20. **Gear Testing.**—It is fairly common practice to purchase gears on specifications stating maximum allowable errors. It is well known that running quietness as well as reliability and service life depends largely on overall gear accuracy. In addition to special tools and instruments

for the measurement of all important dimensions, testing machines are available designed to measure overall gear error. One of these uses a very accurate master gear, supported in movable bearings, which meshes with the gear to be tested, the latter being mounted in fixed bearings. As the

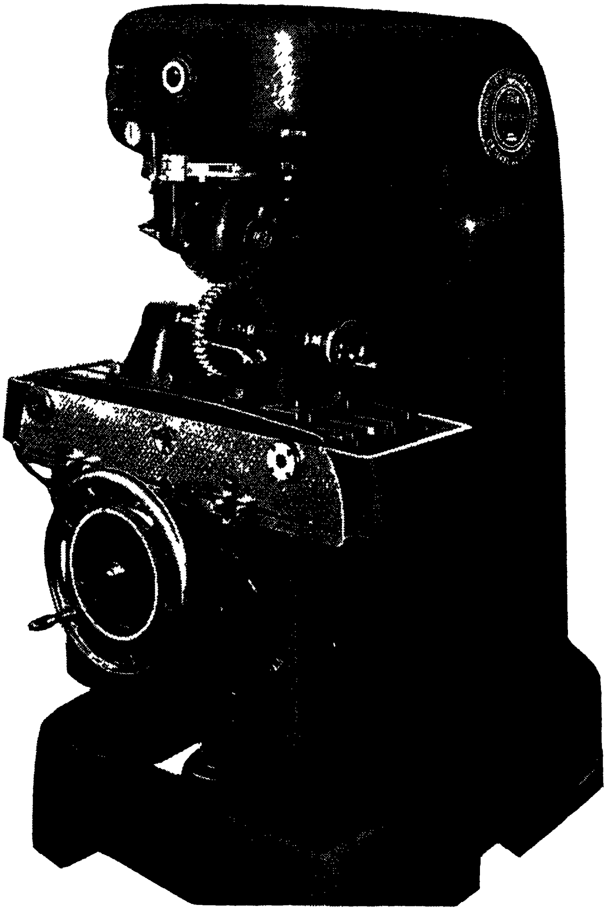


FIG. 9–34. Gear Shaving Machine.
National Broach and Machine Co., Detroit, Mich.

two are revolved slowly, the movement of the master gear, greatly magnified, is recorded as an error curve. These total errors allowable vary from 0.0005 in. for extremely good gears to 0.005 in. for fairly good gears of large size.

The Maag instrument, represented diagrammatically in Fig. 9–35, is typical of profile testers. The head m is translated vertically on sliding

surfaces, carrying the lever *s*, which makes contact with the gear tooth through a rounded point *i*. The gear is mounted in fixed bearings, and is fastened to a ground disk *d* which is the diameter of the base circle. The disk rolls on the straight edge *e* of the head, establishing correct

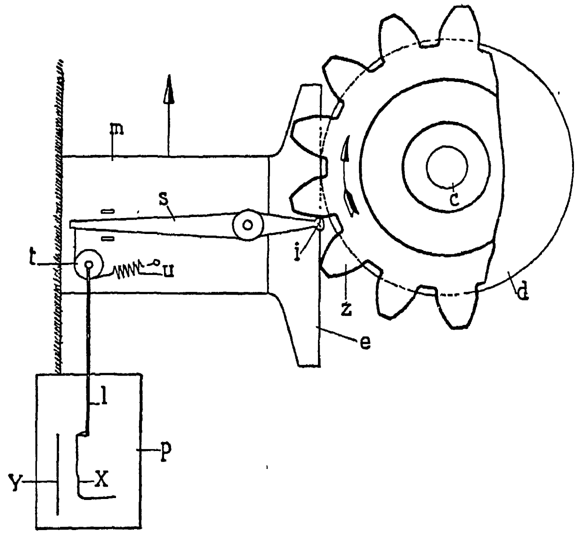


FIG. 9–35. Profile Testing Instrument.
The Maag Gear-Wheel Co, Zurich, Switzerland.

meshing speed with an imaginary rack on the head. Any error in the tooth profile will cause rotative motion of *s* on the head. This motion, greatly magnified at the pencil point on the lever *l*, will cause an irregular curve such as *x* to be drawn on drum *p*. A straight line such as *y* would indicate a perfect involute profile.

QUESTIONS AND PROBLEMS

1. Define diametral and circular pitch and derive the relation between them.
2. Two gears having teeth of circular profile could be placed in mesh so as to transmit torque. In what respect would they be unsatisfactory? Prove graphically.
3. Two involutes are described by the pencil and cord method, one on a 2 in. base circle, the other on a 3 in. base circle.
   (*a*) At the points on each involute where 6 in. of cord has been unwound, which will have the largest curvature?

(*b*) Can these involutes be made to coincide for 1/2 in., assuming that they may be drawn to any finite length?

(*c*) Which property of the involute qualifies it for use as profiles for gear teeth.

4. A certain handbook gives this rule for finding the outside diameter of a gear. "Add two to the number of teeth and divide by the diametral pitch." Is this rule rational or empirical and what are its limits of application?

5. Find the principal dimensions, according to the example in § 9–4, for 20°-involute, standard, stub gears of 8 diametral pitch having 24 and 56 teeth.

6. A pair of gears transmit in a speed ratio of 5 to 3. The diametral pitch is 3, and the pinion has 36 teeth. Find the center distance and the circular pitch.

7. A 30-tooth pinion of 10 diametral pitch meshes with a gear of 8 in. pitch diameter. Their form is $14\frac{1}{2}°$ full depth. Find: (*a*) the addendum, (*b*) working depth, (*c*) base circle diameters, (*d*) the speed of the pinion if the gear turns at 150 rpm.

Ans. (*a*) 0.1 in. (*b*) 0.2 in. (*c*) 2.9043 and 7.7448 in. (*d*) 400 rpm.

8. An 18-tooth pinion of 6 diametral pitch meshes with a gear of 9 in. pitch diameter. Their form is 20° stub. Compute the items asked in Prob. 7.

9. A 20° full-depth gear of 4 diametral pitch has 88 teeth and runs at 600 rpm driving its pinion at 2200 rpm. Calculate: (*a*) the center distance, (*b*) pitch, outside and root diameters.

10. A pair of $14\frac{1}{2}°$, standard, full-depth gears of 2 diametral pitch have 20 and 40 teeth. Calculate the possible increase in the addendum of each gear before interference would occur.

11. A pair of external spur gears of 6 diametral pitch are required to connect shafts at a center distance of $8\frac{1}{4}$ in. so that when one runs at 900 rpm, the other will run at 2400.

(*a*) Find the number of teeth in each.

(*b*) What is the relative sense of rotation?

(*c*) If the gear were internal (annular), what would the center distance and relative sense be?

12. A 15-tooth pinion of 5 diametral pitch drives a rack. When the pinion turns at 30 rpm what is the linear speed of the rack in feet per minute?

13. Insert the appropriate name in each of the spaces of this table, that represent practical gear-finishing processes.

Method	Processes of Metal Removal				
	Shaping	Milling	Grinding	Lapping	Shaving
Forming					
Generating					

14. Compute the values given in Table 9–3 for unequal-addendum gears where the pinion has 16 teeth.
15. Calculate the contact ratio of a 48-tooth annular gear meshing with a 24-tooth pinion. The pressure angle is 20°, and the proportions are those recommended in § 9–7.
16. What is the pitch angle (angle the circular pitch subtends) of a 12-tooth gear, of a 60-tooth gear? How much does this angle change on each with the addition of one tooth? Explain the bearing of this on Table 9–2.
17. A 4-diametral-pitch, 20°, full-depth rack meshes with a 20-tooth pinion. (*a*) What radial length of pinion tooth will be in action? (*b*) What length of side of rack tooth will be in action? Solve analytically. Ans. (*a*) 0.368 in. (*b*) 0.446 in.
18. A pair of 2-diametral-pitch, $14\frac{1}{2}°$, full-depth gears of 30 and 60 teeth are in mesh. What radial length of tooth on each gear will be in action?
19. A standard $14\frac{1}{2}°$ gear of 2 diametral pitch and 40 teeth was generated to be run with a 20-tooth pinion at standard center distance, but the undercutting was found to be excessive. It was decided to use the same cutter to produce another pair with the same tooth numbers and working depth, but using a center distance 0.3 in. in excess of the standard to avoid the undercutting. At what pressure angle would the new gears operate? Ans. 18.36°.

    Note: The gears must share the increase of pitch radii in proportion to their tooth numbers.
20. Find the pressure angle at which gears of 24 and 36 teeth will operate if cut by a standard $14\frac{1}{2}°$ cutter of 4 pitch for standard working depth and a center distance of 7.75 in.
21. Derive an expression for the contact ratio for a rack and pinion, corresponding to equation (7) for two gears. Assume the general case where there is no undercutting and contact does not extend to the interference point.
22. Test the expression developed in answering Prob. 21 by computing the contact ratio for a 20° stub-tooth rack meshing with a 14-tooth pinion. The answer is given in Fig. 9–29.

23. Compute the contact ratio of a 40-tooth, 20°, full-depth gear meshing with a 16-tooth pinion. Check your answer on Fig. 9–28.
24. Find the numbers of teeth in a pair of $14\frac{1}{2}$° standard, external, full-depth gears which transmit in a ratio of 2 to 1 and which have contact up to one interference point but are not undercut. Check your answer on Fig. 9–27.
25. Using the undercut graphs, determine the smallest pinions that could be used with each of the three standard racks without undercutting.
26. A 26-tooth gear of 20° full-depth form is cut with a 40-tooth standard generating cutter. (*a*) Will it be undercut? (*b*) How would this gear perform if meshed with a rack? (*c*) Could it be cut in such manner as to mesh with a rack without interference and, if so, how?
27. Standard generated gears are required to transmit in a speed ratio of 7 to 3. What are the tooth numbers and form of the smallest gears that will (*a*) give a contact ratio of 1.4 without undercutting, (*b*) give a contact ratio of 1.2 without undercutting?
28. Using the methods given in § 9–3, construct the mating sides of two teeth of a pair of 20° full-depth, external gears of 2 diametral pitch and 16 and 32 teeth. Taking about 6 points along their line of action, find the velocity of rubbing for a chosen speed of rotation. Plot these values as vectors on each tooth curve and normal thereto. Join the ends of the vectors in a smooth curve. Compare the curves.
29. Plot rubbing-velocity curves for a pair of internal gears comparable to those of Prob. 28. Use the recommended dimensions for annular gears. Compare results with those for the external gears.

CHAPTER X

BEVEL AND HELICAL GEARING

10–1. Straight-Tooth Bevel Gears.—Bevel gears, excepting hypoids, are used to connect shafts having center lines that intersect. A straight-toothed pair is shown in correct mesh in Fig. 10–1. The most common angle between the shafts is 90° but this is no limitation of the gears. They are adaptable for any transmission angle. While the function of bevel gears, to transmit around a corner, has been an important factor in the development of machinery, they have an inherent characteristic that has made them difficult to produce accurately. That is, the size or pitch of the tooth varies across its length so that a cutter that fits the inter-tooth space at its outer end will not pass through the inner end. Furthermore, if one side only of the inter-tooth space is machined at a time using a narrow cutter, the profile changes so that any formed cutter must be a compromise in shape between that required at the two ends. The earlier bevels were cut in this way and an approach to true profile made by subsequent hand filing. Indeed, they can be finished in no better way except by the use of special and quite expensive bevel gear cutting machines, so the manufacture has become a specialty.

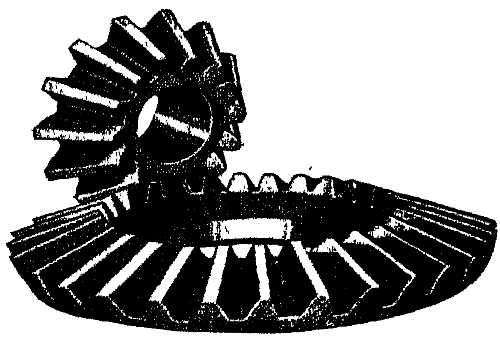


FIG. 10–1. Straight-Toothed Bevel Gears.

From Fig. 10–2 it will be seen that the pitch surface of a bevel gear is a frustum of a cone. For the larger gear the pitch cone is *BOC* with vertex at *O*. *BDC* is called the back cone. The *nominal pitch* is conventionally taken as that at the outer end of the tooth, which fixes the nominal pitch diameter as *BC*.

Bevel gears have relative spherical motion. Therefore the true meshing profile of the teeth on both gears at their outer ends would be found only on the surface of a sphere of radius *OB*. However, a projection of this true shape on the plane *FD* would change it very slightly unless the

teeth were unusually large compared to the sphere. This is called *Tredgold's approximation* and means that the profile of the teeth of the large bevel gear, for example, is obtained as the profile of the teeth of a spur gear of the same pitch, and of pitch radius DB. The circle of radius DB is called the *formative circle*. The formative circle for the small bevel has FB as radius.

We shall next illustrate the method of finding the principal dimensions of a pair of bevel gears, using Fig. 10–2 as reference. Let subscript g indicate the larger gear and subscript p the pinion. Let P represent the nominal diametral pitch which measures the size of the teeth of both gears at the large end B. Then $BC = N_g/P$, and $BE = N_p/P$. These dimensions are laid off normal to the center lines of the shafts which may be at any given angle. If the shaft angle is other than 90°, a large scale layout is the simplest method of determining the angles and dimensions. If the shaft angle is 90°, the following relations obtain:

$$\tan (BOH) = \frac{N_g}{N_p} \tag{1}$$

since pitch radii are proportional to tooth numbers. Similarly,

$$\tan (BOF) = \frac{N_p}{N_g} \tag{2}$$

Triangles BHO and DHB are similar; therefore, calling the angle between an element of a cone and the axis of the cone the cone angle,

$$\frac{HB}{DB} = \cos HOB = \cos \text{ (pitch cone angle)}$$

If N' represents the formative number of teeth, then

$$\frac{HB}{DB} = \frac{N_g}{N_g'} = \cos \text{ (pitch cone angle of gear)} \tag{3}$$

Similarly,

$$\frac{N_p}{N_p'} = \cos \text{ (pitch cone angle of pinion)} \tag{4}$$

Suppose bevel gears are required on normal shafts, having tooth numbers $N_g = 24$ and $N_p = 16$, while the nominal diametral pitch is 4.

$$D_g = 24/4 = 6 \text{ in.,} \quad \text{and} \quad D_p = 16/4 = 4 \text{ in.}$$

$$N_g' = N_g/\cos (HOB) = \frac{24 \times \sqrt{13}}{2} = 43.28$$

$$N_p' = N_p/\cos (KOB) = \frac{16 \times \sqrt{13}}{3} = 19.28$$

Of course these numbers will regularly be fractional, but they are only measures of tooth shape, using spur gears as the standard for comparison.

The Gleason Works has for many years specialized in gear cutting machines. Their straight-bevel-gear generator is illustrated in Fig. 10–4, and its principle of operation can best be understood by a further study of Figs. 10–2 and 10–3. Without changing the pitch diameters of either gear, allow the point O, Fig. 10–2, to move up the line OD until it is on

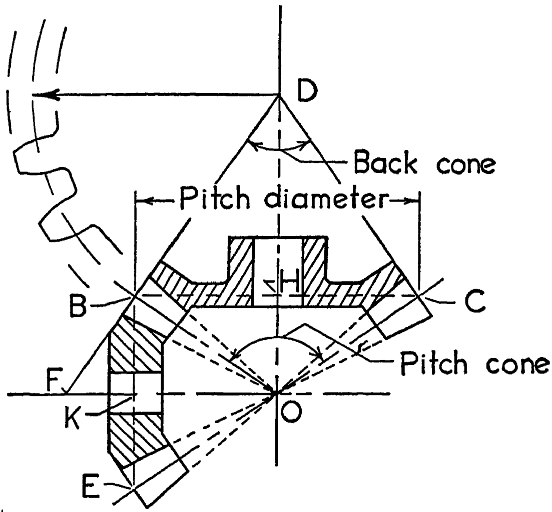


FIG. 10–2. 90° Bevel Gears in Section.

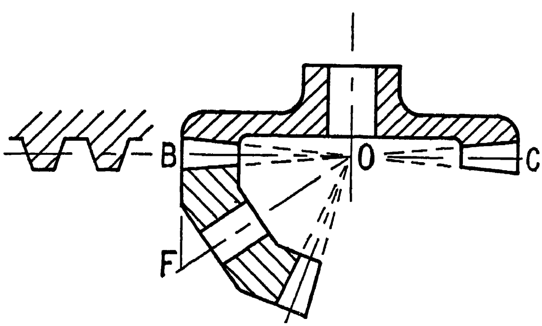


FIG. 10–3. Bevel Crown Gear and Pinion.

the line BC. The pinion now has its axis tipped, Fig. 10–3, to run through the new position of O, but the gear has its pitch surface all in one plane and its formative circle is infinitely large. It has become a "rack bevel gear" but is called a *crown gear*. The sides of its teeth are plane like the teeth of the spur rack. Note that the small gear has been changed in

no respect except in relative position. Its formative circle is unchanged. It will mesh perfectly with the crown gear; hence a straight-edged cutter, given the motion of the crown gear as well as a cutting stroke, will *generate* accurate profiles for the small gear just as the rack cutter generates spur gears.

Now turning to the Gleason machine, the gear being cut is shown mounted so as to mesh with an imaginary crown gear that might be mounted on the revolving face plate that carries the cutting heads. As

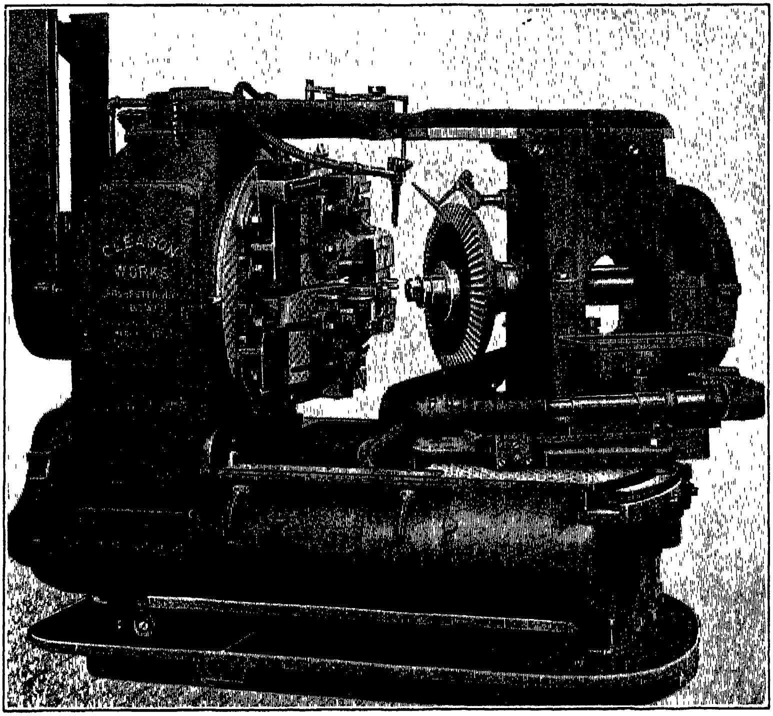


FIG. 10-4. Straight-Tooth Bevel-Gear Generator.

The Gleason Works, Rochester, N. Y.

this face plate revolves slowly with the gear blank, the straight cutting tool is traversed *radially* and generates one side of a tooth. Then the gear blank is indexed one tooth, and the rolling and cutting motion is repeated to generate the side of another tooth.

If two tools are used simultaneously to cut both sides of a tooth at once, they must be mounted on separate heads and set so that both travel radially from the center of the face plate. This is called the "spread blade" method and is the one illustrated. It is good practice to cut the

small end of the tooth a little full (thick), since it deflects more. Thereby full contact over the length of the tooth is achieved at full load.

The standard tooth forms and dimensions for spur gears are used for bevels and the relations governing contact ratio, interference, undercutting, unequal addendum, etc., are the same, within Tredgold's approximation, as for spur gears, provided the relations are considered in a plane normal to the teeth. This means that we must deal with the formative circles and the formative tooth numbers instead of the actual pitch circles and actual tooth numbers.

Referring again to Fig. 10–2, the formative number of teeth for the gear was 43.28, and for the pinion 19.28. Consulting the spur gear graphs, Figs. 9–27 to 29, we see that, if a minimum contact ratio of 1.4 were required, 19.28 with 43.28 teeth would have to be of 20° full-depth form to allow the use of standard teeth without undercutting. This limitation, therefore, applies to these bevel gears. Of course, unequal addendum design is applied to bevel gears in the same manner as to spur gears.

Bevel gears of equal size connecting shafts at 90° are called *miter gears*. The A. G. M. A. recommends that the face width of bevel gears should not exceed one-third of the pitch-cone side for $P \lesseqgtr 3$, or one-quarter of the cone side for P between 3 and 20.

The design of bearings to support bevel gears presents more serious problems than in the case of spur gears. Spur gears produce radial loads, that is, loads normal to the shaft center line; while bevel gears produce both radial loads and thrust loads, the latter being parallel to the shaft. This combination of loads is best taken by ball or roller bearings.

10–2. **Spiral Bevel Gears.**—The straight-tooth bevel gear, like the spur gear, makes contact on the full length of a tooth at the same instant. Under load, this sudden, complete contact causes shock, particularly at high speeds. The spiral bevel gear, Fig. 10–5, has smoother action, because each tooth makes meshing contact first at one end, and the contact passes gradually across the bearing portion of the tooth.[1] Fig. 10–5 also shows that the contact is somewhat localized. Since the small end of the tooth yields more under load, it is good practice to cut that end a little full as with straight teeth.

At first glance it might appear that the cutting of spiral bevel gears would be a difficult problem. The fact is that the operation is more simple than the correct cutting of straight bevel teeth. Returning to the crown gear defined in the last article, it will be recalled that the sides of the teeth were plane, and the pitch line running along the side of a tooth

[1] For a more detailed treatment of the advantages of gradual tooth engagement see Helical Gears, § 10–4.

would be a radius from the gear axis. Now suppose this crown gear tooth is curved across the face of the gear in a circular arc. The side of the tooth would no longer be plane, but a section taken normal to the tooth would have the same shape as before, that is, the side of the tooth would be straight, and could be generated by a straight-sided cutter travelling on a circular arc.

FIG. 10–5. Spiral Bevel Gear and Pinion.
The Gleason Works, Rochester, N. Y.

The gear that would mesh properly with this spiral crown gear would have its intertooth space

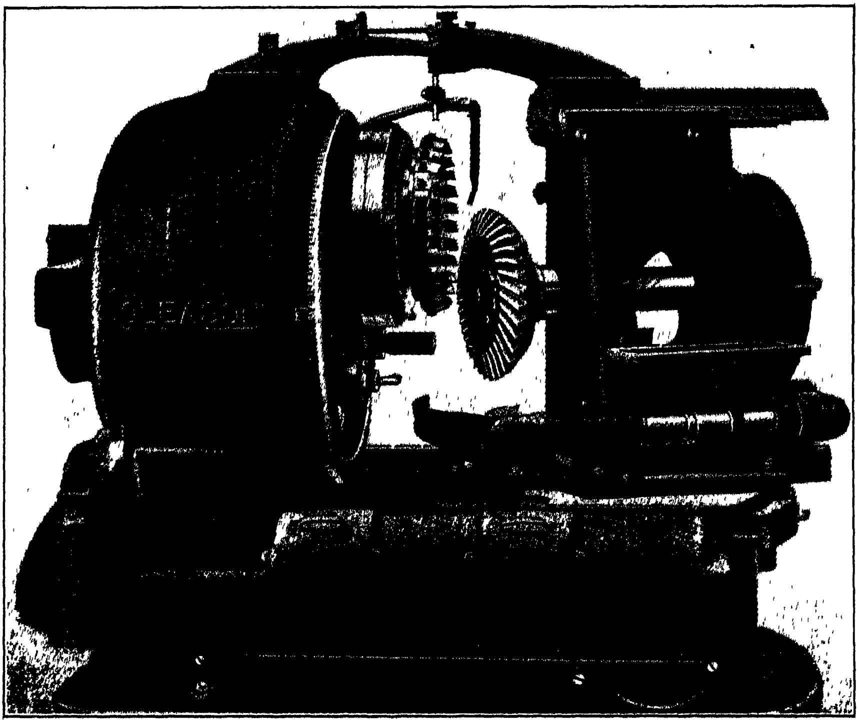
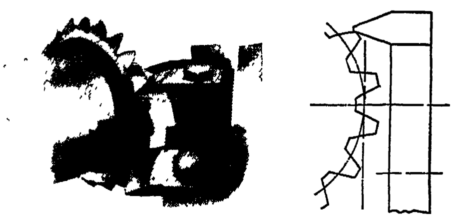


FIG. 10–6. Spiral Bevel Gear Generating Machine.
The Gleason Works, Rochester, N. Y.

generated by a tooth of the crown gear as the two turned in mesh. Accordingly, to cut *one side* of a tooth of a spiral bevel gear, it is necessary to traverse a straight-sided cutter along the side of a tooth of an imaginary, spiral crown gear, as this crown gear is rolled and meshed properly with the gear to be cut.

It is easy to traverse a tool in a circle. Indeed, a whole circle of cutting tools can be so put to work in series as is done in the spiral bevel generator of Fig. 10–6. The gear being cut is mounted at the proper angle with its imaginary crown gear, represented by the face plate on the main spindle of the machine. Mounted eccentrically on the face plate is the axis of the tool disk, carrying its circle of straight-sided cutting tools. The gear blank and face plate are rolled together slowly through about 65°, their motions being controlled by accurate gearing, while the cutter disk, driven at cutting speed, generates one side of a tooth. The same side of each tooth is so generated by indexing at the end of each cut. To cut opposite sides of the teeth requires an adjustment of the blank depending on the width of cutting tools used. On this machine the indexing is automatic. An excellent illustration of the relative positions of cutting tool and work during the generating action is provided by the Gleason Works in Fig. 10–7.



At top of roll with tooth completely generated

At center of roll with tooth partially generated

At bottom of roll where generating action begins

FIG. 10–7. Relative Motion of Cutter and Blank in Generating Spiral Bevels.

From a pamphlet by the Gleason Works, Rochester.

10–3. **Hypoid Gears.**—Two hyperboloids of revolution are shown in contact in Fig. 10–8. Portions, such as the shaded trustrums, could be used as the pitch surfaces of true hyperboloidal gears. These pitch surfaces approximate frustrums of circular cones, and that is the approximation made in the production of hypoid gears.

The outstanding advantage of these gears, shown in Fig. 10–9, is that

the connected shafts do not intersect. This allows either or both shafts to project in both directions. It is of particular value when it is desired to place a bearing on both sides of the pinion. There is seldom room to give the pinion of ordinary bevel gears an inside bearing. For the same

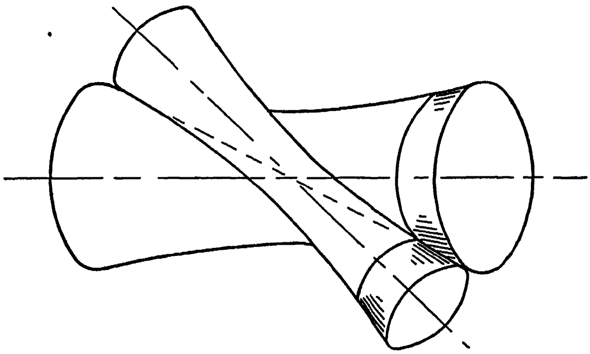


FIG. 10-8. Hyperboloids of Revolution.

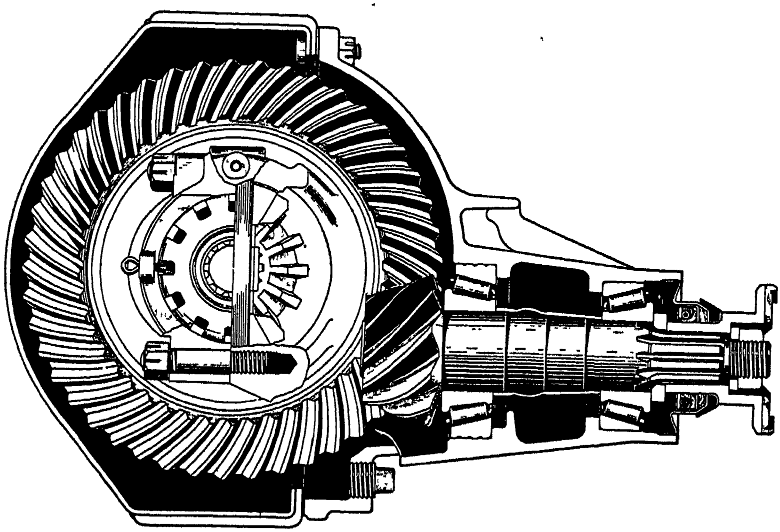


FIG. 10-9. Hypoid Gear and Pinion.

The Packard Motor Car Co., Detroit, Mich.

size of gear and the same speed ratio, the hypoid pinion is larger and stronger than its closest competitor, the spiral bevel pinion. In rear-axle automotive drives, the hypoid gear allows the transmission shaft to be placed lower for the same rear-axle road clearance. This in turn allows the center of gravity of the vehicle to be lowered.

Hypoid gears are cut on spiral-bevel-gear generators using straight-sided cutters as explained in § 10-3.

The nonintersecting shafts of hypoid gears make possible the kind of continuous drive illustrated in Fig. 10-10. This design by the Gleason Works is for a wire-drawing mill. After each successive draw, the wire,

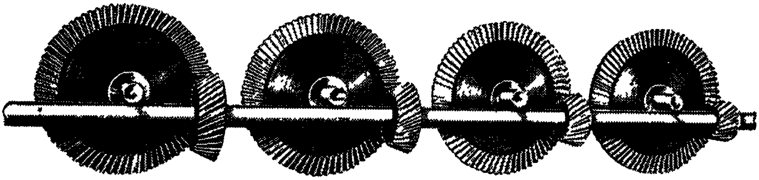


FIG. 10-10. Hypoid Gears for Multiple Drive.

moving from right to left, must be pulled through the dies at increasing speed due to its increasing length. This requires an increasing pinion size. The arrangement assures correct relative speeds and is very compact.

The spot-contact characteristic is a valuable feature of hypoid gears, making them less sensitive to slight shaft deflections. This ability to transmit satisfactorily under conditions of slight misalignment is common to spiral bevels, hypoids, helical gears not on parallel shafts, and certain types of worm and wheel drives, all of which have spot contact.

10-4. Helical Gears on Parallel Shafts—Herringbone Gears.—If one imagines a spur gear to be cut into a large number of slices by planes normal to the axis of the gear, and if, starting at one side, each slice were rotated through a small angle with respect to the next, the resulting teeth would wind around the gear each forming a helicoid. If the number of slices were infinite, the teeth would be smooth and a helical gear would be formed. The center line of each tooth on a helical gear is a true helix on the pitch cylinder, with a definite and uniform *lead,* that is advance per turn. Fig. 9-14 shows helical gear teeth being machined by the formed-cutter milling process. The blank must be slowly rotated, as the cutter advances at proportional speed to give the required helix angle.

The *helix angle* is the angle between a tangent to the tooth center line and a plane through the axis of the gear. The helix angle is often referred to as the *angle of cut.* The helix angle of a spur gear is zero.

Helical gears connecting parallel shafts have the following advantages over spur gears of the same pitch and size:

(1) Greater smoothness of action results from the contact beginning at one end of a tooth and gradually passing to the other end, than with spur teeth which make contact suddenly on their whole length.

(2) Greater strength results from the teeth being wound around the gear somewhat.

(3) For the same pitch and width of face, helical gears have more pairs of teeth in simultaneous contact.

(4) Pitch-line contact exists on one or more pairs of teeth at every instant.

(5) Under the same tolerance as to noise and vibration, higher speeds are possible with helical gears than with spur gears.

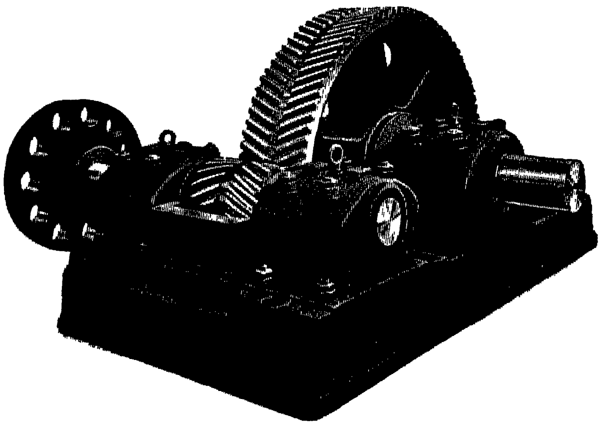


FIG. 10–11. Herringbone Reduction Gears.

The Fawcus Machine Co., Pittsburgh, Pa.

There is one serious disadvantage of simple helical gears; that is the end thrust produced on both driving and driven shafts. This difficulty is completely overcome in the *herringbone gear*, Fig. 10–11, which consists of two helical gears with reversed helix angle, made on the same blank or mounted on the same shaft. To facilitate machining the teeth, the halves are commonly spaced to allow some overtravel of the tool at the center. This is essential if a hob is used. Herringbone gears are used for rolling mill drives, turbine reduction, ship propulsion, etc. Pitch-line velocities as high as 10,000 ft per min, single gear reductions of 10 to 1, and transmitting capacities of many thousand horse power, have been successfully used.

Advantages (3) and (4) are clarified by a study of Fig. 10–12. It represents a portion of the pitch surface of one gear, developed in the plane of the paper. Imagine the gear being driven in the direction of the arrow, so that the upper sides of the teeth, represented by the hatched strips, are in action. The pitch line is b, and the zone of action is that between lines a and c. If this were a spur gear, the contact ratio would

be two, meaning that the average number of pairs of teeth in action would be two. The contact ratio of the herringbone gears in this case is also called two, but the situation is entirely different. Actually six pairs of teeth are simultaneously in action, and each pair has two spots of contact. Of course, the number of pairs of teeth in action is a function of the helix angle and the width of face. (The width of face of herringbone gears is always measured parallel to the axis, excluding the center gap if there is one.) The face width of these gears is regularly much larger in proportion to the pitch than is the practice with spur gears.

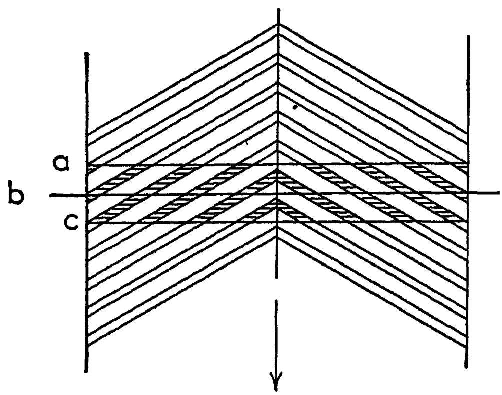


Fig. 10–12. Herringbone-Gear Action.

Another feature revealed in Fig. 10–12, which is even more important than the multiple contact, is the pitch-line contact. In this case four pairs of teeth are simultaneously in contact *at the pitch line*. This is a characteristic common to all helical gears on parallel shafts, namely, that they have at least one pair of teeth in contact at the pitch line, where the velocity of rubbing is zero, if they have a contact ratio of one or more. Some manufacturers contrive to have their gears finished so that the teeth will be a few thousandths high near the pitch cylinder, and otherwise inside the true involute, so that greatest tooth pressures will occur where the rubbing is least. If these conditions are met, the performance of the gears is independent of tooth-profile errors except in the region of the pitch cylinder. This is never true for spur gears and probably explains, more than any other factor, the superiority of helical gears on parallel shafts in smoothness and extremely high-speed performance.

If herringbone gears are hobbed, a center gap is necessary for clearance. To cut the teeth so that they are continuous at the center, as in Fig. 10–13, requires a special shaping process. These are generally called Sykes gears in honor of the developer of the ingenious machine on which they were first produced. The essential mechanism of this machine is represented in Fig. 10–14.

The generating action does not begin until the cutters are fed down to full tooth depth, as in the case of the Fellows shaper. The cutters, which must be right and left hand, are shown separately in Fig. 10–15. In

action, Fig. 10–14, the two cutters are reciprocated in one head with constant spacing as shown. As the head moves to the left, the right cutter takes a chip, but must be turned at proportional speed to produce the helical cut. This right-hand cutter is on the central shaft which passes to the left member of the two "wheels for rotating cutters," where a cam roller on the shaft engages a helical groove on the inside of the hub. While the right cutter is taking its chip, the left cutter must

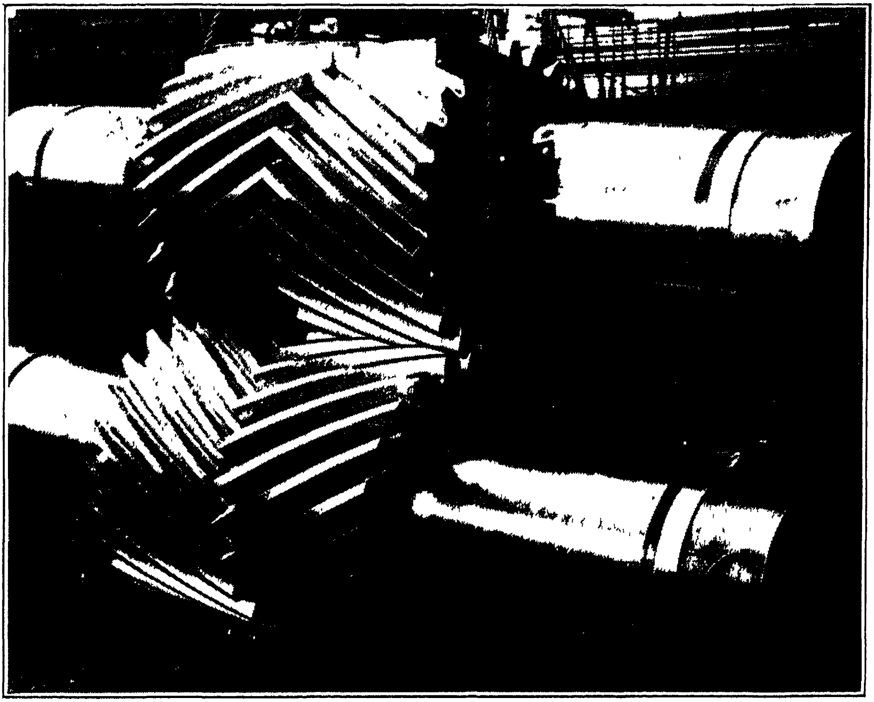


FIG. 10–13. Herringbone Gears for Rolling Mill Drive.

Farrel-Birmingham Co., Buffalo, N. Y.

back out of its half of the blank, being turned in opposite sense. The left cutter is mounted on the hollow shaft which has a roller following a groove in the right hub.

On the first stroke to the left, the right cutter brought a chip to the center. On the return stroke, the left cutter brings the complementary chip to the center thereby freeing both. The cutting proceeds in this manner until full tooth depth is reached. Then the generating feed gears are engaged, and the blank is slowly rotated in geared connection with the wheels for rotating the cutters. The gear is completely generated

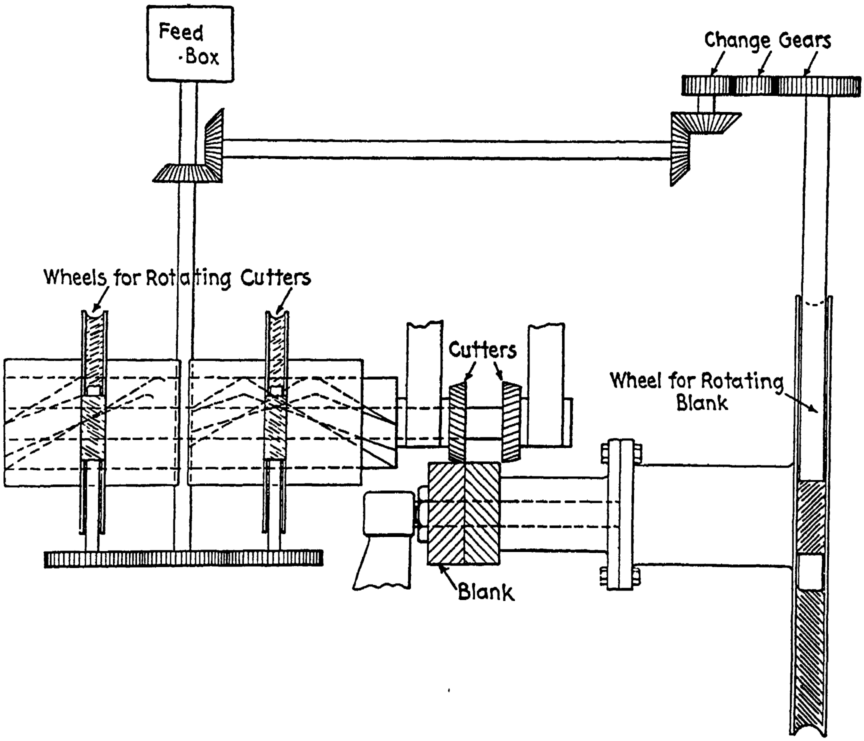


FIG. 10–14. Mechanism for Cutting Sykes Gears.

Farrel-Birmingham Co., Buffalo, N. Y.

FIG. 10–15. Sykes Herringbone Cutters.

Farrel-Birmingham Co., Buffalo, N. Y.

in one rotation of the blank. Elimination of the center gap makes possible shorter and stiffer shafts and more compact machines.

The action of helical gears on parallel shafts takes place in a plane normal to the two shafts. If a drawing showed nothing but such a

normal section, one could not tell whether the gears were helical or spur. So far as the meshing action is concerned, the pressure angle is the pressure angle in this plane, and the meshing profile is *Tdg* in Fig. 10–16. The cutter, however, must operate parallel to the teeth, and must produce the tooth profile *Tef*, in a plane normal to the tooth center line. In this plane, both pressure angle and pitch are different.

Let *VP* represent the meshing pressure angle,

VN, the normal pressure angle,

VH, the helix angle.

In the right-angled wedge, *aT* is part of a radius running to the pitch point *T*. *Tc* is a tangent to the meshing profile, so $\angle\, cTa$ is *VP*. *Tb* is a tangent to the normal profile, so $\angle\, bTa$ is *VS*. *cab* is the helix angle *VH*. Then

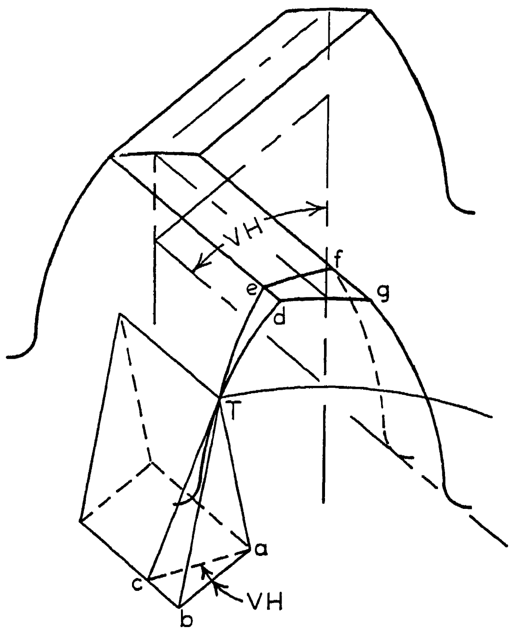


FIG. 10–16. The Three Angles of the Helical Tooth.

$$\tan VN = \frac{ab}{aT} = \frac{ac \cos VH}{ac/\tan VP}$$

or

$$\tan VN = \tan VP \cos VH \tag{5}$$

If P_n represents diametral pitch normal to the tooth, and P the diametral pitch in the meshing plane, it can be seen that

$$P_n = \frac{P}{\cos VH} \tag{6}$$

since the circular pitches p_n/p have the ratio ab/ac. These pitch relations are demonstrated more fully in § 10–5.

It should be noted that, in the relation of the three angles given by equation (5), *VN* and *VP* are measured at the pitch point. Due to the difference in formative tooth numbers, the tooth profile in the meshing

plane resembles the tooth profile in the normal plane only for the smaller helix angles. The two profiles differ more and more as the helix angle increases. This effect is treated more fully in § 10–6.

It is economical to use cutters of standard pitch. In addition, either the helix angle or the meshing pitch can be chosen, but the other will be dictated by relation (6). Similarly, only two of the three angles in (5) can be chosen, the third being dependent. Table 10–1 gives the range

TABLE 10–1

HERRINGBONE GEAR PROPORTIONS

	Helix angle VH	Meshing pressure angle VP	Normal pressure angle VN	Addendum	Clearance
Maximum	45°	27° 14′	20°	$\frac{1}{P}$	$\frac{0.3}{P}$
Minimum	15°	14° 59′	$14\frac{1}{2}$°	$\frac{0.7}{P}$	$\frac{0.157}{P}$

of values of angles and dimensions for herringbone gears. A common helix angle for hobbed gears is 23°, and for Sykes gears 30°.

The speed reducer, Fig. 10–18, is an example of the combination of helical gears with spiral bevel gears in a right-angled drive for large speed change.

10–5. **Helical Gears on Nonparallel Shafts—Spiral Gears.**—When helical gears connect nonparallel and nonintersecting shafts, they are generally called spiral gears but this is a misnomer. When helical gears connecting two shafts revolve in normal planes, they are said to connect shafts at 90°. The angle between nonintersecting shafts is, geometrically, an indefinite term. It is convenient to use the term *shaft angle*, and we shall therefore define it as the angle between the normal planes of the two shafts. It is also the angle between the planes of rotation of the two gears. We shall deal here with the general case of shafts at any angle.

The design of helical gears for nonparallel shafts gives rise to some kinematic relations entirely different from those previously encountered, due to the fact that the diameter of a helical gear depends not only on its tooth size and number of teeth but on its helix angle. Fig. 10–20 shows a helical gear at the left with its pitch cylinder developed on the right. That is, the pitch surface may be imagined as opened on the far side of the gear and laid out flat. All the teeth are therefore seen between H and K since HK is the pitch circumference.

The shape of the intertooth space will be found in a plane normal to the teeth, and the *normal circular pitch* must be measured on a *normal helix*

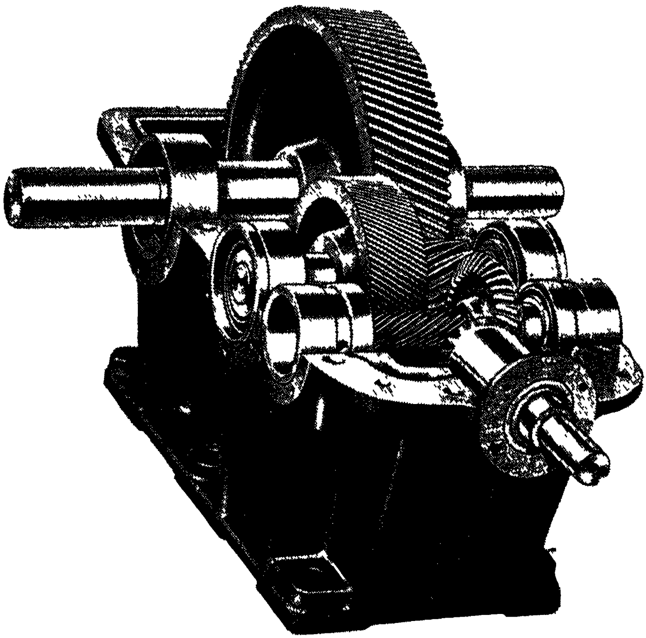


FIG. 10–17. Reduction Gears.
The Falk Corporation, Milwaukee, Wis.

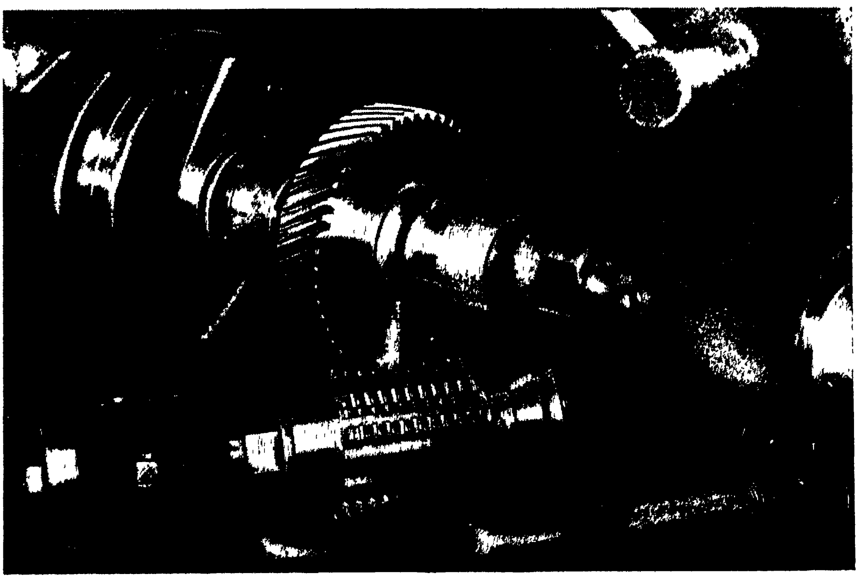


FIG. 10–18. Hobbing a Helical Gear.
Barber-Coleman Co., Rockford, Ill.

such as shown at N. On the developed pitch surface, HF is the length of normal helix that will cut all the teeth once. Consequently if p_n is the normal circular pitch, N the number of teeth, D the pitch diameter, that is the diameter of the pitch cylinder, and θ the helix angle,

FIG. 10–19. Helical Gears for Shafts in Normal Planes (Right Hand Gears).

Farrel-Birmingham Co., Buffalo, N. Y.

$$FH = Np_n = \pi D \cos \theta \tag{7}$$

If P_n represents the diametral pitch of the cutter to be used, then P_n must be the true, normal, diametral pitch such that $P_n = \pi/p_n$, giving

$$D = \frac{N}{P_n \cos \theta} \tag{8}$$

Now consider the general case of two helical gears in mesh, as in Fig. 10–21, connecting shafts at any angle α. It is a principle of action of gears of any kind whatsoever that the teeth of a meshing pair must pass the pitch point alternately. It follows that

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}$$

regardless of how helix angles and pitch diameters may vary. A further fact of some importance is that the sum of the helix angles of a meshing pair must equal the shaft angle, if the direction of the teeth is properly chosen to give least sliding as in Fig. 10–21. Least sliding also requires that the two gears be of the same hand, but not necessarily right hand as in this case.

The kinematic design of helical gears requires different methods for the following cases.

CASE I—NORMAL SHAFT DISTANCE NOT FIXED

The angular-velocity ratio will regularly be specified, and shaft sizes will generally dictate minimum pitch diameters, while considerations of wear resistance will indicate a certain pitch of tooth. Using equation (8), make D_2 the acceptable pitch diameter for the smaller gear, and take for trial value of θ_2 one-half the shaft angle. This gives a trial value for N_2 since P_n has been specified, but N_2 will probably be fractional. Choose the next higher whole number for N_2 that will also give a whole number

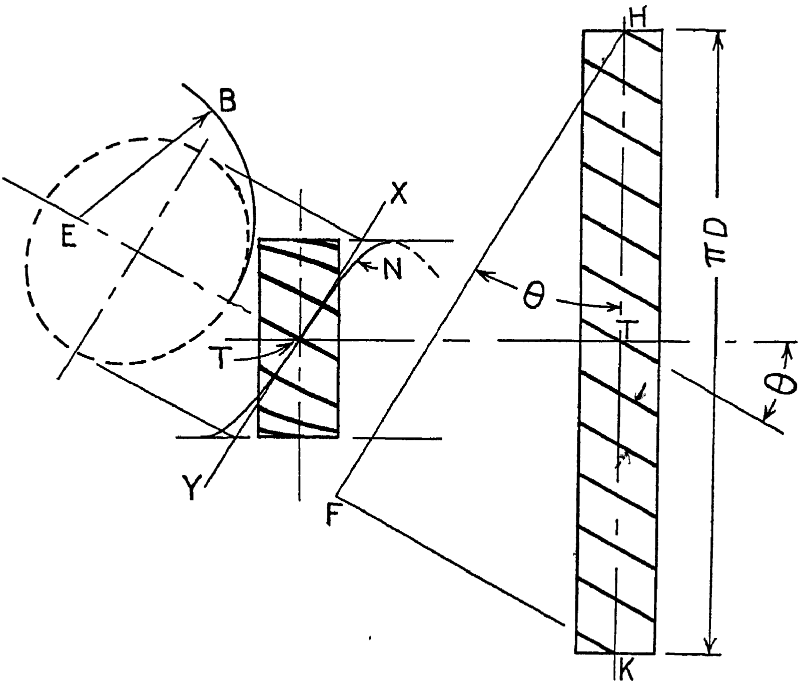


FIG. 10–20. Developed Pitch Cylinder.

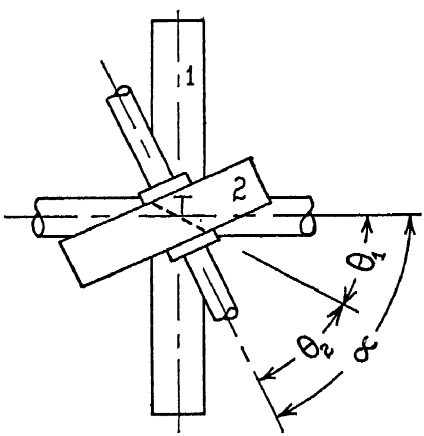


FIG. 10–21. Helical Gears at Any Angle.

for N_1 and satisfy the specified ratio. Place these values for N_1 and N_2 in (8) and solve for final pitch diameters D_1 and D_2.

Suppose now that, due to a large ω_2/ω_1 having been specified, one gear is much larger than the other. Choose new values of θ_1 and θ_2, still keeping $\theta_1 + \theta_2 = \alpha$, the shaft angle, and the size of the gears can be equalized to any extent desired. The cost and space required for the gear box can often be reduced thereby.

Example.—Two shafts at 80° are to be connected in a speed ratio of 11 to 3 by helical gears to be cut with a 2-diametral-pitch cutter. The minimum pitch diameter allowable for the smaller gear is 3 in.

Using (8) and solving for the number of teeth in the smaller, faster gear with $\theta_2 = \theta_1 = 40°$,

$$\begin{aligned} N_2 &= D_2 P_n \cos \theta_2 \\ &= 3 \times 2 \times 0.766 = 4.596 \end{aligned}$$

This points to 6 for N_2 and 22 for N_1 as the smallest numbers that will meet the specifications. Placing these in (8) gives $D_2 = 3.916$ in., and $D_1 = 14.360$ in.

It will be readily seen from Fig. 10–21 that increasing θ_2 and decreasing θ_1 will tend to equalize the size of the gears. If θ_2 is made 60°, θ_1 becomes 20°, and

$$D_2 = \frac{6}{2 \times 0.5} = 6 \text{ in.}$$

$$D_1 = \frac{22}{2 \times 0.9397} = 11.706 \text{ in.}$$

The extent to which this equalization can be carried in practice is limited by the fact that the friction of the increased tooth sliding at high helix angles reduces efficiency. With shafts at 90°, this effect does not become serious until the larger helix angle goes above 75°.

If the gears are to be cut with formed cutters, it will be necessary to determine the number of 2-pitch cutter to be used for each gear. As in the case of bevel gears, the shape of the intertooth space *normal to the teeth* must be considered. In Fig. 10–20, the plane XY is normal to the tooth at the pitch point T. The plane XY cuts the pitch cylinder in an ellipse, projected in broken line. The normal profile of the tooth and intertooth space is the same as the profile of the teeth of a spur gear of pitch circle curvature equal to the curvature of this ellipse at the end of its minor axis. The formative spur gear, therefore, has a pitch radius equal to EB.

The radius of curvature of the ellipse at the end of its minor axis is

$$EB = \frac{(\frac{1}{2} \text{ major axis})^2}{\frac{1}{2} \text{ minor axis}} = \frac{(\frac{1}{2}D \div \cos \theta)^2}{\frac{1}{2}D} = \frac{D}{2 \cos^2 \theta} \tag{9}$$

The formative number of teeth

$$N' = 2(EB)P_n = \frac{DP_n}{\cos^2 \theta}$$

From (8), $D = \dfrac{N}{P_n \cos \theta}$, from which

$$N' = \frac{N}{\cos^3 \theta} \tag{10}$$

In the example, $N_1 = 22$, and $\theta_1 = 20°$, so $N_1' = 22/0.9397^3 = 26.5$. Table 9–2 indicates that a No. 4 cutter is required. For the pinion, $N_2' = 6/0.5^3 = 48$, which indicates a No. $2\frac{1}{2}$ or No. 3 cutter.

If helical gears are to be cut by hob or generating shaper, the spur gear graphs of Figs. 9–27 to 9–29 can be used to determine *approximately* whether undercutting will result from the use of standard cutters. The generating cutters mesh in the normal plane of the helical teeth, but the curvature of the pitch surface changes on the ellipse, Fig. 10–20. Also at the higher helix angles there will be considerable side cutting which alters the resulting profile.

The problems of interference of a meshing pair of so-called spiral gears, and of their contact ratio, are entirely different from the corresponding spur-gear problems, because the meshing action does not take place in a plane normal to the teeth. For data on these points the reader is referred to specialized publications on gearing.[1]

Case II—Normal Shaft Distance Fixed

Applications where considerations other than the design of the connecting gears dictate the shaft spacing are not infrequent. Let the specified normal shaft distance be C inches. It is normal to both shafts and is the shortest distance between them. Then from (8),

$$D_2 + D_1 = 2C = \frac{N_2}{P_n \cos \theta_2} + \frac{N_1}{P_n \cos \theta_1} \tag{11}$$

Observing the relations $N_2/N_1 = \omega_1/\omega_2$, and $\theta_1 + \theta_2 = \alpha$, and using the subscript 2 for the gear having fewest teeth, to correspond with Fig. 10–21,

$$2C = \frac{N_2}{P_n \cos \theta_2} + \frac{\frac{\omega_2}{\omega_1} N_2}{P_n \cos (\alpha - \theta_2)} \tag{12}$$

This is the design equation for case II. All quantities in (12) will be

[1] Buckingham, Earle, "Manual of Gear Design, Section Three, Helical and Spiral Gears."

specified except N_2 and θ_2, and there is the further limitation that N_2 and N_1 must be whole numbers. Manifestly, the procedure is to get an approximate value for N_2, choose an adjacent whole number that will also make N_1 a whole number, and solve for the value of θ_2 that will satisfy the equation.

A direct algebraic solution of (12) will not be attempted. The engineering method of plotting $2C$ against θ_1 on cross-section paper is feasible and can be made as accurate as desired.

Example.—Design helical gears to connect two shafts at an angle of 85° in a speed ratio of 3 to 1. The gears are to be cut with a cutter of 4 diametral pitch and the normal shaft distance must be exactly 4 in.

To get the trial value of N_2, make the helix angles of the two gears equal, or $42\frac{1}{2}°$. Under this condition, the pitch diameters will be in the same ratio as the numbers of teeth.

$$\frac{D_1}{D_2} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = 3$$

also

$$D_1 + D_2 = 2C = 8$$

from which

$$D_2 = 2$$

and

$$N_2 = P_n D_2 \cos \theta_2 = 4 \times 2 \times 0.7373 = 5.898$$

Hence 6 and 18 would appear to be acceptable values for N_2 and N_1.

Placing these values in (4) gives

$$2C = \frac{6}{4 \times \cos \theta_2} + \frac{18}{4 \times \cos (85° - \theta_2)}$$

and the possibilities in this relation can be read from the plot, Fig. 10–22. There are two possible solutions. Either $\theta_1 = 24° \, 20'$ or $\theta_1 = 40°$ will make the shaft distance 4 in. The $24° \, 20'$ value gives a smaller size for the larger gear, and, as it is in the good efficiency zone, should be used. D_1 becomes 4.938 in., and $D_2 = 3.062$ in.

Fig. 10–22 further reveals that the least shaft distance possible with the above tooth numbers is 3.90 in. at $\theta_1 = 32°$. For any closer spacing it would be necessary to change to 5 and 15 teeth.[1]

Fig. 10–23 shows the Lees-Bradner grinder set up to finish a helical gear. The cover is removed to afford a view of the work and grinding wheels. The latter are trued to plane surfaces representing the sides of rack teeth, and set at the proper angles to grind opposite sides of teeth

[1] Another graphical solution for Case II is given by J. K. Olsen in Product Engineering, Oct. 1933.

simultaneously. The work-carrying shaft is set at the helix angle and is supported in the sliding carriage B which is traversed horizontally by the connecting rod C. On the upper end of this shaft is the master gear which is rolled on the stationary rack to provide the generating motion.

The method of indexing is simple and ingenious. At the end of each stroke the master gear goes out of mesh with its rack and into mesh with an indexing rack which turns it one circular pitch.

For grinding spur gears, the work-carrying shaft is set vertically and the master rack has straight teeth.

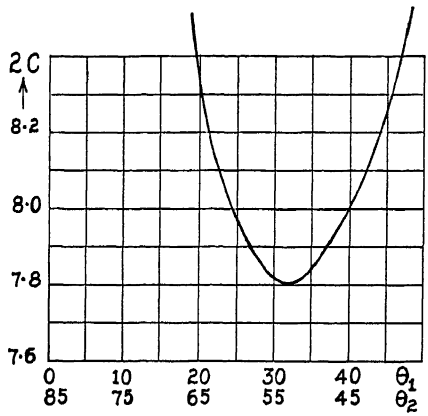


FIG. 10–22. Relation of Angle of Cut to Shaft Distance.

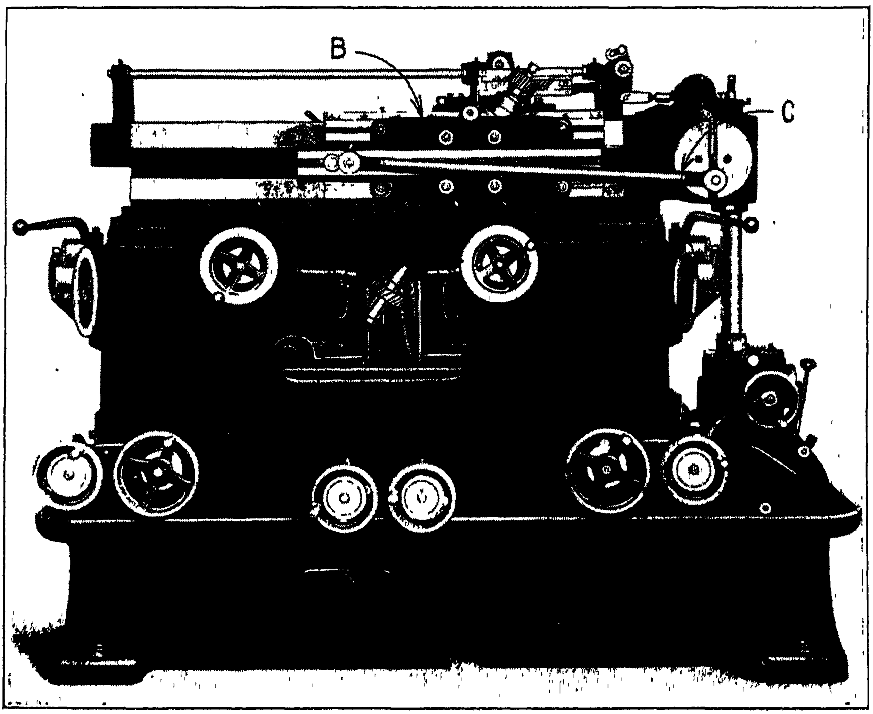


FIG. 10–23. Gear-Tooth Grinding Machine.

The Lees-Bradner Co., Cleveland, Ohio.

Fig. 10–24 shows another design of grinding machine. Each head carries a plane-surfaced grinding wheel so that both sides of a tooth are ground at once. The head column is swiveled to accommodate different helix angles, and the heads are tipped for different pressure angles. The gear being ground is rotated by a gear train as it is rolled under the grinding wheels.

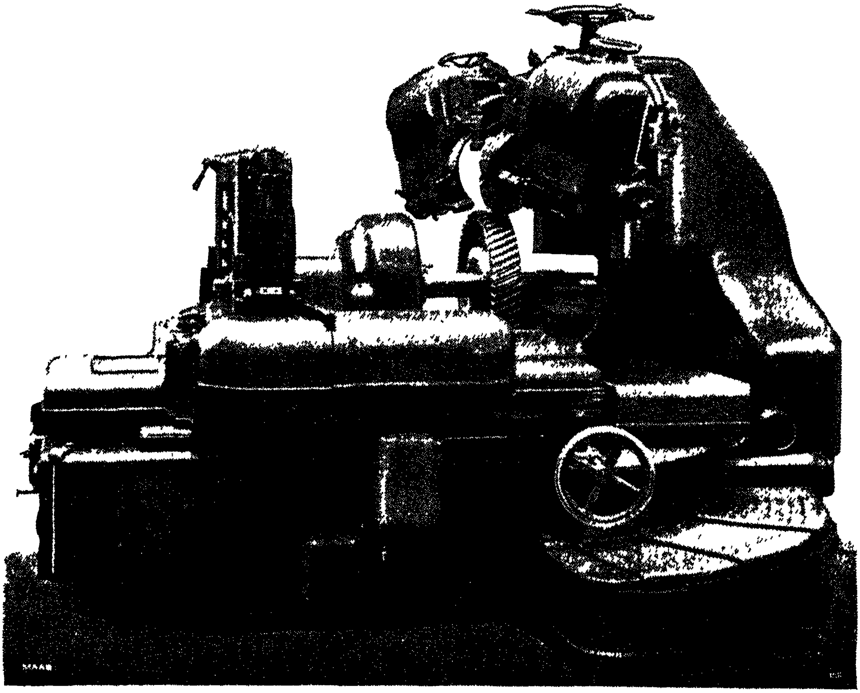


FIG. 10–24. Maag Grinding Machine.

Triplex Machine Tool Corp., New York.

10–6. **The Worm and Wheel.**—In simplest form, the worm and wheel are helical gears, one of which has few teeth, possibly one, in which case it is called a single-threaded worm. Such a pair have theoretical point contact and are really helical gears. However, when used to transmit considerable power, the worm-wheel face is made concave to fit the curvature of the worm as in Fig. 10–25. The worm wheel is said to envelop the worm. In this way a greater area of contact with the worm-wheel tooth is effected. In fact, the theoretical contact is now a line instead of a point.

In the Hindley worm, the same principle is applied to the worm, Fig. 10–29. It is given hour-glass contour thereby enveloping a portion of

the mating wheel and extending the contact over a considerable arc of the worm wheel. However, any wear or lack of adjustment allowing endwise motion of the worm causes the drive to bind, whereas the straight worm is not subject to this difficulty.

The *lead* of a worm, as for any screw, is the axial advance of the driven member per turn. On a double-threaded worm (having two teeth) the lead is twice the axial pitch. The *lead angle* is 90° minus the helix angle. The axial pitch of the worm must equal the circular pitch of the worm wheel. The latter pitch is measured in the center plane of the worm wheel. A plane section through the worm axis and the center plane of the wheel gives the profile of a rack and spur pinion. The worm and wheel is no exception to the rule that, in gearing, the numbers of teeth alone decide the angular velocity ratio, $\omega_1/\omega_2 = N_2/N_1$.

The rack-and-pinion relation can be better visualized with the aid of Fig. 10–25. Suppose that the worm is moved axially without rotation.

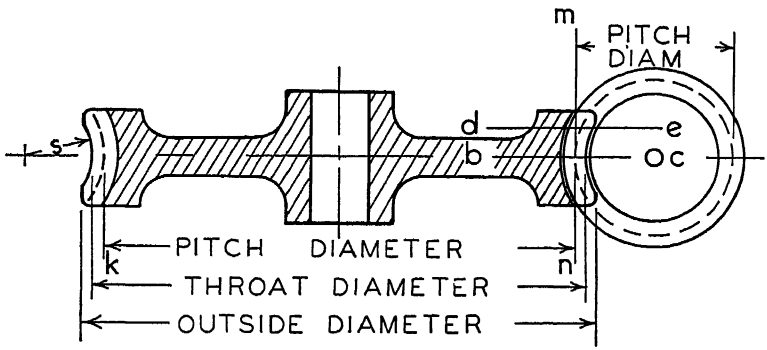


FIG. 10–25. Worm and Wheel in Mesh.

It would rotate the wheel, and the meshing action would be no different from the usual kind, except for the absence of sliding tangential to the worm. This further shows that in all planes such as *de*, parallel to the center plane, the pitch points must be on the straight line *mn*. It is therefore inaccurate to speak of the pitch cylinder of a worm, though the term often appears in the literature of the subject. It is not an axode. The true pitch surface of the worm is the plane *mn*. The pitch surface of the worm wheel is the cylinder *kn*, which rolls on this plane without sliding. It is convenient, however, to call the diameter of the broken line circle, the pitch diameter of the worm.

10–7. **Producing the Worm and Wheel.**—The tooth profiles and dimensions of worms and wheels are not well standardized, because the methods of producing them are not standardized, particularly in the case of the worm. There is a tendency to standardize the tools rather

than the tooth contours. In fact, to standardize the latter in the case of the worm would be impracticable.

Consider the straight-sided rack profile, which is the simplest to produce and the most used tool form for worms. A simple formed threading tool, such as is commonly used in a lathe, is the only kind that will produce an axial intertooth profile of the same shape as the tool. To do this, the tool must be set so that its cutting edges are in the axial plane of the worm. A milling cutter of rack shape (straight sided) fed at the correct lead angle, produces a tooth with curved sides in the normal plane, and if the diameter of the milling cutter is increased, the curvature changes, due to the increased side cutting action. Increase of lead angle greatly increases the side cutting, unless the thread angle of the tool (pressure angle) is correspondingly increased. The results with hobbing are similar. A glance along the intertooth space of a worm with large lead angle will demonstrate how the warping helicoids get in the way of the entering and leaving teeth of the milling cutter or hob. Furthermore, all these tooth curves in the normal plane become different

TABLE 10-2

A. G. M. A. Standard Worms

Term	Symbol	Single and Double Threaded	Triple and Quadruple Threaded
Pressure angle of cutter	VN	$14\frac{1}{2}°$	20°
Axial pitch	p		
Pitch diameter	D_w	$2.4p + 1.1$	$2.4p + 1.1$
Outside diameter	OD_w	$3.036p + 1.1$	$2.972p + 1.1$
Root diameter	RD_w	$1.664p + 1.1$	$1.726p + 1.1$
Teeth in worm wheel	N		
Length of worm	L_w	$p\left(4.5 + \frac{N}{50}\right)$	$p\left(4.5 + \frac{N}{50}\right)$
Lead angle	VL	$\tan VL = \frac{\text{lead}}{\pi D_w}$	$\tan VL = \frac{\text{lead}}{\pi D_w}$
Top round (tooth corner) radius		$0.05p$	$0.05p$

TABLE 10-3

A. G. M. A. Standard Worm Wheels

Term	Symbol	Single and Double Worm	Triple and Quadruple Worm
Circular pitch	p		
Number of teeth	N		
Pitch diameter	D_g	Np/π	Np/π
Throat diameter	TD_g	$D_g + 0.636p$	$D_g + 0.572p$
Outside diameter	OD_g	$TD_g + 0.4775p$	$TD_g + 0.3183p$
Face width	F	$2.38p + 0.25$	$2.15p + 0.20$
Radius of wheel face	s	$0.882p + 0.55$	$0.914p + 0.55$
Radius of wheel rim		$2.2p + 0.55$	$2.1p + 0.55$
Radius of outer corner		$0.25p$	$0.25p$

curves in the axial plane, slightly different for low lead angles, greatly different for large lead angles. It should be clear why it is economical to standardize the tool rather than the tooth profile.

The A. G. M. A. recommends as standard tooth form, that produced by a straight-sided milling cutter of outside diameter, between the outside diameter of the worm, and $1\frac{1}{4}$ times the outside diameter. Also, the following axial pitches are recommended as standard: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2 in.

For general industrial applications, the A. G. M. A. has recommended standards based on only two pressure angles for the milling cutters used, and designed to cover all speed ratios from 10 to 1 up to 100 to 1. The principal dimensions are given in Tables 10–2 and 10–3.

Worm wheels are regularly cut with hobs, Fig. 10–26. The closer the hob is in its dimensions to the worm to be used, the better will be the

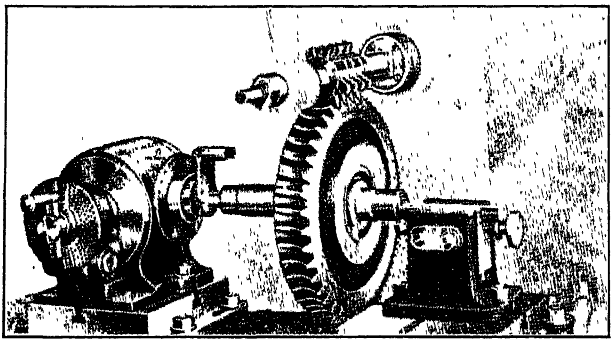


FIG. 10–26. Hobbing a Worm Wheel.

results. The hob should never be shorter than the worm, otherwise the conjugate profile will not be completely cut and interference will result. The hob should never be smaller than the worm, otherwise bearing will be concentrated at the ends of the teeth of the wheel. High lead angles accentuate these difficulties.

The above A. G. M. A. recommended pitches and standards cover a wide range of applications, and all those wheels can be cut with a set of 44 hobs. However, the **highest efficiency** in worm drives can only be obtained when the rubbing velocity is low compared to the pitch velocity of the wheel. That requires high lead angles and usually many more teeth than four (quadruple) on the worm. This calls for a special design for each important drive.

As previously mentioned, high lead angles require higher pressure angles [1] on the tools to be used:

[1] Kent's M. E. Design Handbook, p. 14–29.

TABLE 10-4

Pressure Angle	Maximum Lead Angle
$14\frac{1}{2}°$	15°
20°	25°
25°	35°
30°	45°

Face widths are made narrower than the values in Table 10-2 for high speeds and high lead angles. It has been found that finer pitches give smoother performance at high speed. With high lead angles and finer pitches, it becomes more economical and satisfactory to hob the worms instead of using milling cutters. Worms are regularly made of steel, sometimes hardened and the teeth ground. The best worm wheels are of bronze. Cast iron is used for low speeds.

10-8. **Worm and Wheel Applications.**—Well designed worm and wheel applications give large speed changes with high efficiencies. The best lead angles are from 45° to 30°, the efficiency dropping fast as the lead angle goes below 30°. With good bearings, especially with ball or roller bearings, drive efficiencies over 90% are possible.

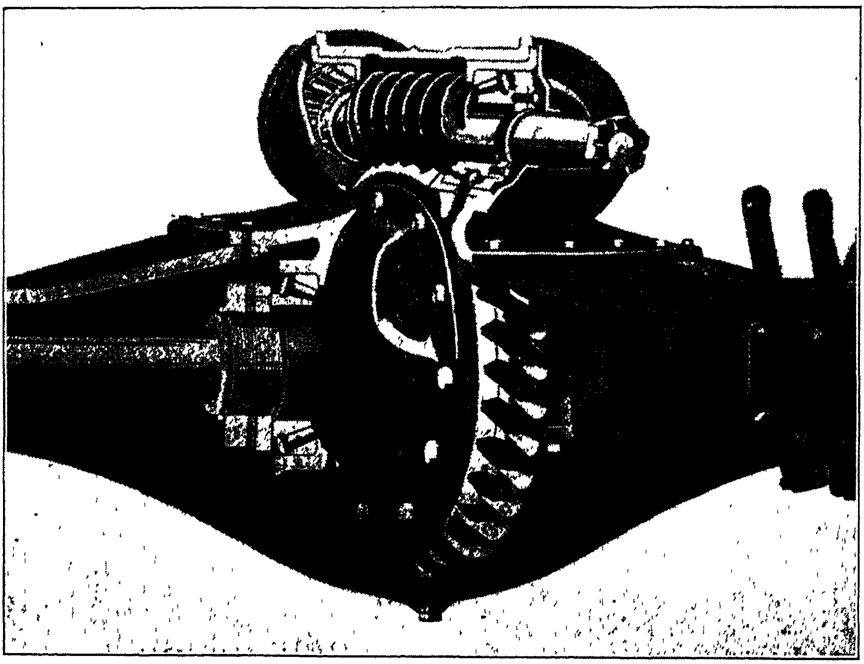


FIG. 10-27. Worm and Wheel Drive for Truck.

The Studebaker Corp., South Bend, Ind.

As the helix angle of a worm wheel is decreased approaching zero, there comes a point where the worm wheel can no longer drive the worm due to the tooth friction and the friction of the bearings supporting the worm. This condition arises when

$$\mu = \frac{\text{friction force}}{\text{normal force}} > \tan \theta_1$$

where μ is the coefficient of friction, and θ_1 is the helix angle of the worm wheel (also the lead angle of the worm for 90° drives). With ball or roller bearings and well lubricated teeth, there will be reversibility with values of θ_1 as low as 5°.

Fig. 10–27 shows a worm and wheel drive for a truck. Such drives must be not only reversing but highly efficient. Conversely, worms used on steering columns should approach irreversibility to avoid road shock at the steering wheel. Fig. 10–28 is a section through such a worm and wheel sector.

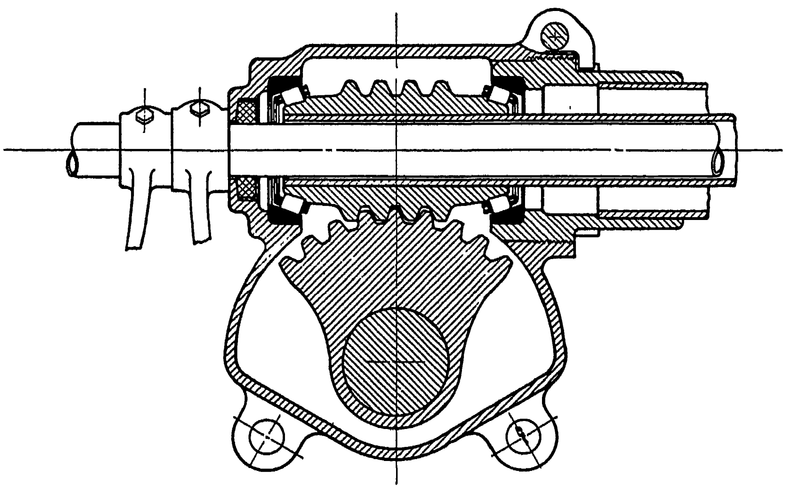


FIG. 10–28. Steering Worm and Wheel Sector for Automobile.

An improvement is seen in the Ross patent of Fig. 10–29. Two worm-wheel teeth having zero helix angle are cut on the roller. Of course a Hindley type worm is required. Tooth rubbing is almost eliminated, and any wear on the roller is distributed around its circumference. In Fig. 10–28 the wear is certain to be greatest on the center tooth of the sector, and it is generally made high (the sector cut eccentrically) to avoid tightness on sharp turns after there has been adjustment for wear.

Fig. 10–29 also suggests the fact that a plain spur gear can be meshed with a worm, the shaft angle being 90°, and this is often done. Correct

theoretical action still results. In fact, correct theoretical action is not spoiled by a slight change of shaft angle with any straight worm and wheel. The side of the worm thread is not a plane surface as in the case

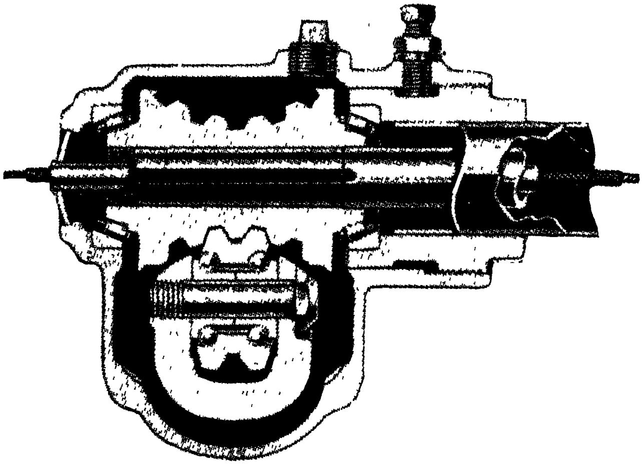


FIG. 10–29. Worm and Roller Steering Gear.
Oldsmobile Division of General Motors Corp., Detroit, Mich.

of a rack tooth, but a curved helicoid. Hence if the worm is rocked sideways, changing the shaft angle slightly, other portions of the helicoidal surfaces come into action, requiring only a slight increase of shaft spacing.

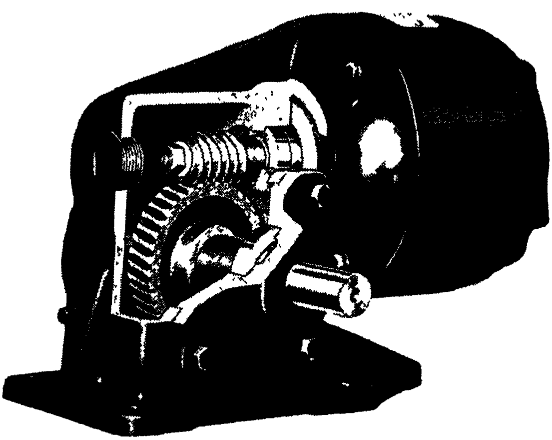


FIG. 10–30. Right-Angle Gear-Motor Drive.
The General Electric Co., Schenectady, N. Y.

A very compact motor reduction drive for right-angle applications is given in Fig. 10–30.

The testing machine of Fig. 10–31 is for all errors that would produce a change in shaft center-line distance with the gears in full mesh. These include eccentricity and variations in pitch of the wheel and lead of the worm. The bearings of the worm are fixed, while the worm-wheel shaft is carried on a carriage free to slide on the frame as shown.

A spring acts on the carriage tending to keep the gears closely meshed. Any errors tending to change the center-line distance as the gears revolve, will move the floating carriage. This motion, magnified by levers, is

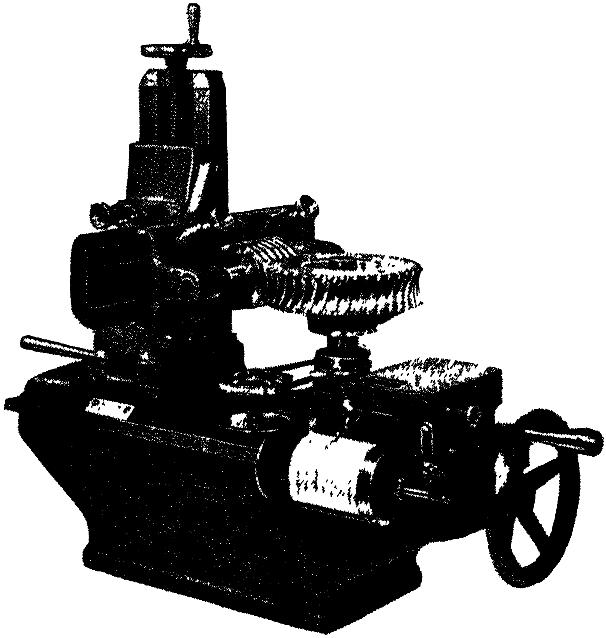


FIG. 10–31. Checking Machine for Worm and Wheel.

Maag Gear-Wheel Co., Zurich, Switzerland.

transmitted to the pencil point, bearing on the recording drum. The drum is connected through a belt and friction wheels with the worm-wheel shaft. In this way, the location of errors, indicated by irregularities in the traced curve, can be fixed on the gears.

QUESTIONS AND PROBLEMS

1. A pair of straight-tooth bevel gears have 48 and 64 teeth, are of 8 nominal diametral pitch, and one-in. face, and connect shafts at 90°. The tooth form is $14\frac{1}{2}°$ full depth. Make a sketch of these gears calculating the sizes and angles of the pitch cones and the overall diameters of the two blanks. What is the diametral pitch of the teeth at their small ends?
2. Find the formative tooth numbers of the gears of Prob. 1.
3. Straight-tooth bevels of 6 diametral pitch having 48 and 24 teeth are required to connect normal shafts. If the teeth are given $14\frac{1}{2}°$ standard form, being cut by Gleason rack-shaped cutters, will

they be undercut, and what will be their approximate contact ratio?

Ans. The formative tooth numbers are 26.8 and 107.4. The pinion will be undercut. Contact ratio is about 1.9 (Fig. 9–27).

4. Straight-tooth bevel gears of 5 diametral pitch, having 10 and 15 teeth, are required to connect normal shafts. Can the teeth be given any of the standard forms without being undercut by rack-shaped cutters? If so, what would be their form and approximate contact ratio?
5. Shafts at 60° are to be connected by 4-nominal-pitch straight-tooth bevels, the tooth numbers being 52 and 39. Find the formative-tooth numbers. What tooth form would you use and what face width?
6. A helical gear was cut with a $14\frac{1}{2}°$ standard cutter, fed to standard depth, and the helix angle was 20°. If this gear is used with a like gear on a parallel shaft, at what pressure angle will they mesh? Ans. 15° 23′.
7. Herringbone gears are hobbed with a 20°, full-depth, standard hob. At what pressure angle will they mesh if the helix angle is (*a*) 23°, (*b*) 30°?
8. A 12-tooth helical gear has a helix angle of 36°, and is to be cut with a standard, composite, formed cutter of 4 diametral pitch.
   (*a*) What number of cutter should be used?
   (*b*) What is the pitch diameter?
   (*c*) To what outside diameter should the blank be turned?
9. A pair of 10-pitch herringbone turbine gears has a helix angle of 30° and a total face width for both halves of 8 in. Find the fewest pairs of teeth in mesh at the pitch line for any phase.
10. Set forth clearly the kinematic advantages of helical gears on parallel shafts over spur gears.
11. Design helical gears to connect shafts at 75° in a speed ratio of 7 to 3, to be cut by an 8-diametral-pitch cutter. The pitch diameter of the smaller gear should be at least $2\frac{1}{2}$ in.
12. Helical gears are required to connect two shafts at 90°, having a shaft distance of exactly $3\frac{1}{2}$ in., in a speed ratio of 4 to 1. Recommend gear sizes and the numbers of the cutters, assuming that the teeth are to be cut with formed composite cutters of 4 diametral pitch.
13. Design helical gears to be cut by a 10-pitch hob and to connect shafts at an angle of 90° in a speed ratio of 5 to 1. The pitch diameter of the smaller gear must be 2 in. or larger.

14. Two shafts having normal planes which make an angle of 60° must have a fixed normal distance of 3 in. Design helical gears to be cut with a 6-pitch hob, to connect these shafts in a speed ratio of 5 to 2.
15. The axial pitch of a single-threaded worm is 0.75 in. What is the pitch diameter (at the throat) of its worm wheel which has 24 teeth?
16. A triple-threaded worm has a lead angle of 30°, an axial pitch of 0.5 in., and is cut by a milling cutter of 25° pressure angle. It drives a worm wheel having 48 teeth. Find the pitch diameters of each, also the helix angle, the normal pitch, and meshing pressure angle of the worm wheel.

    Ans. Pitch diam. of worm 0.827 in., of worm wheel 7.6394 in. Helix angle of worm wheel 30°, normal pitch 0.433, meshing pressure angle 28° 18′.
17. A quadruple-threaded worm has a pitch diameter of 4 in. and an axial pitch of 1 in. The normal, thread, pressure angle is 20°. The worm wheel has 40 teeth. Find the pitch diameter of the worm wheel, also its meshing pressure angle.
18. A single-threaded worm of 0.75-in. axial pitch and 2-in. pitch diameter is cut by a $14\frac{1}{2}$° milling cutter, and drives a 60-tooth worm wheel. Find the normal pitch of the worm and its axial pressure angle, also the pitch diameter of the worm wheel.
19. Compare the size of the worm and wheel of Prob. 18 with that of a spur pinion and gear of the same pitch, that would give the same speed ratio. A 12-tooth pinion may be used.
20. A double-threaded worm of one-inch axial pitch drives a 50-tooth wheel. Find the principal dimensions according to Tables 10–2 and 10–3, and make a drawing of the pair in mesh.
21. Design and draw a quadruple-threaded worm of $\frac{3}{4}$-in. axial pitch meshing with a 40-tooth gear.

CHAPTER XI

GEAR COMBINATIONS

11–1. Simple and Compound Gear Trains.—A succession of meshing gears having only one gear with one set of teeth on each of the shafts connected, is called a *simple gear train*. In the simple train of Fig. 11–1, let N_a be the number of teeth on the left gear, N_b the number of teeth on gear b on shaft b, etc. Since, when any two gears mesh, the teeth on each must pass the pitch point alternately, it follows that their *relative angular velocities must be inversely as the number of teeth*. In Fig. 11–1, therefore,

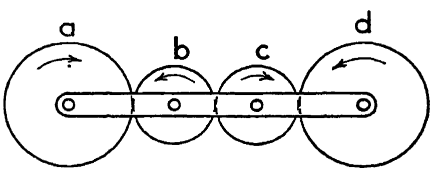


FIG. 11–1. Simple Gear Train.

$$\frac{\omega_b}{\omega_a} = \frac{N_a}{N_b}, \qquad \frac{\omega_c}{\omega_b} = \frac{N_b}{N_c}, \quad \text{and} \quad \frac{\omega_d}{\omega_c} = \frac{N_c}{N_d} \tag{1}$$

Now

$$\frac{\omega_d}{\omega_a} = \frac{\omega_b}{\omega_a} \times \frac{\omega_c}{\omega_b} \times \frac{\omega_d}{\omega_c}$$

and substituting,

$$\frac{\omega_d}{\omega_a} = \frac{N_a}{N_b} \times \frac{N_b}{N_c} \times \frac{N_c}{N_d} = \frac{N_a}{N_d} \tag{2}$$

It is thus plain that those gears in a train, that receive and transmit motion on the same set of teeth, have no influence on the overall speed ratio, and for this reason are called *idlers*. Every added idler changes the sense of rotation however, and they are also useful in getting a desired spacing between the end shafts without the use of large expensive gears.

It is often convenient to indicate the sense of rotation in gear trains by use of the + and − signs. If clockwise rotation be given the + sign, counterclockwise rotation must have the − sign, and, in the train of Fig. 11–1, it will be noted that the sign of ω changes at every successive shaft. This is the rule with external but not with annular gears. It should also be noted that when the end gears of a train revolve in the *same* sense, the *relative* speed ratio is positive regardless of the sense in which the two end gears rotate.

Train value is the angular velocity of the last gear of a train divided by that of the first. Now returning to equations (1) and (2), ω_a and ω_d

are of opposite sense so that the train value becomes

$$\frac{\omega_d}{\omega_a} = -\frac{N_a}{N_d} \tag{3}$$

A *compound gear train* is one where more than one set of teeth *fastened together* on the same axis enter into the train. For the case shown in Fig. 11–2 where c and b are rigidly fastened together, the train value is

$$\frac{\omega_d}{\omega_a} = \frac{N_a}{N_b} \times \frac{N_c}{N_d} \tag{4}$$

Both ω_d and ω_a being negative, the train value is positive.

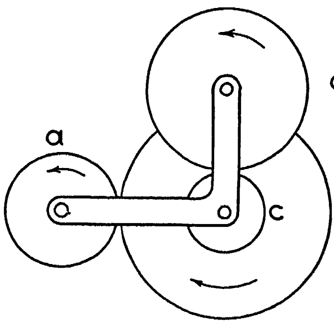


FIG. 11–2. Compound Gear Train.

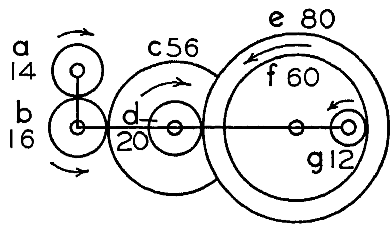


FIG. 11–3. Seven Gear Train.

If a is the initial driver, b is a follower, c a driver, and d a follower. Hence train value is often expressed thus,—

$$\textit{Train value} = \frac{\text{product of teeth on drivers}}{\text{product of teeth on followers}}$$

In practice, a single pair of spur gears or a simple train is seldom used for a speed reduction greater than 7 to 1, for the reason that one of the gears must be so large that economy in both space and first cost would result from the use of a compound train. A worm and wheel might be used where larger reduction ratios are demanded.

The compound gear train of Fig. 11–3 yields the following train value,

$$\frac{\omega_g}{\omega_a} = -\frac{14 \times 20 \times 60}{56 \times 80 \times 12} = -\frac{5}{16}$$

That is, the last driven gear g will turn 5/16 of a turn while the first driver a turns once, and in opposite sense.

11–2. **Epicyclic or Planetary Gear Trains.**—A *reverted gear train* is one where the end driven gear is coaxial with the initial driver. Thus,

the compound train of Fig. 11–4 has been reverted in Fig. 11–5. While gears a and d are now on the same axis, they cannot be pinned together, as c and b are, but will turn at relative speeds dictated by the train value.

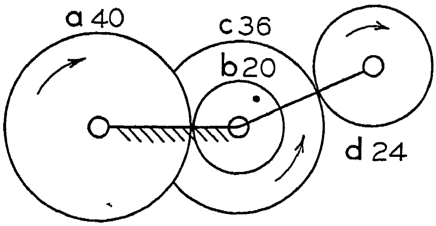


FIG. 11–4. Compound Train.

Any train can be reverted by the use of idlers. In a four-gear train, without idlers, such as that being considered, it can be reverted only if the sum of the pitch radii of gears a and b equals the sum of the pitch radii of gears c and d. The back gears of a lathe, Fig. 11–15, together with the gears on the spindle axis constitute a reverted train. In an automobile, the transmission in intermediate, low speed, and reverse, Fig. 11–10, is through reverted trains so that transmission in high can be effected by simple clutch engagement.

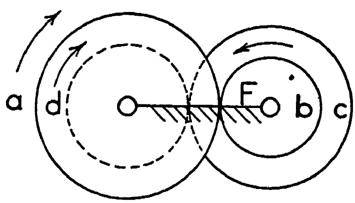


FIG. 11–5. Reverted Train.

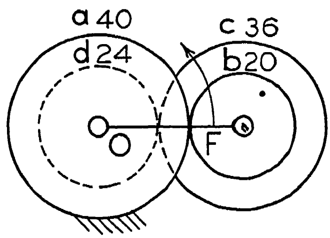


FIG. 11–6. Epicyclic Train.

The train value is not affected by the change from Fig. 11–4 to Fig. 11–5.

$$\frac{\omega_d}{\omega_a} = + \frac{N_a \times N_c}{N_b \times N_d} = + \frac{40 \times 36}{20 \times 24} = + 3$$

In Fig. 11–4 and Fig. 11–5, the fixed link of the mechanism is F. Fig. 11–6 shows an *inversion* of this mechanism, accomplished by fixing gear a and unfixing link F. The result is an epicyclic gear train in which b rolls about a.

An *epicyclic gear train* is a reverted train in which one of the gears has been made the fixed link. We shall always consider that the gear fixed in the epicyclic train is the first driver in the reverted train.

Now, in order to produce the identical *gear action* that resulted, in the reverted train, from turning gear a once clockwise, it is necessary in the epicyclic train to turn F once counterclockwise.

Suppose next that both a and F are temporarily unfixed, and, with the gears locked, the whole mechanism is given one turn counterclockwise

about O. Every link, including d, will receive one revolution counterclockwise, without any gear action. Now fix a and revolve F once counterclockwise. There will be the same gear action in magnitude and sense as occurred in the reverted train. Therefore d will receive one turn in the same sense as the driver F in addition to the reverted train value. For the same gear action, the reverted train gave d clockwise rotation. Hence, *the epicyclic train value is one minus the reverted train value.*

For the above case, the epicyclic train value $= 1 - (+3) = -2$. This means that if the driver F is given one turn, d will be driven two turns in opposite sense.

Observe that the gear that becomes the fixed link of the epicyclic train must be considered the first driver in the reverted train. This is a necessary premise in getting the correct reverted train value.

Since the epicyclic train value is one minus the reverted train value, it will be readily seen that to get a very large reduction ratio in the epicyclic train, it is only necessary to have a reverted train value very close to one. For example, if, in Fig. 11–6, $N_a = 61$, $N_b = 61$, $N_c = 62$, and $N_d = 60$, the reverted train value $\frac{\omega_d}{\omega_a} = +\frac{61 \times 62}{61 \times 60} = +\frac{31}{30}$. The epicyclic train value is then

$$\frac{\omega_d}{\omega_F} = 1 - \frac{31}{30} = -\frac{1}{30}$$

Suppose a positive value is desired so that driver and driven, F and d, will revolve in the same sense. Make

$$\frac{\omega_d}{\omega_a} = \frac{N_a \times N_c}{N_b \times N_d} = \frac{61 \times 60}{61 \times 62} = +\frac{30}{31}$$

This gives an epicyclic value,

$$\frac{\omega_d}{\omega_F} = 1 - \frac{30}{31} = +\frac{1}{31}$$

Consider the case where

$$\frac{N_a \times N_c}{N_b \times N_d} = \frac{61 \times 61}{60 \times 62} = +\frac{3720}{3721}$$

Here $N_a + N_b = 121$, and $N_c + N_d = 123$, so, if the train were reverted and c and d were properly spaced, a and b would have backlash. However, using the variable center distance method, § 9–13, gears a and b can be enlarged to have the same center distance as the standard gears c and d, without change of tooth numbers. Such a combination made

epicyclic gives an astonishing speed reduction.

$$\text{Epicyclic value} = 1 - \frac{3720}{3721} = + \frac{1}{3721}$$

Not often are such large gear reductions required, but when they are, nothing approaches the epicyclic train in compactness and effectiveness. Even where only moderate speed reduction is desired, it is widely used, as in hoists and several standard speed reduction units. The famous Model-T Ford automobile had an epicyclic gear transmission giving two forward speeds and one reverse speed.

Epicyclic gear trains always involve the rotation of gears on axes that are also travelling in circles. The analogy with the motions of the planets in the solar system has led to the expression "sun and planet gears" or "planetary" gears.

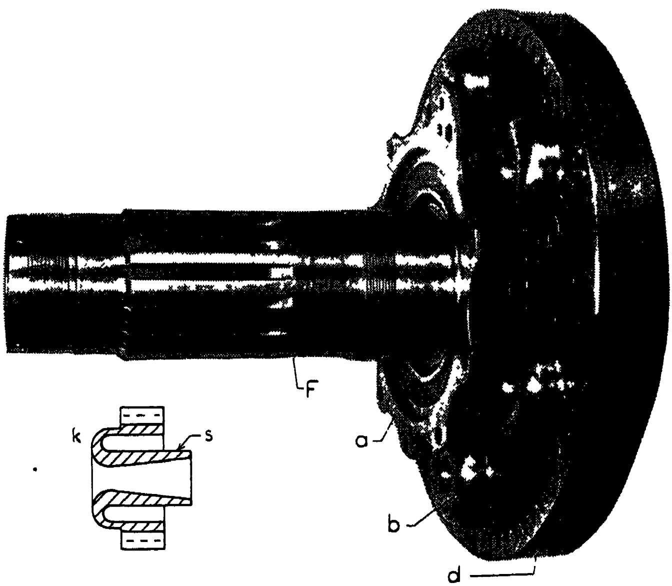


FIG. 11–7. Reduction Gears of Wright Cyclone 9 Aircraft Engine.

Wright Aeronautical Corp., Paterson, N. J.

The use of the annular gear makes it possible to construct a reverted train, and therefore an epicyclic train, with three gears only. Consider the reduction gears used between the engine and propeller shaft of the Wright Cyclone, nine-cylinder, aircraft engine, Fig. 11–7. The center gear of 40 teeth has its flange bolted to the frame. We shall call it a since it corresponds to a in Fig. 11–6. The planet gears b, each having 20

teeth, turn on axes which are fixed in the flange of the propeller shaft F. There are six planet gears to distribute the load, but kinematically only one is required. The annular gear is on the engine shaft and will be called d. It has 80 teeth.

The fixed gear a must be considered the driver of the train before it is made planetary. This reverted train value, with F fixed, is

$$\frac{\omega_d}{\omega_a} = -\frac{N_a}{N_b} \times \frac{N_b}{N_d} = -\frac{N_a}{N_d} = -\frac{40}{80} = -0.5$$

Fixing a and unfixing F gives the planetary train value

$$\frac{\omega_d}{\omega_F} = 1 - (-0.5) = +1.5$$

When the engine is running at 1800 rpm, it will drive its propeller at 1200 rpm in the same sense.

In dealing with epicyclic trains, one should be careful to relate the arrangement logically to the original theory and particularly to Fig. 11–6, disregarding the direction in which energy may be flowing through the train in the particular case. Of course, it is possible to compute the relative speeds in Fig. 11–7 by considering gear b as a lever fulcrumed on a, but the epicyclic train occurs in a great variety of forms, and the general solution will prove adequate for them all.

At k in Fig. 11–7, a sectional view of the planet gear b has been drawn. This is the aircraft designer's idea of a spur gear. Note the large size of the bearing which is the cylindrical surface s. This part, and the large annular gear d, are models of light-weight design.

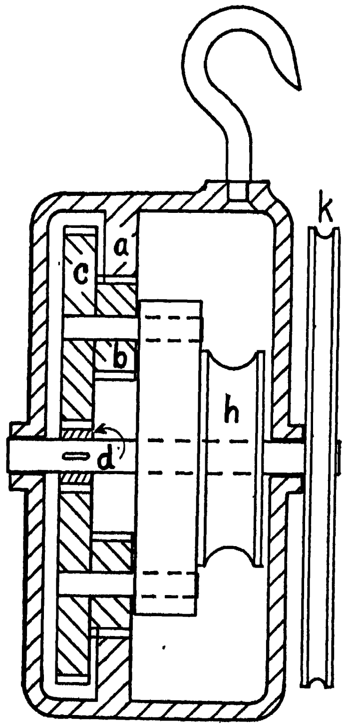


FIG. 11–8. Epicyclic Hand Hoist.

11–3. Epicyclic Hoists.—The large mechanical advantage required in hoists must be combined with lightness of weight and ruggedness. The compact epicyclic train meets the requirements admirably. In Fig. 11–8, the large sprocket wheel k takes the hand chain while the load chain grips the heavy sprocket h. The shaft of K turns freely in h, and is keyed to

gear d. Gear d meshes with c which is attached to b, and b meshes with the teeth on a, the frame of the hoist. The shaft on which c and b turn is firmly fastened in h. Gears c and b are duplicated for balance and greater strength. Sometimes three sets are used spaced 120° apart.

The fixed gear is a, so it must be considered the first driver in the reverted train from which the epicyclic train may be considered derived, and in which the order of drive was a, b, c, d, with h the fixed link. The reverted train value is

$$\frac{\omega_d}{\omega_a} = -\frac{N_a \times N_c}{N_b \times N_d}$$

$$= -\frac{88 \times 48}{24 \times 16} = -11$$

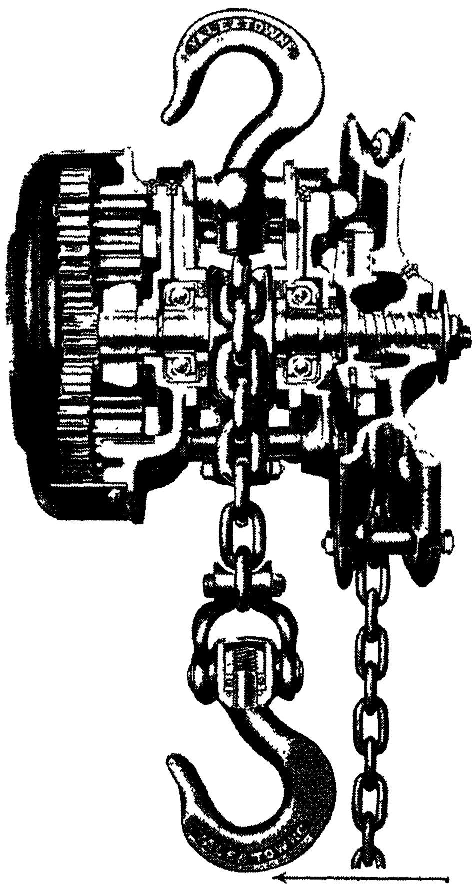


Fig. 11–9. Epicyclic Geared Hoist.

The Yale and Towne Mfg. Co., Philadelphia, Pa.

Now fix a and allow h to turn, and the train becomes epicyclic with a train value $\omega_d/\omega_h = 1 - (-11) = +12$. Hence to turn the load wheel once, the hand wheel must be turned 12 times and in the same sense. The actual mechanical advantage of the hoist is further increased as the pitch diameter of k is larger than that of h. The faces of the chain wheels k and h are formed to grip the chain links and prevent slipping, as clearly shown in the cut of the Yale and Towne hoist, Fig. 11–9. This view also shows how the load can be lowered under the control of a brake located in the hand sprocket wheel.

11–4. **Automotive Selective Transmission.**—Fine examples of the reverted gear train are to be found in automotive gear boxes. The term "selective" connotes that the operator can select and use any speed without going through any other speed. Some machine-tool gear boxes are not selective in this sense.

An automotive transmission stripped to the essential parts is shown in Fig. 11–10. Shaft *m* is connected to the engine through a clutch. Gear *a* is an extension of *m*. Gear *b* is always in mesh with *a*, so these are called the constant-mesh gears. Gears *b*, *c*, *e*, and *g* are the back shaft gears, are fastened together, in modern practice are cut on one forging. They turn whenever *m* turns. Gears *f* and *d* are keyed to the splined shaft *k*. They are free to slide on *k* under control of the shift lever, but must always turn with *k*. *k* connects, through a universal joint, with the transmission shaft that runs to the rear-axle gears. There is no connection between *k* and *m* except that *m* supports the right end of *k* through the small roller bearing inside *a*.

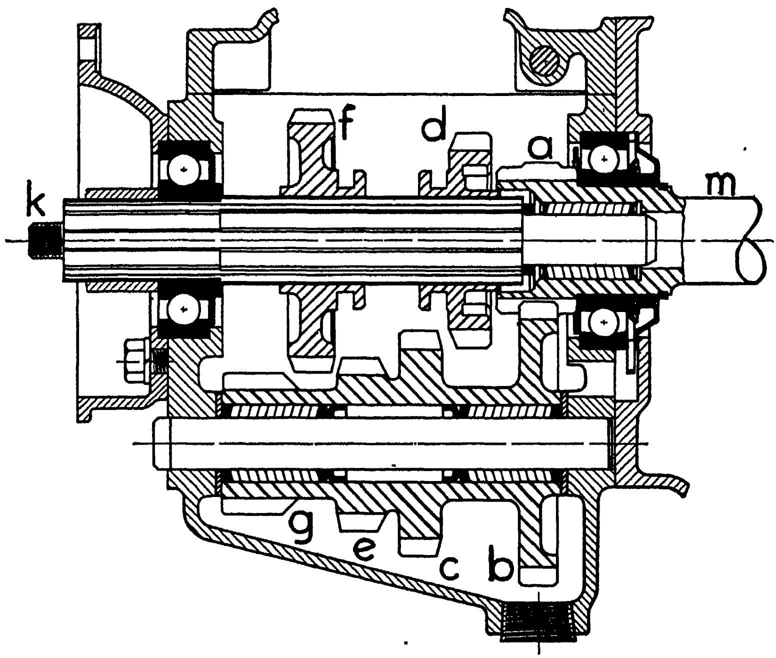


FIG. 11–10. Four-Speed Automobile Transmission.

High speed is obtained by sliding *d* to the right so that it is connected to *a* through a positive gear clutch. The ends of the external teeth on *a* fit into internal teeth of the same pitch on *d*. This is a direct drive without benefit of gear action.

Intermediate speed is obtained by moving *d* to the left so that it meshes with *c*. Then transmission is through the reverted train *abcd*.

Before *f* can be placed in gear, *d* must be returned to neutral position, in which it appears in Fig. 11–10. This is insured by the design of the

shifting mechanism. When f is moved into mesh with e, low speed is obtained through the train $abef$.

For reverse speed, f is moved to the left. f and g clear each other, and f meshes with an idler gear r, not shown, which is in constant mesh with g. The idler accounts for the reversed sense, and the train involved is $abgrf$. The idler shaft can be seen in Fig. 11–11.

The speed ratios will now be computed from the tooth numbers.

$$\text{Intermediate} \quad \frac{\omega_m}{\omega_k} = \frac{N_b}{N_a} \times \frac{N_d}{N_c} = \frac{35}{18} \times \frac{26}{27} = 1.87$$

$$\text{Low} \quad \frac{\omega_m}{\omega_k} = \frac{N_b}{N_a} \times \frac{N_f}{N_e} = \frac{35}{18} \times \frac{33}{20} = 3.21$$

$$\text{Reverse} \quad \frac{\omega_m}{\omega_k} = -\frac{N_b}{N_a} \times \frac{N_f}{N_g} = -\frac{35}{18} \times \frac{33}{16} = -4.01$$

The reverse gear, having no numerical influence on the result, is omitted from the computation. The engine-to-rear-wheel speed ratio is further increased by the ratio of the bevel gears in the rear axle.

To insure that gears will slide into mesh axially, it is necessary to round the ends of the teeth of both gears, and special machines have been developed for this operation. It is difficult, even with the best rounding of teeth, to mesh gears unless their pitch line velocities are nearly equal. With the simple transmission of Fig. 11–10, clashing often results, when changing from high to intermediate speeds or vice versa. The difficulty is surmounted by the use of the friction-synchronizing principle popularly known as syncro-mesh.

The Cadillac transmission, Fig. 11–11, illustrates this principle as well as the use of helical gears throughout for greater smoothness and freedom from gear noise. The symbols correspond as far as possible with Fig. 11–10. The clutch and engine end is m, but here there are two sets of constant-mesh gears (in addition to the constant-mesh reverse pair), namely, a with b, and c with d. The tail shaft k ends in a pilot bearing inside a, as in Fig. 11–10. Gear d *rides freely* on k, but the syncro-mesh hub 2 has a sliding-key connection to k through axial splines.

Suppose that intermediate speed is desired. The shifter on 2 (10 on the lower view) is moved toward the right. The expanding friction clutch first makes contact, and synchronizes the speed of 2 and d instantly. As 2 moves farther to the right, its spur teeth engage the internal teeth (not shown) on d, giving a gear-clutch connection between d and 2. The transmission is then $m\ a\ b\ c\ d\ 2\ k$.

For high speed, 2 is moved to the left, and connection is made to a through synchronizing cones and gear clutch in similar manner.

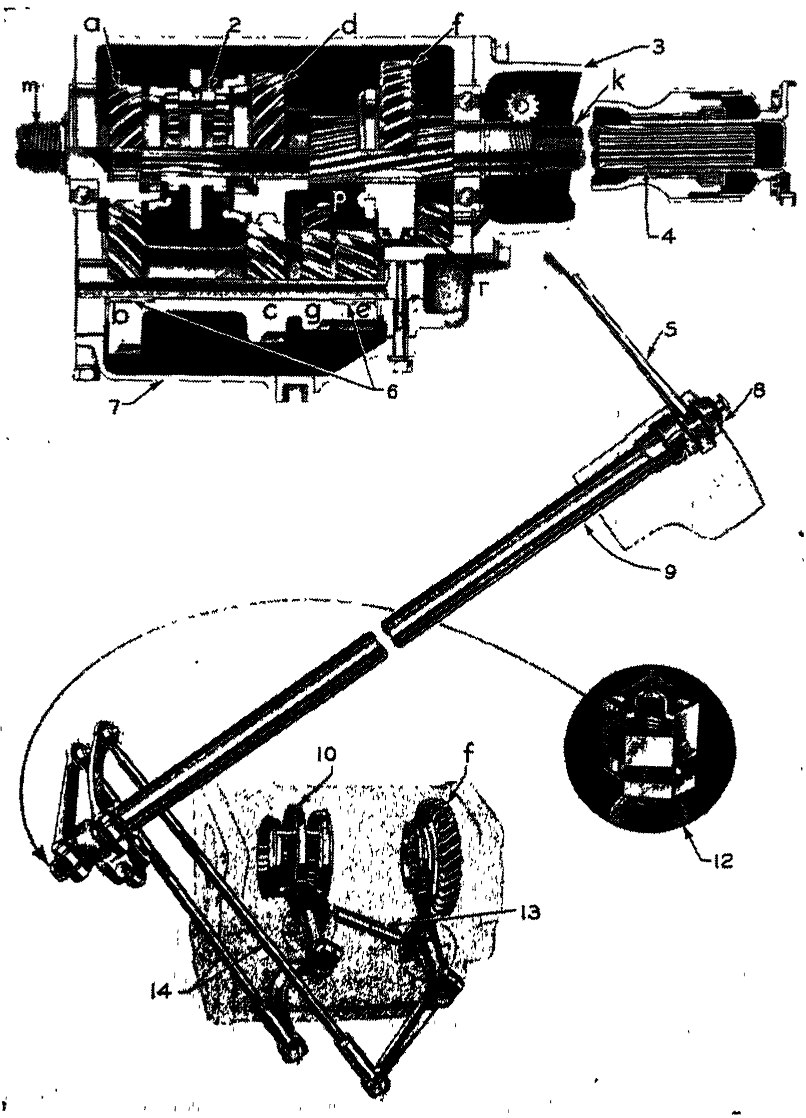


FIG. 11–11. Cadillac Transmission with Controls.

Cadillac Motor Car Division, General Motors Corp., Detroit, Mich

Low and reverse speeds are obtained by direct engagement of the helical gears. If the splines that key f to k were axial, the end thrust of the helical gears would tend to make f jump out of meshing position. This is avoided by the helical spline which is given such sense (f and k

both left hand) as to balance end thrust. Fortunately, this relation also produces the smoothest meshing action.

In this view, the reversing gears *r* and *p* can be seen behind the shaft carrying gears *b*, *c*, *g*, and *e*. *p* and *g* are in constant mesh. For reverse, the transmitting train is *a b g p r f*.

In order to place the shift lever 5 in a handy position on the steering column 9, it is necessary to use the rather complicated system of links shown in the lower view. The short shaft 13, operating between the two cams, makes it impossible to place either *f* or 2 in gear unless the other is in neutral (center) position.

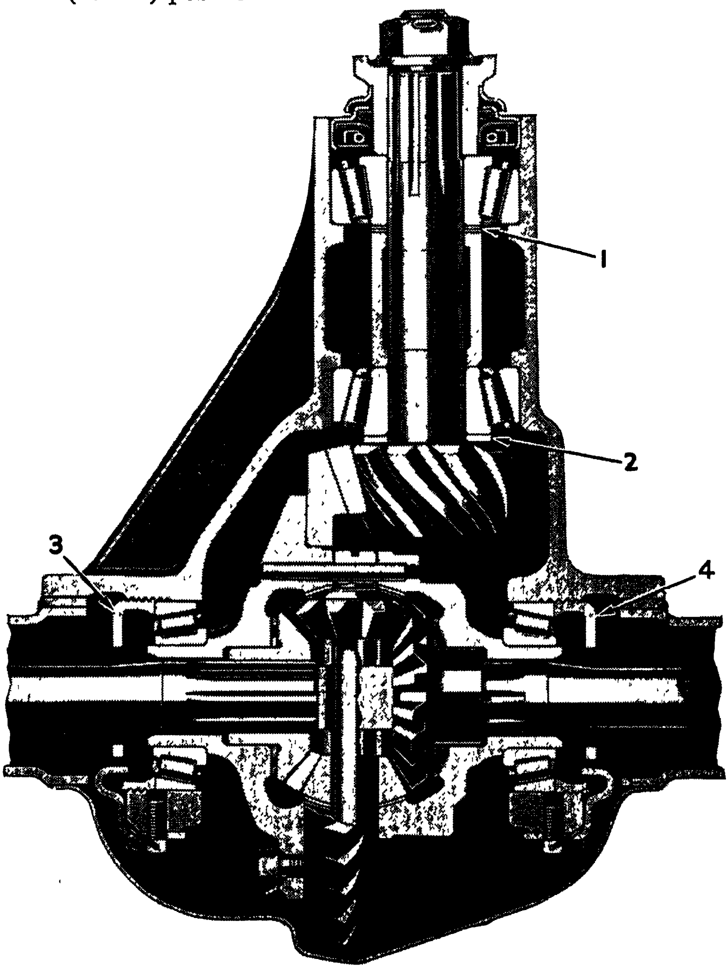


FIG. 11–12. Rear Axle Assembly.
Chrysler Corporation, Detroit, Mich.

11–5. **The Differential Gear.**—While the differential gear train has had a variety of uses for several generations, notably in textile machinery, its application in the rear axle transmissions of automobiles and trucks is probably the most widely known. While it fulfils the definition of an epicyclic train, it is much more than this, and its unique properties warrant consideration.

Fig. 11–12 shows the Chrysler differential and rear axle assembly. Note that the two rear-wheel driving shafts are splined into two differential pinions and have no direct connection to the large hypoid bevel gear. The adjusting nuts, 3 and 4, not only control the running clearance of the two adjacent tapered-roller bearings, but also position the large hypoid gear for proper mesh with its pinion. Similarly, the thickness of the washers at 1 and 2 position the pinion and adjust its supporting bearings.

For consideration of the theory Fig. 11–13 will be used. *H* represents the housing or frame of the whole rear-axle assembly. *d* is the pinion

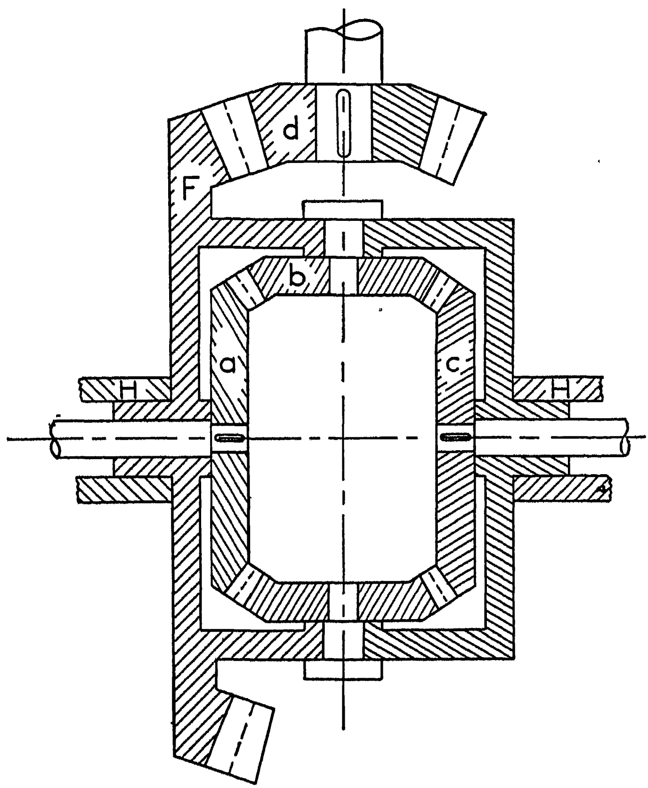


Fig. 11–13. The Differential Gear in Principle.

on the rear end of the transmission shaft running from the gear box, and d drives F, which is the large bevel gear and also the revolving box or housing of the differential gears a, b, and c. The gears marked b may be two, three, or four in number, but kinematically they all perform the same function so only one need be considered.

If F is temporarily fixed, a, b, and c constitute a reverted train of value

$$\frac{\omega_c}{\omega_a} = -\frac{N_a}{N_b} \times \frac{N_b}{N_c} = -\frac{N_a}{N_c} = -1$$

and it can have no other value, since a and c must always be equal in size. The epicyclic train is obtained by fixing a, considered the first driver in the reverted train, and making F the new driver. The epicyclic train value is

$$1 - \text{reverted train value} = 1 - (-1) = +2$$

That is, F will drive c at double speed and in the same sense.

When the rear axle of an automobile is jacked up, the above two conclusions can be quickly demonstrated. First, with the transmission shaft which drives F held stationary, turning one rear wheel forward will cause the other to turn backward at the same speed. Secondly, if the car is in gear and the engine is turned over so as to drive F, then holding one rear wheel stationary will cause the other to rotate at double the speed of F.

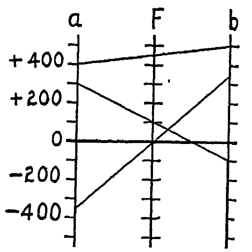


FIG. 11–14. Relative Rotation of the Three Elements of the Differential.

So far, the mechanism has performed as a reverted train and as an epicyclic train, but in service, if there is any differential action, all four links a, b, c, and F move with respect to the frame. The result is, of course, that the effect of the epicyclic action is added to the motion of a, and that complicates matters.

For this case there is a much simpler approach. The gear b, whether turning or not, acts continuously as a lever with fulcrum at the center line of its bearing, and arms of equal length bearing on a and c. The motion of its fulcrum governs the motion of F, and the two ends of the lever govern the speeds of a and c. It follows from the properties of the lever that if the rotative speeds of these three elements are laid off to scale on three equally spaced parallel lines, Fig. 11–14, the three speeds obtaining at any instant must lie on a straight line. For example, if a revolves at 300 rpm and F at 100 rpm in the same sense, projecting the straight line gives the speed of c as $-$ 100, or 100 rpm in opposite sense. If the right wheel is going forward at 500 rpm and the gear F

is being driven forward at 450 rpm, the left wheel must have a forward speed of 400 rpm. Thus the speeds of any two of the elements determine the speed of the third. If the rear wheels have equal speed, as when the vehicle is on a straight course, there is no gear action in the differential train. If the rear wheels have unequal speeds, due to a curved course or slippage of one wheel, there will be differential action. Neglecting the effect of friction, there will always be equal driving torque on the two wheels, whatever their relative speeds may be.

11–6. **The Back-Gear Train.**—The back-gear train is simply the spur-gear reverted train of Figs. 11–5 and 11–10. One of the earliest applications was to the lathe, where, as in Fig. 11–15, it is usually combined with the cone pulley to give the large number of cutting speeds required by the various materials and sizes to be machined.

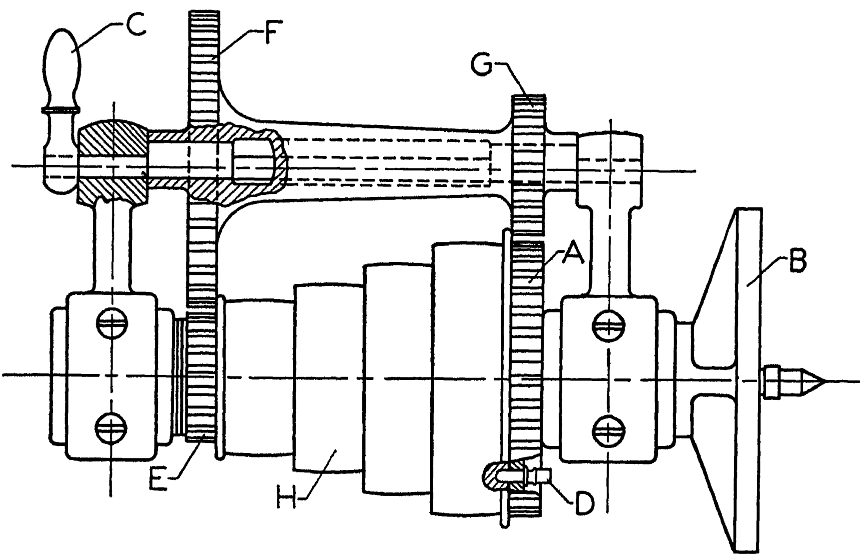


FIG. 11–15. Lathe Back-Gear Train.

The gear A is always keyed to the spindle shaft. The cone H carries the gear E, and these ride freely on the spindle shaft except when fastened to A by the pin D. In the position shown in Fig. 11–15, the back gears are out of mesh, pin D is in, and the drive is direct from the several steps of the cone to the spindle.

For the lower speeds, D is disengaged. Lever C, attached to its eccentric shaft, is turned through 180° and locked, bringing the two back gears into mesh, F with E, and G with A. Then the transmission train

is $H\ E\ F\ G\ A\ B$ and

$$\frac{\omega_B}{\omega_H} = \frac{N_E}{N_F} \times \frac{N_G}{N_A}$$

This method of placing gears in mesh by radial rather than axial movement was employed on some of the very early automobile transmissions. It is a satisfactory method if the gears are not turning when engaged, and if the spacing can be accurately controlled, as on the lathe.

11–7. **The Thread-Cutting Gear Train.**—Henry Maudslay developed and built the first tool-slide rest in England about 1790. Prior to that

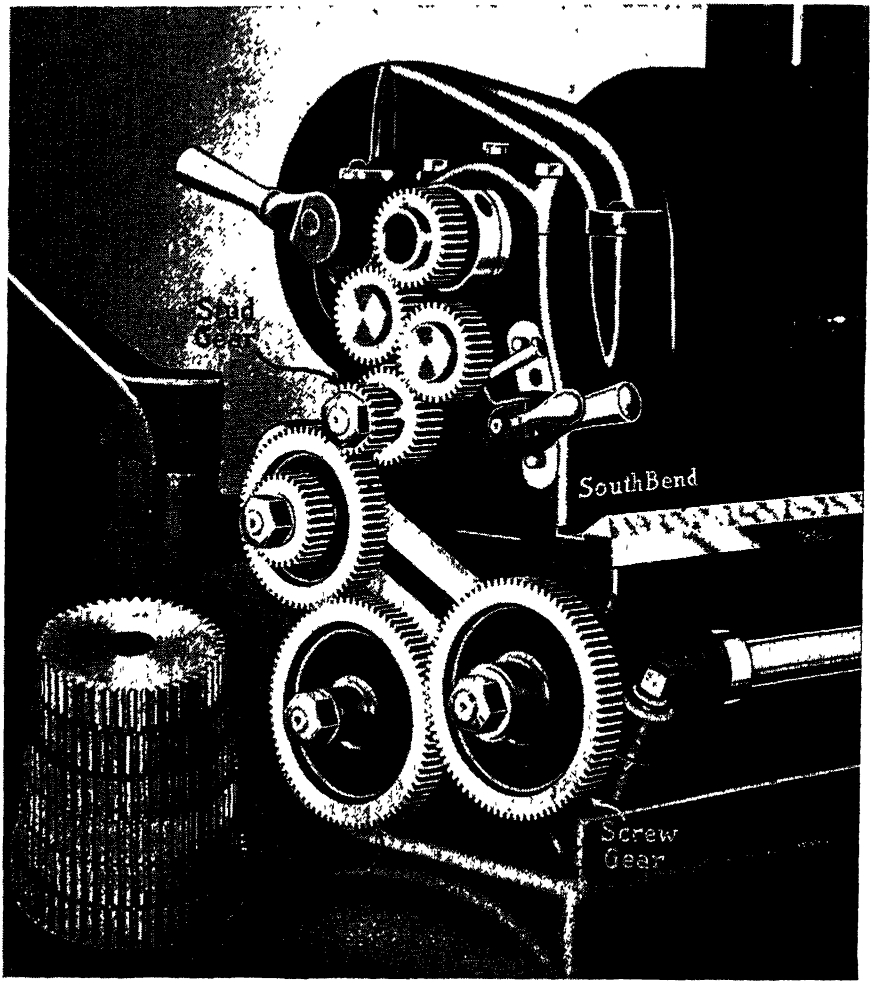


FIG. 11–16. The Thread-Cutting Gear Train.
South Bend Lathe Works, South Bend, Ind.

time, the tool was held and guided by hand. He later hit upon the idea of a lead screw driven by gears connected to the spindle, and thereby laid the foundation for the modern screw-cutting lathe, commonly called the engine lathe.

In Fig. 11–16, the upper gear is keyed to the rear end of the spindle. Next come the reversing pair which are mounted on a bracket that turns on the stud-gear shaft. As the handle is raised, the left reverse gear goes out of mesh, and, with the handle in top position, the right reverse gear goes into direct mesh with the spindle gear. Various change gears can be assembled on the stud and the slotted radial arm, either in simple or compound trains, to give a great variety of speed ratios between lead screw and spindle.

For thread cutting, the split nut in the apron, Fig. 11–18, is closed on the lead screw. Suppose it is desired to cut 13 threads per inch. If the lead screw has 5 threads per inch, the required gear train value, screw gear to spindle gear, must be 5/13.

11–8. **Quick-Change Gear Train.**—A practical disadvantage of the gear train described in § 11–7 is the time required to change gears. It is particularly wasteful when the lathe is used for ordinary machining, and a variety of feeds is necessary. The quick-change gear box, while usually providing fewer train values, is a valuable time saver.

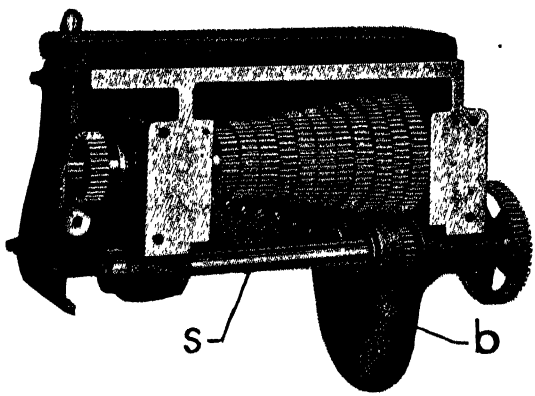


FIG. 11–17. Quick-Change Gear Box.

Reed-Prentice Corp., Worcester, Mass.

A rear view is provided in Fig. 11–17. The stepped gears are all keyed to the upper shaft. The shifter arm *b* carries a gear having a sliding-key connection to the splined shaft *s*, and, in mesh with this, an idler gear to mesh with any of the stepped gears desired. The shifter-arm handle on the front of the gear box (the back of Fig. 11–17) carries a latch pin which is engaged in holes in the casing. These holes, as can be seen, are positioned to give proper spacing between the idler gear and the selected step gear.

11–9. **Apron Gearing for Lathe.**—A rear view of a lathe apron is given in Fig. 11–18. The tapped holes in the top of the main casting are used

to fasten the apron to the saddle. On the saddle is mounted the cross slide, and on that the compound rest swivels. The compound rest carries the tool post. All these parts can be moved along the lathe bed longitudinally by rotating pinion *d*, which engages a rack on the under side of the lathe shear. Turning the hand wheel gives hand feed, and longitudinal power feed is obtained by connecting *d* with *w* through a clutch and spur gear train. The worm wheel *w* is driven by its worm which is splined to the lead screw as clearly shown. These are the longitudinal feeds for ordinary cutting.

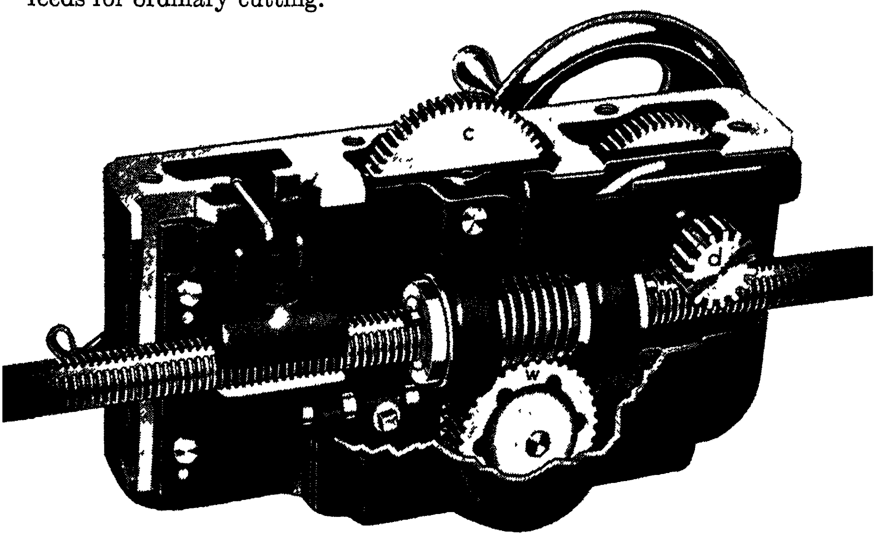


FIG. 11-18. Mechanisms of Lathe Apron.
The South Bend Lathe Works, South Bend, Ind.

Thread cutting requires a definite geared connection between the spindle (work) speed and the longitudinal feed, without any uncertainties such as a friction clutch introduces. The required conditions are supplied by the thread-cutting gears and the split nut. The two halves of the split nut move in the dovetailed grooves and are drawn together to engage the thread of the lead screw by a double acting cam. An interlock prevents closing of the split nut unless the friction power feed is in neutral position.

The power cross feed is driven by gear *c*. The power-feed lever on the front of the apron has three positions. One connects *w* to *d* as described above. One is neutral, and the third connects *w* to *c*.

Some larger lathes have an angle power feed for moving the compound rest on the cross-feed member. The principle of operation of such feeds is presented in § 11-10.

Some lathes have a separate power feed shaft (plain splined shaft) in addition to the lead screw. That avoids the difficulty of cutting a spline in the threaded shaft.

11–10. **The Universal Power Feed.**—A general view of a planer was given in Fig. 1–2. Fig. 11–19 shows the tool heads or rail heads as they are supported and controlled from the cross rail. The internal mecha-

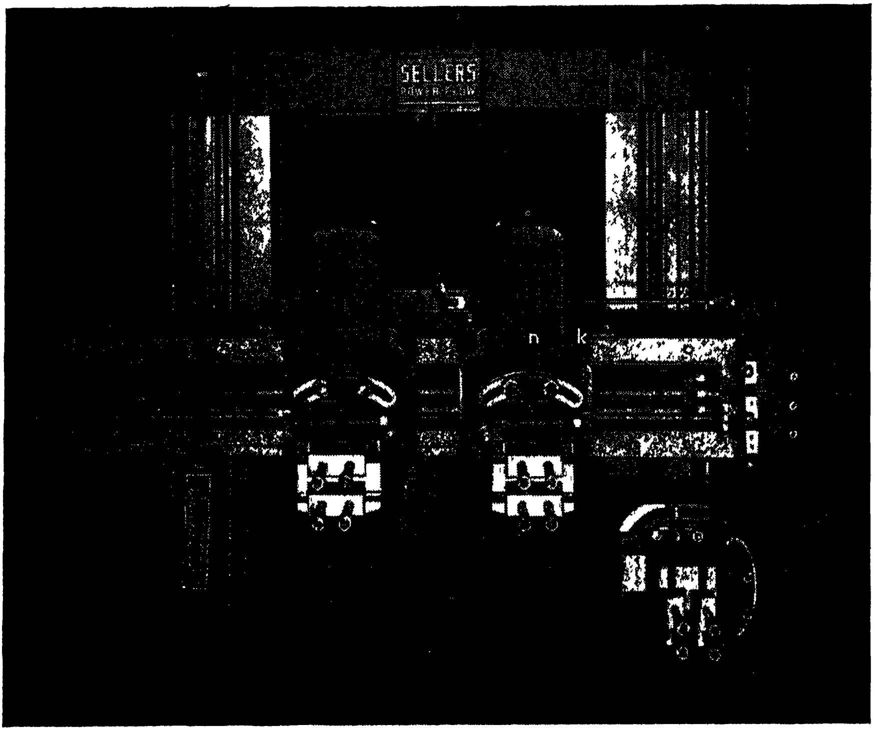


FIG. 11–19. Tool Heads and Cross Rail of Planer.

William Sellers and Co., Philadelphia, Pa.

nism, by means of which this control is accomplished, is illustrated in Fig. 11–20, the basic drawing of which was supplied by courtesy of William Sellers and Company.

In order to machine surfaces at any angle, it is necessary to be able to feed the cutting tool on a path at any desired angle with the vertical. This is accomplished by swiveling the index plate k, on the saddle b. The circular T-groove g serves both for guide and clamping device. With the angle slide n now pointing in the desired direction, the saddle b carries the whole head to the desired position along the cross rail a, either

by hand or power feed. This cross-feed motion is accomplished by turning the feed screw *m*, which passes through a nut (not shown) in the saddle. With the head clamped in position, the tool could now be fed by turning the screw *f* through the stationary nut *e* by a hand crank on the squared top of *f*. To apply a power feed to *f* directly, in any angular position, would be a difficult problem, so a different method must be used.

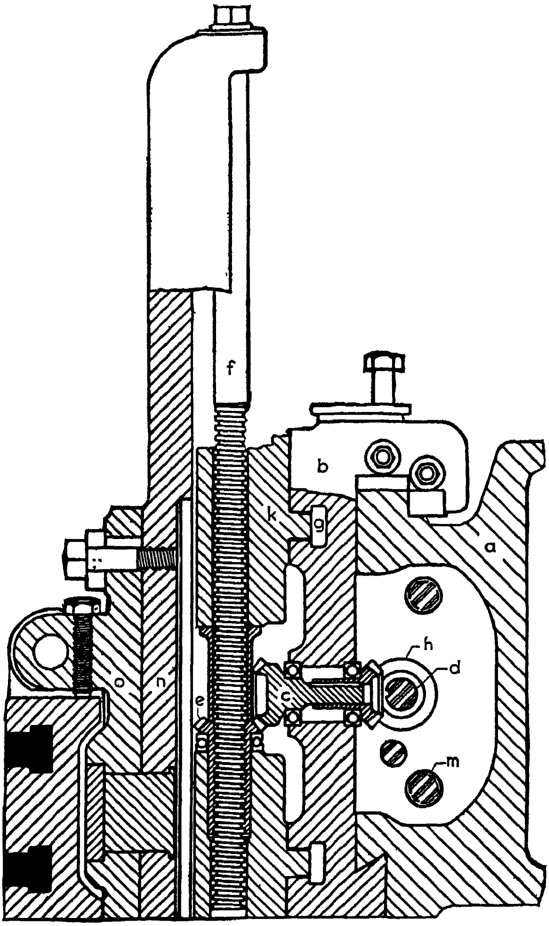


FIG. 11–20. Power Angle-Feed Mechanism for Rail Head of Planer.

Applying power to move a slide in varying directions of travel is called **universal power feed.** The solution generally adopted throughout the machine tool field is the one illustrated here. The screw *f* is locked in the slide *n*, and power is applied to the nut *e* by shaft *c*, which *must be located on the swiveling axis.*

The power is transmitted along the rail on the splined shaft *d*, which turns miter gear *h*. This engages *c*, which has a bevel gear on each end, and *c* drives the nut *e*. In the Sellers design, the tool frame *o* also swivels on the slide *n*, but that has no relation to the universal power feed. The tool frame can be swiveled at such an angle as will give best action to the tool apron (clapper head). Where two tool heads are mounted on the same rail, as in Fig. 11–19, it is usual to incorporate independent cross feed and universal feed for each head.

This universal-feed mechanism is applied in principle on the table of

the universal milling machine, the compound-rest slide of lathes, and to many other machines.

11–11. **Controlled Differential.**—The differential gear train of Fig. 11–21 is of special interest because the differential action is accomplished by the use of spur gears only, also because of the unique control, through the two brake drums (10), of the action of the mechanism. Many applications [1] to vehicular and other machinery are possible.

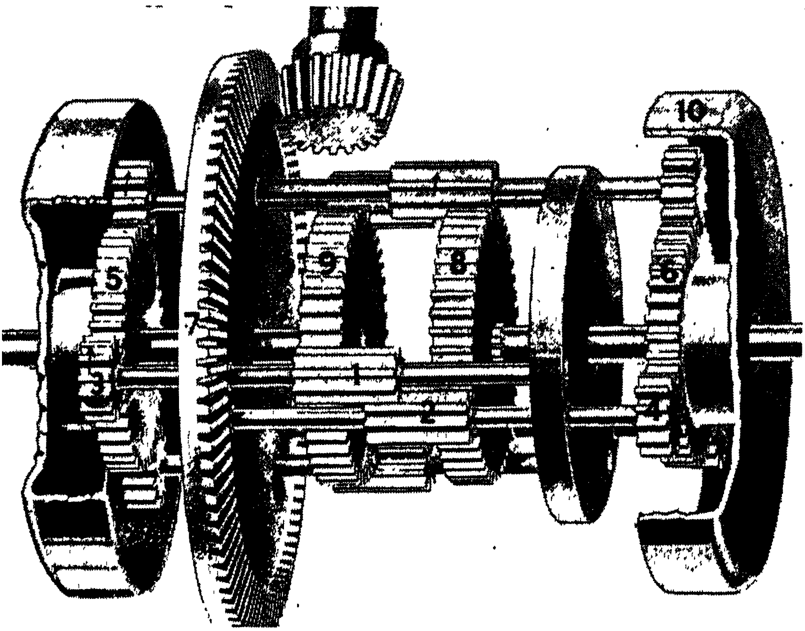


FIG. 11–21. Schematic Diagram of Controlled Differential.
$N_5 = N_6 = 52$ teeth, $N_3 = N_4 = 24$, $N_1 = N_2 = 16$, $N_9 = N_8 = 60$.

Assuming that power is applied through the bevel pinion, and that gears 8 and 9 are connected to the two driving wheels of the vehicle, these driving wheels would be driven at the same speed if there were no spur gear action. This would correspond to the situation in the ordinary bevel-gear differential, Fig. 11–13, for straight travel of the vehicle.

Suppose the mechanism is used to drive a crawler-type tractor. Consider the results of holding the left brake drum (attached to gear 5) stationary, while bevel gear 7 is driven clockwise. Evidently gears 7, 3, 5, 1, 9 constitute an epicyclic train. What is this train value, ω_9/ω_7? We must first find the reverted train value, considering 5 to be the driver

[1] At the time of writing it is not permissible to describe the applications.

with 7 fixed.

$$\frac{\omega_9}{\omega_5} = \frac{N_5}{N_3} \times \frac{N_1}{N_9} = \frac{52}{24} \times \frac{16}{60} = +\frac{26}{45}$$

The epicyclic train value

$$\frac{\omega_9}{\omega_7} = 1 - \frac{26}{45} = +\frac{19}{45}$$

Meanwhile with 5 still fixed, gear 8 is driven by the train 7, 3, 5, 1, 2, 8, another planetary train. Its reverted train value is

$$\frac{\omega_8}{\omega_5} = -\frac{N_5}{N_3} \times \frac{N_1}{N_2} \times \frac{N_2}{N_8} = -\frac{52}{24} \times \frac{16}{16} \times \frac{16}{60} = -\frac{26}{45}$$

Its epicyclic value with 5 fixed is

$$\frac{\omega_8}{\omega_7} = 1 - \left(-\frac{26}{45}\right) = +\frac{71}{45}$$

While 7 is driven clockwise 45 turns, 9 goes 19 turns clockwise, and 8 goes 71 turns clockwise. This establishes the minimum turning radius of the tractor. It assumes that brake drum 10, attached to gear 6, is free to turn. If equal friction force is applied to both drums, ordinary braking results, and the vehicle maintains a straight course. More braking applied to one drum than the other causes the vehicle to turn on a radius greater than the minimum. The result is an effective steering control.

11–12. **Aircraft-Engine Starter.**—With the increase in size of aircraft engines, designers were confronted with the problem of providing starters that would deliver the very high torques required for cold-weather starting, in units of light weight. The ordinary electrical system, such as used on automobiles, proved much too heavy, and there was the added problem of starting when the electrical system was dead.

Both of these problems have been satisfactorily solved by starters of the type shown in Fig. 11–22. The principle of operation is the application of energy at low rate, by starting motor or hand crank, over a sufficient period of time to store a large amount of energy in a light high-speed flywheel. When the jaws T are moved forward, they engage the engine shaft, and the flywheel supplies energy at the rate required to provide the large initial "breakaway" torque. After the engine is turning, the motor O, if live, can supply energy to keep it turning at starting speed for some time.

To attain the desired light weight (in the case of this starter 55 pounds for an 1800 horsepower engine), the speed of the flywheel and motor armature must be very high. This requires some interesting gear trains.

If hand cranking is required, torque is applied at *A*, and transmission is through the train *B C D E S P M K J F G H O*. Gear *E* is keyed to *S*. Gears *M* and *J* turn freely on *S*. The epicyclic trains will presently be traced in detail. The motor *O* and flywheel *H* revolve on the same shaft. While the flywheel is being accelerated, the jaws *T* are turning but are disengaged.

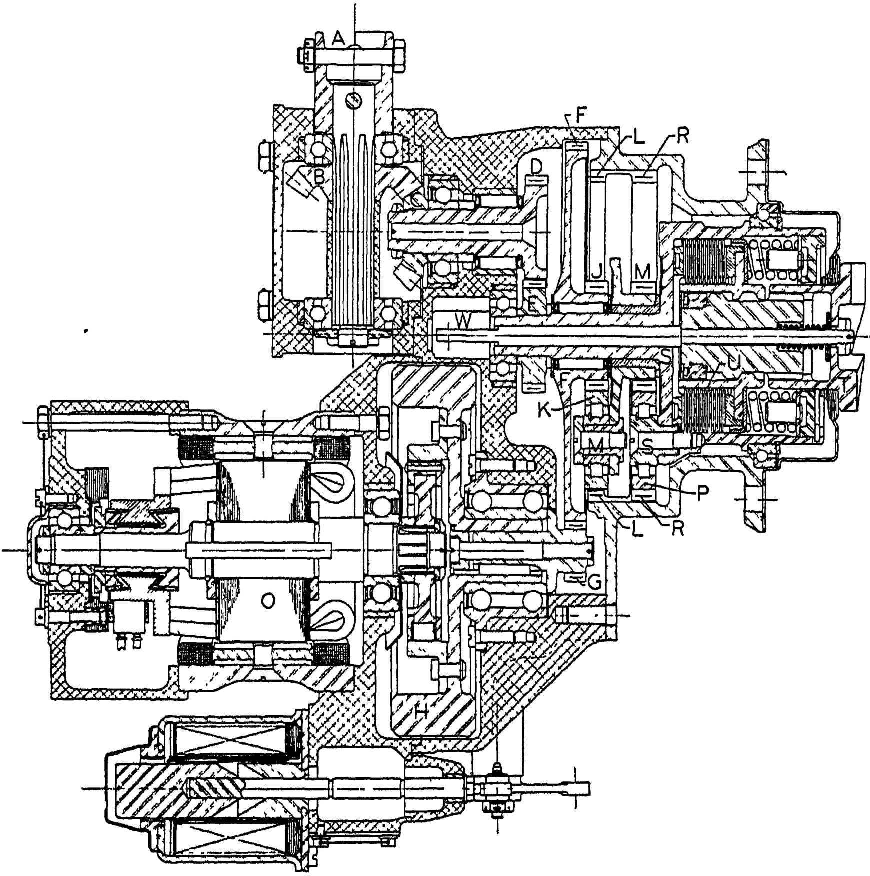


FIG. 11–22. Direct-Cranking Electric and Inertia Starter.
Eclipse Aviation Division of Bendix Aviation Corp., Bendix, N. J.

When the flywheel has been accelerated to its normal speed, the jaws *T* may be engaged for starting, and transmission is from the flywheel through the train *G F J K L M P R S T*. Teeth *F* and *J* are on the same gear (compound), which rides freely on a roller bearing. *K* is a planet gear in an epicyclic train, engaging fixed teeth *L*. This drives body *M*, which rides freely on its shaft and carries teeth *M*. *M* drives the planet

gear P of another epicyclic train, the stationary teeth being R. Hence S is rotated, driving T at the same speed.

Each of the two epicyclic trains, thus in series, have an odd number of planet gears, so none appear above the main shaft in sectional view. U is a plate clutch set to slip at about 900 pound feet of torque, thus protecting the gear train in case of unusual engine resistance. W is a solenoid which provides push-button control of engagement. If left-hand cranking is desired, bevel B can be placed at the other end of its splined shaft so as to engage the other side of gear C.

As an example, the speed ratio for the first epicyclic train will be developed. L, the fixed gear, must be considered the driver of the reverted train, while M becomes fixed. Then the reverted train value is

$$\frac{\omega_J}{\omega_L} = -\frac{N_L}{N_K} \times \frac{N_K}{N_J} = -\frac{N_L}{N_J}$$

The epicyclic train value is

$$\frac{\omega_J}{\omega_M} = 1 - \left(-\frac{N_L}{N_J}\right) = 1 + \frac{N_L}{N_J}$$

A further study of these gear trains will be afforded by Problem 20.

QUESTIONS AND PROBLEMS

1. (*a*) In what respect does a compound gear train differ from a simple train?

   (*b*) What two purposes can be served by the use of idler gears? Give a practical example of each.

2. What is the train value of a simple train of 15, 12, 20 and 30-toothed gears? The first mentioned gear is the driver. All are external spur gears.

3. Find the train value of a compound train of external-toothed gears represented thus,

   40 meshes with 30

   24 with 60

   triple-threaded worm with 36-toothed worm wheel. Show the sense relations by a perspective sketch.

4. Prove that the epicyclic train value is one minus the value of the reverted train from which it is derived.

5. If the epicyclic train, Fig. 11–6, had $N_a = 20$, $N_b = 56$, $N_c = 28$ and $N_d = 48$ what would be its value? Ans. $+\frac{19}{24}$.

6. An epicyclic train has, for the fixed gear $N_a = 50$, meshing with $N_b = 20$. $N_c = 48$ and $N_d = 16$ are connected by an idler. Find its train value.

7. A lathe has a back-gear train composed of a 16-toothed pinion on the cone pulley meshing with a 64-toothed gear keyed to the back shaft. Keyed to the other end of the back shaft is a 20-toothed pinion driving a 60-toothed gear on the live spindle as in Fig. 11–15. If the diametral pitch is 4 find the shaft spacing, also the train value.
8. In an automotive transmission, Fig. 11–10, $N_a = 20$, $N_b = 35$, $N_c = 28$, $N_d = 27$, $N_e = 18$, $N_f = 37$, $N_g = 14$. What are the four speed ratios?
9. If the hoist, Fig. 11–8, had no gear b, but had c meshing with the annular gear a, how many teeth should a have, and what would the train value of the hoist then be?
10. A hoist similar to that of Fig. 11–8 has $N_a = 78$, $N_b = 18$, $N_c = 48$, $N_d = 12$. The pitch diameter of the load sprocket h is 8 in., and of the hand sprocket k, 20 in. How far must the hand chain be pulled to raise the load one foot? Ans. 550 in.
11. Revert the following compound gear train, make it epicyclic by the addition of
    (*a*) one idler,
    (*b*) two idlers,
    and determine the epicyclic train value in each case, with the 40-tooth gear fixed:
    40 meshes with 30
    20 meshes with 27.

    Ans. (*a*) $+\frac{161}{81}$, (*b*) $+\frac{1}{81}$.

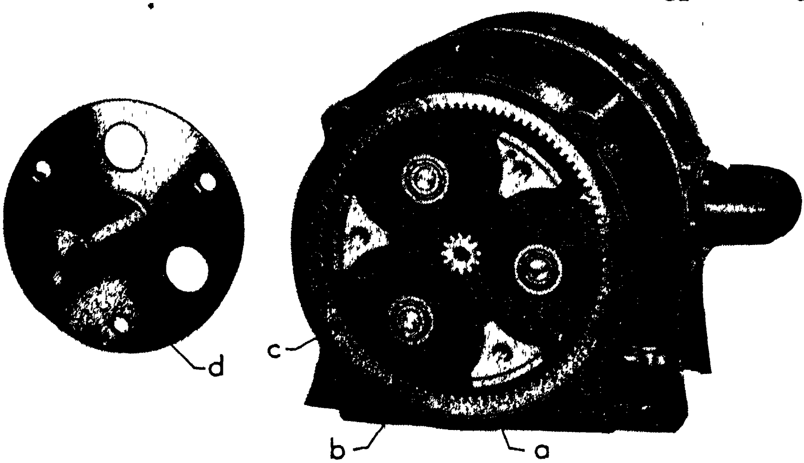
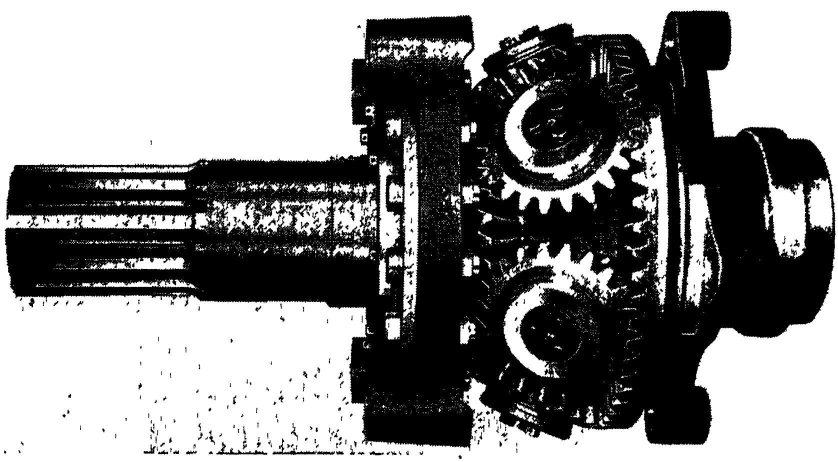


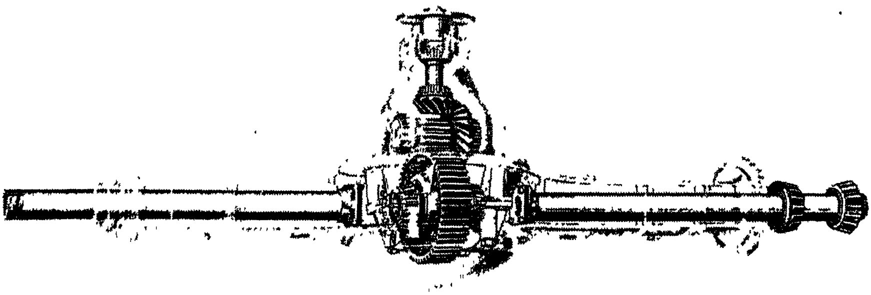
Figure for Problem 13.
The General Electric Co., Schenectady, N. Y.

12. A certain machine requires a speed reduction of exactly 10,000 to 1 between intersecting shafts at 90°. Develop the most compact design you can devise.
13. This gearmotor has its center gear c connected to the motor. The planet gears b carry the output shaft d, its three holes fitting over the three ball bearings so that the output shaft is in line with the motor shaft. $N_a = 96$, $N_b = 42$, $N_c = 12$. What is the speed reduction? (Fig. on p. 263.)
14. In the automobile transmission of Fig. 11–11, take $N_a = 16$, $N_b = 20$, $N_c = 16$, $N_d = 20$, $N_e = 16$, $N_f = 32$, $N_g = 16$, $N_p = 18$, $N_r = 16$. Find the four transmission ratios.
15. Would the end thrust on gear f, Fig. 11–11, be better balanced if the helix angle of the splined shaft were the same as the helix angle of its teeth?
16. While travelling on a curve, gear F of the differential, Fig. 11–13, had a speed of 100 rpm, and the left wheel was travelling at 96 rpm. The center planes of the rear wheels were 4 ft 8 in. apart. What was the radius of the curve measured to the left wheel?
17. In this marine reduction unit, the engine drives the left shaft which carries the 36-tooth bevel gear. The planet gears have 21 teeth each and are carried on the spider of the propeller shaft which extends to the right through the stationary 52-tooth crown bevel gear. What is the speed reduction? (These gears are cut with circular cutters as in Fig. 10–6.)



The Gleason Works, Rochester, N. Y.

18. This is a compound-geared rear axle for a heavy Autocar truck. The spiral bevels have 12 and 24 teeth respectively. The spur gears have 14 and 50 teeth. If the gear-box ratio in low speed is 6.2 to 1, and the rear tire is 48 in. in diameter, how far does one revolution of the engine move the truck in low gear?



The Autocar Co., Ardmore, Pa.

19. In the controlled differential of Fig. 11–21, take $N_5 = N_6 = 27$ teeth, $N_3 = N_4 = 36$, $N_1 = N_2 = 15$, $N_9 = N_8 = 48$. Consider that this is used to drive a tractor having crawler treads 6 ft apart (center distance).
    (*a*) What is the minimum turning radius measured to the center of the vehicle?
    (*b*) When drum 5 is locked by the brake, how many turns does 10 make for one turn of 7?
20. In the airplane starter of Fig. 11–22, calculations will be based on the following gear tooth numbers: $N_B = 20$, $N_C = 16$, $N_D = 24$, $N_E = 24$, $N_F = 90$, $N_G = 12$, $N_J = N_M = 16$, $N_K = N_P = 18$, $N_L = N_R = 52$.
    (*a*) For an engine cranking speed of 100 rpm, what is the speed of the flywheel and motor?
    (*b*) What speed of the hand crank at A is necessary to give the flywheel this speed?

Ans. (*a*) 13,546 rpm, (*b*) 80 rpm.

CHAPTER XII

FLEXIBLE CONNECTORS

12–1. **Classification of Flexible Connectors.**—Belts, ropes, and chains are the flexible connectors used to transmit power. Their distinguishing characteristic, which results from their flexibility, is that they can transmit by tension forces only. Their natural field of application lies in connecting rotating parts that are a considerable distance apart; however, with proper design they can be applied to fairly short spans.

Flexible, or wrapping connectors as they are sometimes called, may be divided into two classes depending on whether they can transmit in a velocity ratio that is not subject to cumulative error, or whether slipping or creep can occur which will make the velocity ratio of the two shafts connected, uncertain. Chains running over sprockets give a nearly constant velocity ratio not subject to cumulative error and, in this respect, are the equivalent of gears. Belts running on pulleys and ropes on sheave wheels fall in the second class. For example, consider the probable performance of an internal-combustion engine that had its valve shaft driven by a belt. After a short period of operation the valve timing would be out of phase and the engine would lose power and stop. However, there is a very large field of transmission applications where exact velocity ratio is entirely unnecessary.

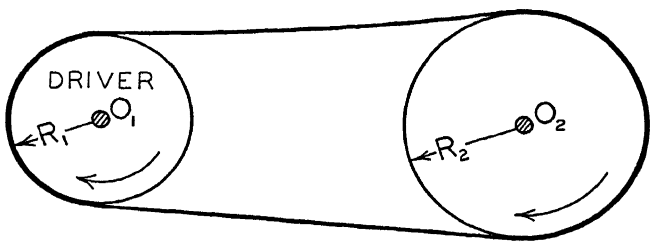


Fig. 12–1. Open Belt Connecting Parallel Shafts.

12–2. **Flat-Belt Drives.**—Where a belt is used to connect two parallel shafts, as in Fig. 12–1, the relative speeds of rotation will be inversely as the radii of the pulleys. If theoretical exactness is necessary, the pitch radii (including a part of the belt thickness) should be used. In view of the fact that a two-per-cent slip of a belt under full load is not uncommon, this refinement is generally omitted unless the belt is very thick.

Letting V represent the belt speed in ft per sec, R_1 the radius of pulley 1, and R_2 of pulley 2, both measured in ft,

$$V = R_1\omega_1 = R_2\omega_2$$

since the linear speed of the belt over both pulleys must be the same, and

$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \frac{\text{rpm}_1}{\text{rpm}_2} \tag{1}$$

In case the same belt runs over many pulleys, whether on parallel shafts or nonparallel, the linear speed of the belt will govern the rotative speeds of all the pulleys according to the above relation.

In Fig. 12–1, pulley 1 should be the driver for best results, since this will make the top the slack side, thus increasing the arc of contact on both pulleys.

The open belt drive, Fig. 12–1, causes the two shafts to rotate in the same sense. A reversal in sense can be easily obtained by the use of a crossed belt, Fig. 12–2. In order to effect reasonable clearance for the crossing portions, each must be given a 180° twist, which increases the stress at the edges. Narrow crossed belts wear fairly well but wide belts should not be crossed unless the span—the distance between shaft centers—is quite long.

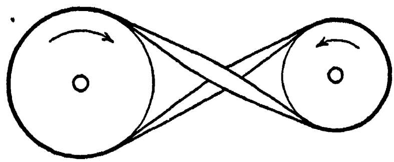


FIG. 12–2. Crossed Belt.

If a belt is allowed to run over the edge of a pulley, a bending action occurs in the overhanging portion that causes it to become quickly frayed and injured beyond repair. It is good practice to make the width of the pulley face 20 per cent larger than the belt width.

Crowning of pulleys is a construction for keeping a flat belt running centrally on its pulley. The pulley face is made higher in the center, as in Fig. 12–3. A flat "V" is the easiest form to machine, but a smooth curve is easier on the belt. The principle of the action of crowning is illustrated in Fig. 12–3. Suppose that, for any reason, the belt is forced from central position so that it occupies the broken-line position. Then the right side of the belt is on a larger pulley diameter than the left side, and will be stretched more. The belt will be curved throughout its free span, but the effective part of this curvature is at *the point where it approaches the pulley.* This action steers the approaching part of the belt continuously toward the highest part of the pulley. When the belt is central, the two sides work against each other and the action is balanced.

Flanged pulleys are generally unsatisfactory for belting. When the edge of the belt makes contact with the flange there is a tendency for

it to climb, or, at best, it rubs and rapid wear ensues. If special considerations, such as the necessity of shifting from tight to loose pulleys, call for flat faced rims, guides must be used. They should be placed to control the on-running side of the belt. To control wide belts, roller guides are recommended.

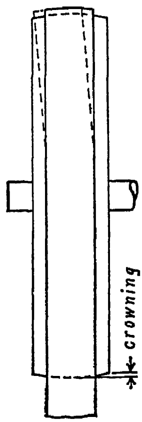


FIG. 12–3. Crowning of Pulley to Make Belt Run True.

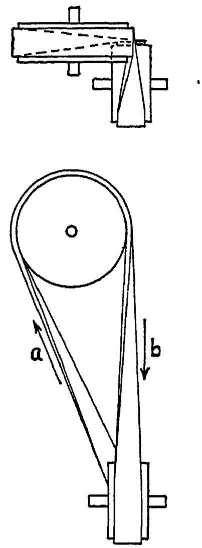


FIG. 12–4. Quarter Turn Belt Drive.

Belts can be used to connect shafts that are not parallel as illustrated by the quarter-turn drive of Fig. 12–4. The principle to be observed in designing all such drives is that *the on-running side of the belt must be in the plane of the pulley it approaches.* If the sense of rotation were reversed (Fig. 12–4) for as much as half a turn of the pulleys, the belt would come off. Such a drive could be made reversible by placing a guide pulley for the side marked *"a"* so as to bring that side into the planes of both pulleys, while, at the same time, moving the upper pulley along its shaft to bring the *b* side of the belt into the plane of both pulleys. By the use of guide pulleys it is possible to connect shafts in any relative position whatsoever, but good design will generally avoid such complications. Simple direct drives give longest belt life and best efficiency.

The tight-and-loose pulley arrangement is much used on the drives for machine tools. For a reversing drive, two belts are used, one open and one crossed. Each belt must be controlled by a shifter to guide it onto either its tight pulley (keyed to the shaft) or its loose pulley which

rides freely on the shaft. In another arrangement, clutches are employed to attach either the open-belt pulley or the crossed-belt pulley to the driving countershaft of the machine.

12–3. Computation of Belt Length.—For the general case of pulleys of unequal diameter, the length of the open belt can be seen from Fig. 12–5 to be

$$S_0 = 2\sqrt{L^2 - (R_1 - R_2)^2} + \pi(R_1 + R_2) + 2(R_1 - R_2)\sin^{-1}\frac{R_1 - R_2}{L} \quad (2)$$

noting that

$$\sin\theta = \frac{R_1 - R_2}{L}.$$

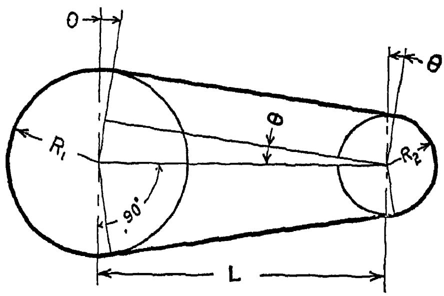


FIG. 12–5. Open-Belt Length.

Similarly from Fig. 12–6, noting that

$$\sin\theta = \frac{R_1 + R_2}{L}$$

the length of the crossed belt is

$$S_c = 2\sqrt{L^2 - (R_1 + R_2)^2} + (R_1 + R_2)\left(\pi + 2\sin^{-1}\frac{R_1 + R_2}{L}\right) \quad (3)$$

Best results are obtained when belts are made endless, that is the ends tapered (scarfed) and glued together to avoid the necessity for joints, laced or otherwise fastened. With endless belts, in order to allow for belt stretch and keep the tension properly adjusted, it is essential that one of the pulleys be held adjustably or that idler pulleys be used. The idlers should be placed on the slack side, and the belt run over them so as to increase the arcs of driving contact. Idlers increase the length of belt required but the advantage of easy adjustment and the greater durability of the endless belt more than offset the increased first cost where considerable power is to be transmitted. Electric motors are regularly supplied with subbases on which they can be adjusted to give the required tension to endless belts without the use of idlers.

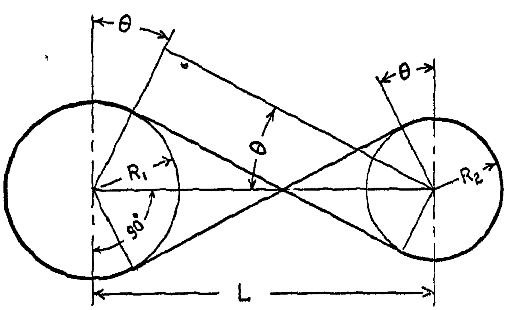


FIG. 12–6. Crossed Belt Length.

In ordering endless belts, the manufacturer is given the "tape line length" around the pulleys, equation (2) or (3). He then makes allowances for the belt thickness and the stretch necessary to produce initial tension.

12–4. **Stepped or Cone Pulleys.**—In order to have available a variety of spindle speeds for machine tools so that suitable cutting speeds can be had for various diameters of work, cone pulleys either alone or in conjunction with back gears or speed change gear boxes are much used. It will be noted from Fig. 12–7 that the belt length for opposite pairs of steps should be the same. If a crossed belt is used, it is only necessary to keep the sum of the radii of all pulley pairs the same, i.e. $R_a + R_b = R_c + R_d = R_e + R_f$, etc., since from (3) the value of S_c depends only on L and $R_1 + R_2$. However, if an open belt is used, the expression that applies, equation (2), is quite unwieldy. The following graphical method will give results quickly and the accuracy possible on large sharp-lined drawings is sufficient for practical purposes.

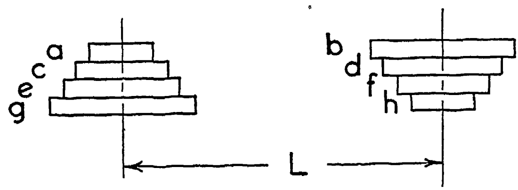


FIG. 12–7. Cone Pulleys.

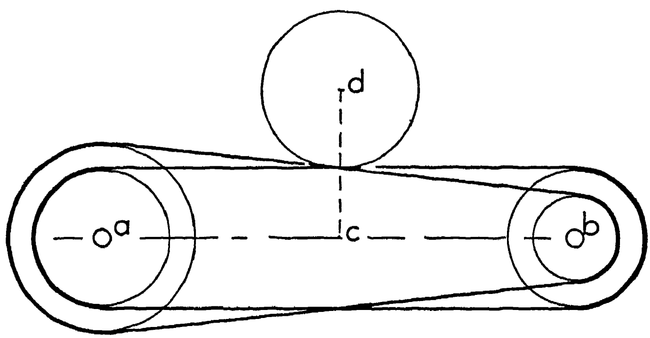


FIG. 12–8. Graphical Method for Sizes of Cone-Pulley Steps.

C. A. Smith [1] devised the approximate method illustrated in Fig. 12–8. The shaft distance of the cone pulleys is ab to scale, with c the midpoint. cd is 0.314 multiplied by the length ab and normal thereto. To use the diagram draw a tangent to the upper sides of any given pair of pulleys and draw a circle with center d and tangent to the same line. Then any other straight line, tangent to this circle, will be tangent to a pair of pulley diameters requiring the same belt length.

[1] Smith, Trans. A. S. M. E., Vol. X, p. 269.

The inaccuracy of the method increases with the belt angle. If the angle that one belt line makes with ab exceeds 18°, first draw a tangent to the circle making 18° with ab, then locate d' at a distance 0.298 (ab) above c. Draw a circle about d' tangent to the 18° line, and make all the steeper belt lines tangent to this last arc.

As a check, or for cases requiring extreme accuracy, the results may be placed in equation (2), compared and adjusted to the required accuracy. An approximate analytical method developed by Kent[1] from Burmeister's graphical method is also available.

The pairs of radii so obtained are theoretically the radii to the pitch line of the belt, that is the line which, neglecting slip, establishes relative speeds. For single-ply leather belt, subtract 70% of the belt thickness to get the pulley radii. For multiple-ply leather belts, also those of rubber, canvas, etc., subtract 50%. The reason for the first value is that the hair (grain) side of a leather belt is used as the pulley-contact side, and the flesh-side fibers are stronger and stiffer. This moves the pitch line nearer the outside for single-ply leather. Multiple-ply leather belts are made with the grain sides on the two outside surfaces, giving a balanced effect and a central pitch line.

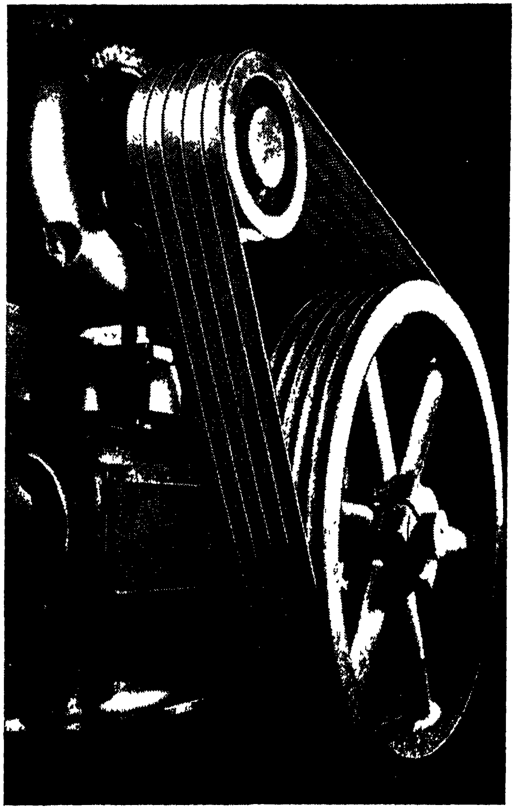


FIG. 12-9. V-Belt Motor Drive.
The Dayton Rubber Mfg. Co., Dayton, Ohio.

In choosing a set of speeds to cover a required range, a **geometrical progression** is the best theoretical solution as well as the most practical, for then each speed is the same percentage increase over the next lower

[1] Kent's M. E. Design Handbook, p. 24-12.

speed. Let r equal this common ratio of successive speeds. Then, if N_1 is the lowest required speed, the next higher speed $N_2 = N_1r$, $N_3 = N_1r^2$, and for h speeds, the highest speed $N_h = N_1r^{h-1}$, from which

$$r^{h-1} = N_h/N_1 \tag{7}$$

This gives r when the speed range and number of speeds are given, or it gives all speeds when r and the number of speeds and one end speed are given.

For example, suppose four speeds are required for the spindle of a milling machine, ranging from 125 rpm to 216 rpm. Using (7), $r^3 = 216/125$ or $r = 1.2$. Then $N_2 = 125 \times 1.2 = 150$, $N_3 = 125 \times 1.2^2 = 180$ rpm.

12–5. **V-Belt Drives.**—The V-belt is used in single or multiple strands and is specially adapted for short spans. Fig. 12–9 shows a 5-belt drive connecting an air compressor with its motor. The belts bear on the sides of the grooves, not on the bottom. They are sometimes called "wedge belts," emphasizing their wedging action.

This wedging action is more than at first appears. In Fig. 12–10, *a* is the usual section of a V-belt when straight. (The serrations on the under sides of the belts in Fig. 12–9 are peculiar to the Dayton construction.) When bent around a grooved pulley (sheave wheel), there is an enforced bulging of the sides as at *b*. Since the belt is made of rubber reinforced by cords and canvas, it can be used on a fairly small sheave. Fortunately the bulging takes place as the belt enters the groove, and the sides straighten as the belt leaves. This results in a partial locking and unlocking action, which is quite effective. It also reduces friction and wear at entrance and exit. Different groove angles are necessary on sheaves of different size for the same belt because of its change of shape with curvature.

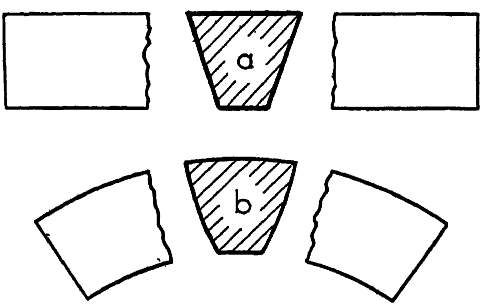


FIG. 12–10. The Gripping Action of a V-Belt.

The net result is that V-belts can be used on quite short spans without special tightening devices, and will perform fairly well even after stretch has reduced the tension considerably. Flat belts can be used on short spans also, but require some device that will insure proper tension after stretch occurs.

12–6. **Special Tension-Adjusting Belt Drives.**—The Rockwood drive, Fig. 12–11, uses the weight of the motor to maintain belt tension prac-

tically independent of stretch. The motor is bolted to an intermediate base which is pivoted to the subbase as shown. Ceiling and wall mountings are also available.

A later development by the American Pulley Co. depends mainly on reactive torque to adjust belt tension to suit the requirements of the transmission. Reactive torque is felt in the arms when one holds an electric drill in action. It is equal and opposite to the applied torque.

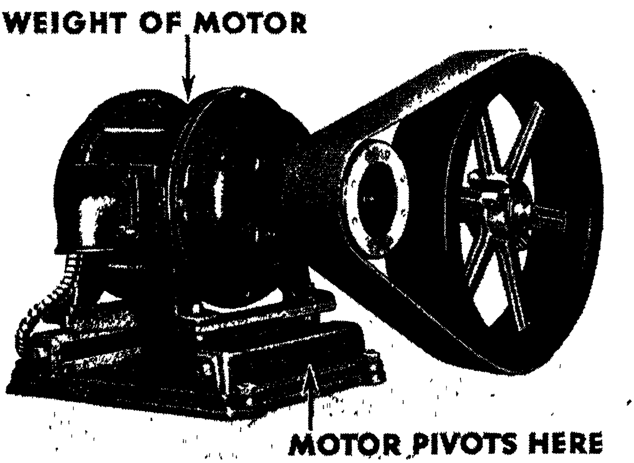


FIG. 12–11. Rockwood Drive.

The Rockwood Mfg. Co., Indianapolis, Ind.

In this device the pivot is placed close to the motor shaft, Fig. 12–12, so that gravity causes only a small initial belt tension. At (*a*) is shown the motor position when no power is being transmitted. The belt is quite slack. At (*b*), the motor and belt are fully loaded but the tight side of the belt must be below as indicated by the arrow, and the motor must run counterclockwise so that the reactive torque on the motor frame will tend to rotate it clockwise, thereby tightening the belt. At (*c*) a partial load is being transmitted, with intermediate tension and therefore intermediate stretch in the belt. At (*d*) is shown the arrangement for vertical drive with the motor above. Here the motor must run clockwise so that the reactive torque will assist the counterweight in producing the required belt tensions. This device, like the Rockwood drive, is limited to motors, where the power can enter through flexible leads. Such drives are great savers of space and belting, because they make very short drives possible with flat belts and plain pulleys. In the American Pulley Co. drive, large belt tensions are only applied when needed.

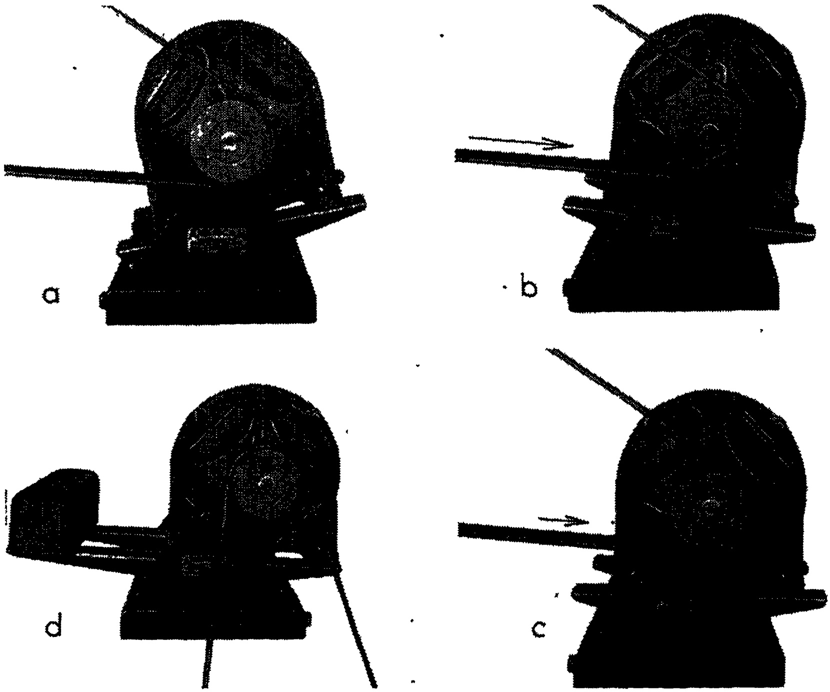


FIG. 12–12. Reactive-Torque Drive.
The American Pulley Co., Philadelphia, Pa.

Short-span drives are also possible using idler pulleys to adjust the tension. Either weights or springs can be used on the frame of the idler to adjust the belt tensions. Idler pulleys can be applied to fixed-center

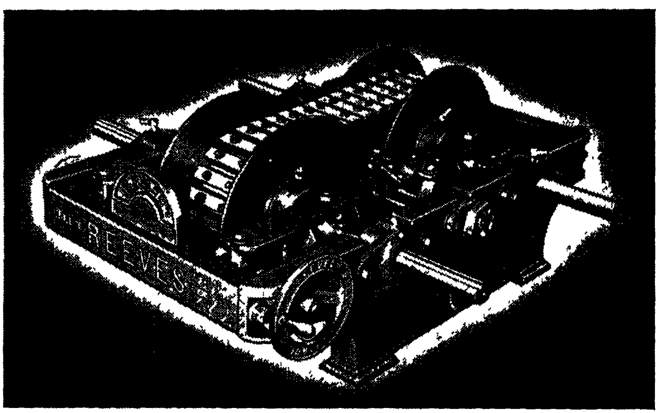


FIG. 12–13. Reeves Speed Controller.
Reeves Pulley Co., Columbus, Ohio.

drives, but do not have the advantages of the above special devices, for motor drives.

12–7. **Variable-Speed Drives.**—The Reeves speed control device, Fig. 12–13, allows continuous variation in the relative speeds of its two shafts by the operation of the handwheel. Speed adjustment can be made while power is being transmitted.

Each pulley consists of two steep cones keyed to the shaft with a sliding key. It will be seen from the cut that the belt is made of a

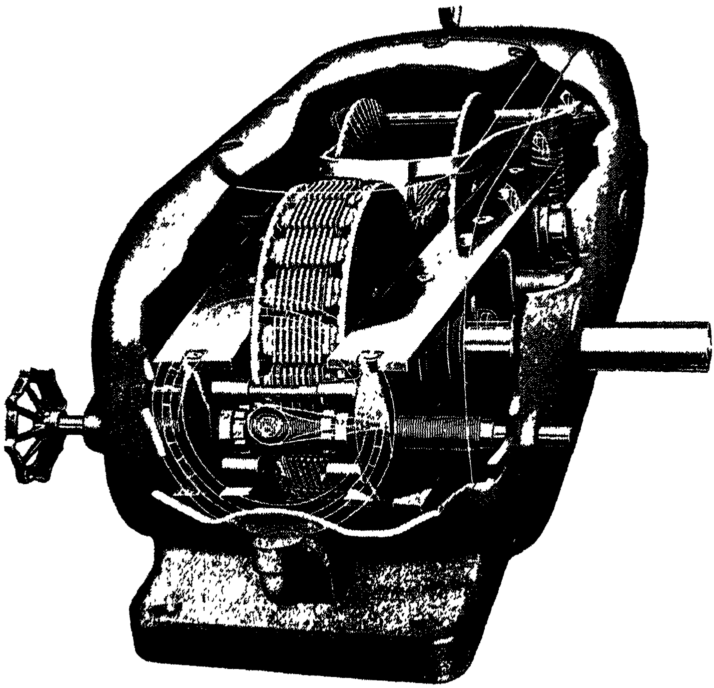


FIG. 12–14. P. I. V. Speed Controller.
The Link Belt Co., Chicago, Ill.

succession of wood blocks riveted to a continuous flexible belt. The blocks are cut at the angle of the pulley cones, so their ends, which are rubber covered, bear evenly on the cones and wedge between them. This results in a firm grip so that high belt tensions are possible with low slippage.

The varying of the speed ratio is accomplished by changing the axial distance of each pair of cones. Suppose it is desired to increase the relative speed of the nearer shaft. The cones on this shaft are drawn apart and the belt rides on a smaller diameter. Now to maintain proper

belt tension the other cones must be drawn together. Both motions are accomplished by a linkage controlled by the handwheel, and the resulting speed ratio is indicated by the pointer on the dial. The drive has been applied to conveyors, sawmills, and a variety of process machinery.

The Link Belt Co. manufactures the **P. I. V.** (positive infinitely variable) speed controller illustrated in Fig. 12–14. The principle of approaching and receding cones used on the Reeves drive is utilized here, but slippage between belt and cones is eliminated by the use of an ingenious positive engagement. The belt is really a chain and each contacting block consists of a sheaf of thin steel strips, free to slide endwise. As shown in Fig. 12–15, each tooth on a cone is opposite an intertooth space on the mating cone. As a block of the belt comes into action, the steel strips are forced into the form of the mating cone grooves. This occurs as readily when contact is near the periphery of the cones where the grooves are wide as when contact is at the smaller radii. The speed ratio is changed by turning the handwheel which controls the spacing of the cones through the linkwork as shown.

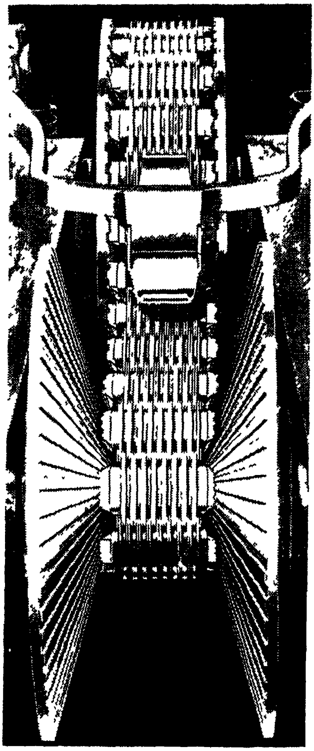


FIG. 12–15. Tooth Formation on P. I. V. Drive.

12–8. **Ropes and Cables.**—Before the development and general distribution of electric power with cheap and reliable motors, both textile and wire ropes were used for the transmission of power. They were more economical than belting for spans in excess of about 40 feet. At present, ropes and cables have a broad field of application in pulley blocks, hoists, drag buckets, elevators, etc. Wire rope is favored where length is great and loads are heavy, also where sharp bends are not required.

Fiber ropes are made of Manila hemp or cotton by twisting fibers into cords, cords into strands, and strands into rope. An application to the pulley block is given in Fig. 12–16. The purpose is to gain mechanical advantage in raising a weight or exerting a force in any direction. If there were only one pulley on each axis, it would be called a single pulley block, and the mechanical advantage would be 2, because the

lower axis would rise at half the speed of the moving rope. Each added pair of pulleys (one on each axis) multiplies the mechanical advantage by 2. Consequently, with the compound pulley block shown, a force of one pound at F will balance a weight of 4 pounds at W, neglecting friction, but would have to be exerted through 4 feet to raise W one foot.

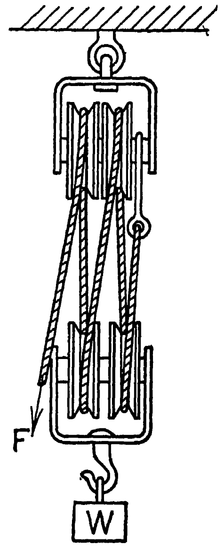


Fig. 12–16. Compound Pulley Block.

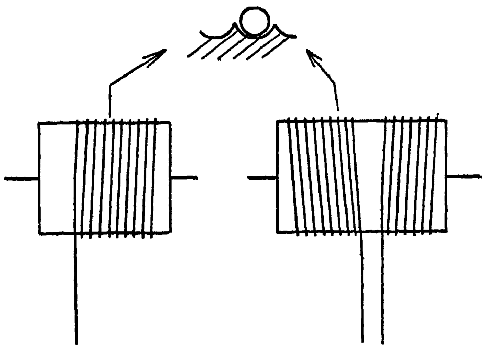
Wire rope is made by twisting wires into strands and strands into rope. Fig. 12–17 shows a 6 × 19 plow-steel rope. The first number indicates the number of strands, the second gives the number of wires in a strand. For the same diameter of rope, the flexibility increases as the size of the individual wires decreases. When a hemp core is used it is charged with grease to prevent rust and decrease friction.

Many applications requiring lifting and pulling demand that the rope be wound on or over a drum. When wound on the drum, one end of the rope must be securely fastened thereto, Fig. 12–18, and a smooth shallow helical groove guides the rope while winding on. Overwinding (two layers) injures the outer wires. To gain the safety and strength of two ropes, the arrangement shown at (*b*) is used. Two helical grooves of opposite hand make it possible for the ropes to lie close together when the elevator or hoist is at the top. In this way both ropes can be attached above the center of mass of the load. Four ropes can be centered by using two drums, one above the other, driven in the same sense, or two drums side by side driven in opposite senses.

Fig. 12–17. 6 × 19 Hoisting Rope (Hemp Core).
The Bethlehem Steel Co., Bethlehem, Pa.

The high lifts required in mines and tall buildings would require excessively large and expensive winding drums. A better solution is found in the **traction sheave,** Fig. 12–19. As the name indicates, friction is relied upon entirely to transmit the driving force. In the simplest arrangement, the rope makes 180° of driving contact with a single sheave. One end is fastened to the elevator, the other to a counterweight. The width of sheave wheel (called a drum if wide) is independent of the height of the lift. It can accommodate several ropes, Fig. 12–21, and still be quite narrow.

Obtaining sufficient traction to work a wire rope up to its capacity without danger of slippage presents a problem. When fiber ropes are used for traction drive, they are pinched in a V-groove similar to that



(a) (b)

FIG. 12–18. Cable Winding Drums.

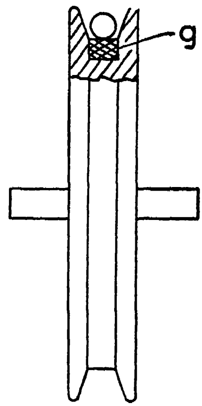


FIG. 12–19. Traction Sheave for Single Rope.

used for a V-belt, but wire ropes are injured by such treatment. The construction shown in Fig. 12–19 has been found satisfactory when the loads are not excessive. The groove g is filled with rectangular blocks of leather or rubber into which the rope seats. Hardwood blocks are also used.

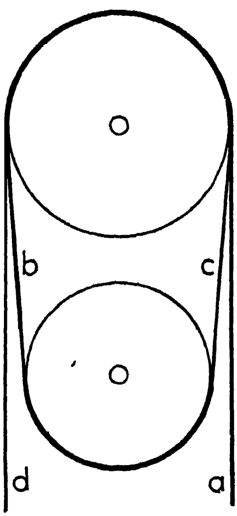


FIG. 12–20.

For high-lift high-speed elevators, the smooth circular metal groove like that in Fig. 12–18 is used with the arrangement shown in Fig. 12–20 for increasing the traction. One rope only is shown. It runs from the elevator to a, then around the traction (upper) drum to b, then around the idler drum to c, then around an adjacent groove on the traction drum, emerging at d, and thence to the counterweight. In this way the one rope has two 180° laps on the traction drum and as many might be added as necessary. The grooves on both drums must be circular, not helical.

The drive and complete cable arrangement for a traction type elevator is illustrated in Fig. 12–21. At the top is the traction sheave driven by an electric motor through a worm-and-wheel reduction. The five cables, which operate in parallel,

are anchored at the upper left corner, pass down around the counterweight sheave, up over the traction drum, down around the elevator sheaves, and up to another anchorage in the upper right corner. The main brake is located at the left of the motor and an overspeed braking device operates on the guide ways. This is a "two to one" drive, meaning that the ropes travel at twice the speed of the elevator car.

12–9. **Chains.**—On the basis of use, chains may be classified as

(*a*) hoisting and hauling,
(*b*) conveying,
(*c*) power transmitting.

On the basis of action the classes would be

(*d*) simple link chains without fixed bearings such as the familiar oval-link welded variety, Fig. 12–22.
(*e*) the Ewart square link chain, Fig. 12–23, the links of which can be easily detached,

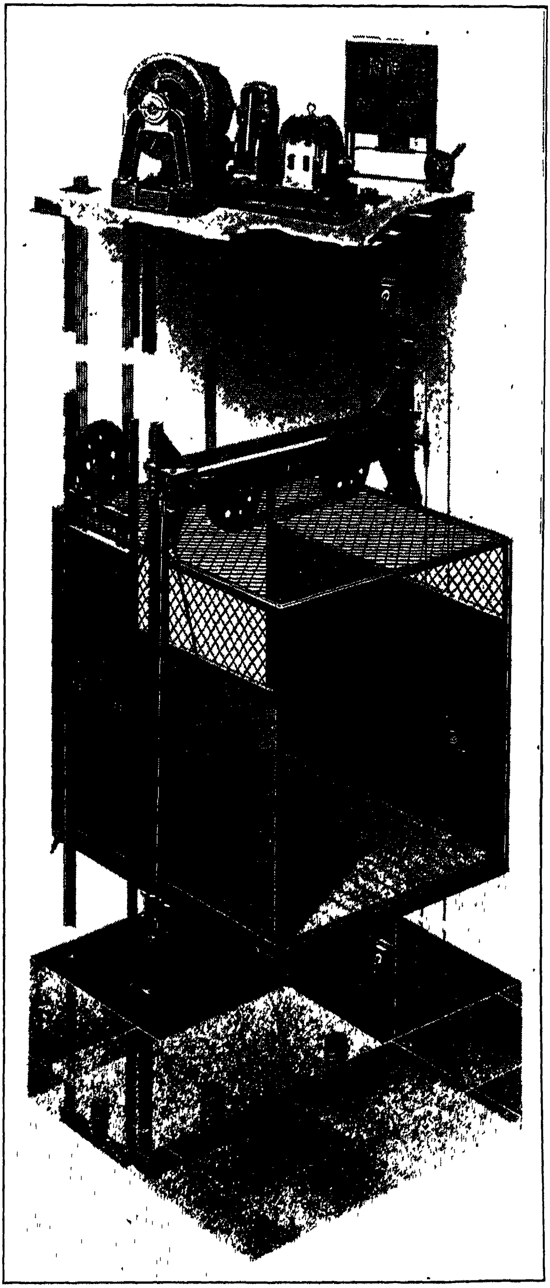


FIG. 12–21. Cable Traction Elevator.
The Otis Elevator Co., Philadelphia, Pa.

(*f*) all forms of block chain, Fig. 12–24, including pintle chain.
(*g*) roller chains, Fig. 12–25,
(*h*) silent chains, Fig. 12–27.

The block chain was the earliest to be developed especially for power transmission, and came into wide use with the popularizing of the low-wheeled bicycle. Like the Ewart chain it rubs the teeth of the sprocket wheels as it grips and again as it leaves. The bearings between links hold lubricant only a short time and if lubricated collect dirt. Indeed, for certain services, the Ewart chains wear better if unlubricated or if graphite only is used. Ewart and block chains are noisy, but have the advantage of low first cost, and are used for a wide variety of purposes where speeds and power to be transmitted are both low. The block chain has a much better bearing, particularly when hardened steel pins are used.

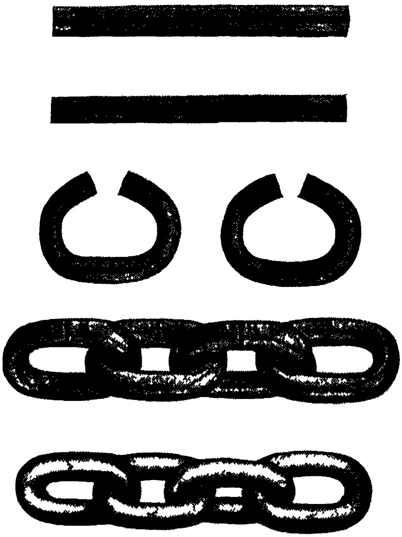


FIG. 12–22. Welded Oval-Link Hoist Chain.

The Yale and Towne Mfg. Co., Philadelphia, Pa.

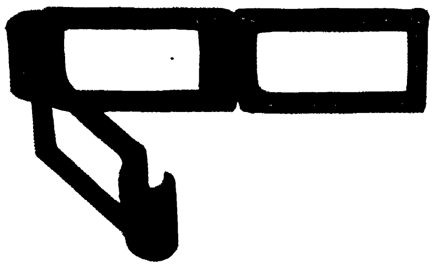


FIG. 12–23. Ewart Detachable-Link Chain.

The roller chain, as the name implies, has rollers riding on the outside of the pins or bushings that form the link bearings, Fig. 12–25. The "cotter-pin link" can easily be removed from the chain to shorten it. The value of the roller is that it converts what, in the block chain, is rubbing as the tooth enters or leaves the sprocket, to bearing action inside the roller, where it can be more effectively lubricated. On properly designed sprocket teeth this chain is quite silent for medium speeds.

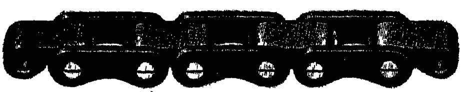


FIG. 12–24. Block Chain.

Sprocket wheels for the three varieties of power transmitting chains so far considered, must have teeth small enough in width and breadth to project through the links. The contour of these teeth in the plane of the wheel, and also their pitch in relation to the pitch of the chain, are important. Fig. 12–26 shows the profile of sprocket which the A. S. A. (American Standards Association) has adopted as standard for roller chains. A bottom arc of radius $E/2$ is followed by a straight portion ec, then another circular arc to the top of the tooth.

$$E = 1.005D + 0.003 \text{ in.}$$

where D is the diameter of the chain roller to be used. The standard values for the other dimensions are empirical [1] and unimportant in developing the principle of operation of the roller chain.

So long as wear cannot be entirely eliminated on rubbing surfaces, chains cannot be made that will not increase their linear pitch continuously even though slightly. If the new chain has the same pitch as the sprocket, the worn chain will have larger pitch. When this happens, the only way to avoid having all the load come on one sprocket tooth (the leaving tooth of the driven sprocket and the entering tooth of the driver) is to make it possible for the

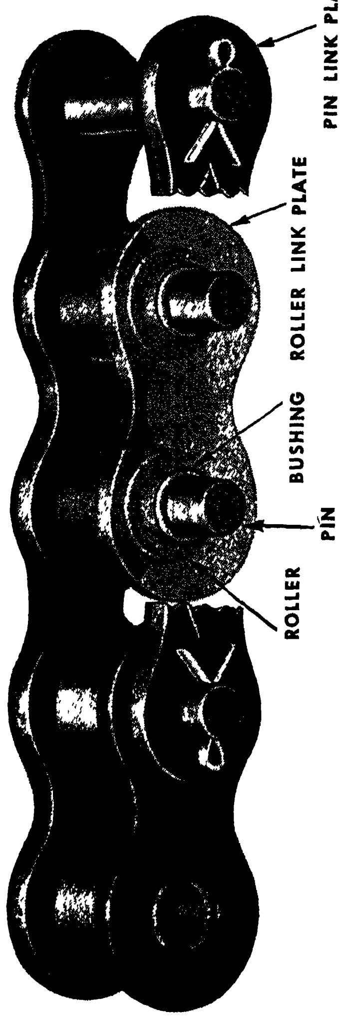


Fig. 12–25. Construction of the Roller Chain.
The Diamond Chain and Mfg. Co.

[1] American Standards Association standard No. B29a—1930.

chain to ride higher on the sprocket as it wears. In this way the circular pitch of the sprocket can match the linear pitch of the chain throughout its life. The profile of Fig. 12–26 has been found well adapted to this adjustment. Further, the sides of the teeth are less likely to wear out of shape when the transmitting force is distributed, and the entire action is smoother. The front elevation of a tooth is shown at *h*.

Conveyor chains are made in a great variety of constructions although the link joints used generally conform to either the block, roller, or Ewart design. Buckets, scrapers, hooks, etc., are connected to the links depending on the material or objects to be moved. Gray and malleable

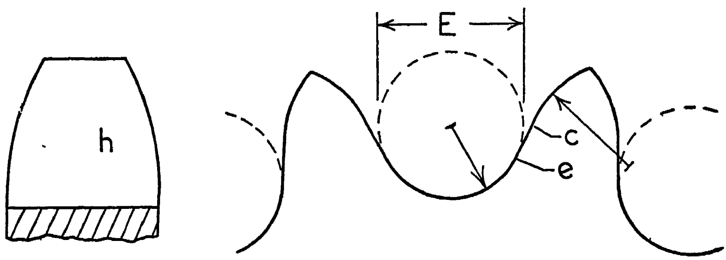


FIG. 12–26. Standard Sprocket for Roller Chain.

iron castings and steel punchings are used with a minimum of machine work, giving low cost conveyors which are satisfactory for the speeds used (generally less than 200 ft per min).

Conveyor sprocket teeth are seldom machined, and the increase in chain pitch due to wear is allowed for by making the pitch of the chain initially shorter than the circular pitch of the sprocket.

Power transmitting chains are built for high-speed performance. The link bearing surfaces are machined, often hardened and ground, and have provision for lubrication.

The **speed ratio** of chain-connected rotating parts is inversely as the number of teeth in the two sprockets. Chains can therefore be used interchangeably with gears where definite speed ratios must be maintained. If the span is large, chains may be more adaptable than gears. Fig. 12–27 shows a roller-chain drive connecting the valve cam shaft (upper shaft) with the crank shaft of a large compression-ignition engine.

All link chains, however, share one **defect** compared to belts and ropes. As the chain passes around on its sprocket, the centro axes of the turning pairs of the links travel at higher velocity than other points on the kinematic center lines of the links, being at different radial distances. If the sprocket turns at uniform speed, the chain is delivered at a velocity containing cyclical variations. The longer the links and the smaller the

sprocket, the more pronounced will these variations be. The possibility that this action on one sprocket would be cancelled by a reverse action on the other sprocket, is remote. The defect is inherent, but is not sufficiently troublesome to prevent the use of chains for quite high-

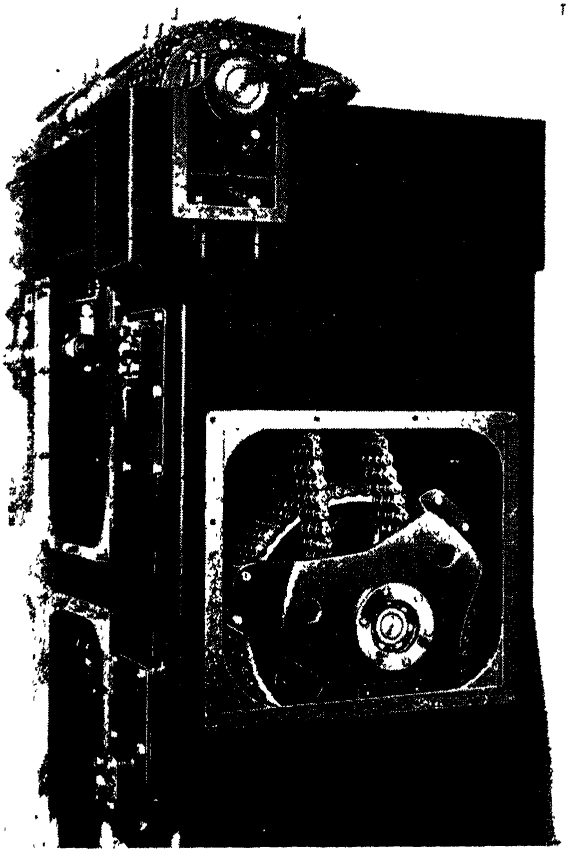


FIG. 12–27. Roller Chain on Diesel-Engine Valve Drive.
The Diamond Chain and Mfg. Co.

speed transmission. It does indicate that high-speed chain should have short links, and sprockets not too small.

12–10. **The Renold Chain.**—The so-called "silent chain" was invented by Hans Renold of Manchester, England. The links are flat steel stampings as may be seen in Fig. 12–28, which also shows the form of the sprocket wheels and the center grooves thereon, into which project special links at the center of the chain for the purpose of holding it in proper axial position.

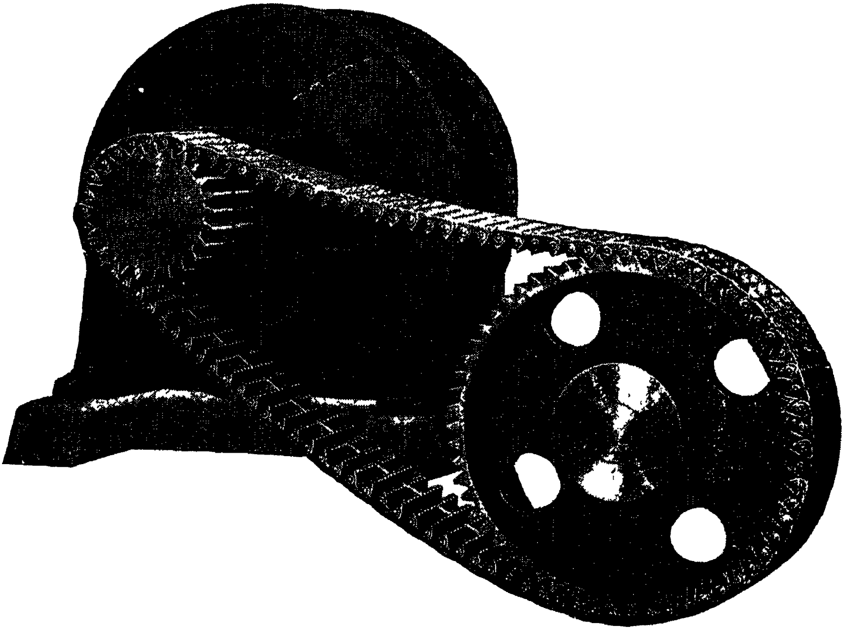


Fig. 12–28. Silent-Chain Drive.
The Link Belt Co., Chicago.

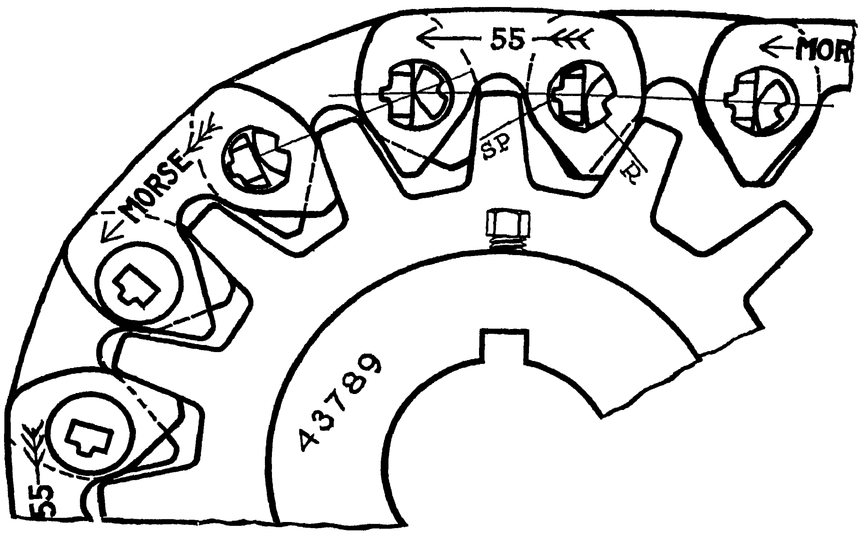


Fig. 12–29. Silent-Chain Action.
The Morse Chain Co., Ithaca, N. Y.

Renold's chain has two features that have made it undoubtedly the finest transmission chain available, capable of high speeds and large powers. These features can best be seen in Fig. 12–29. The first is the scissors action of any two adjacent links, causing the parts that make

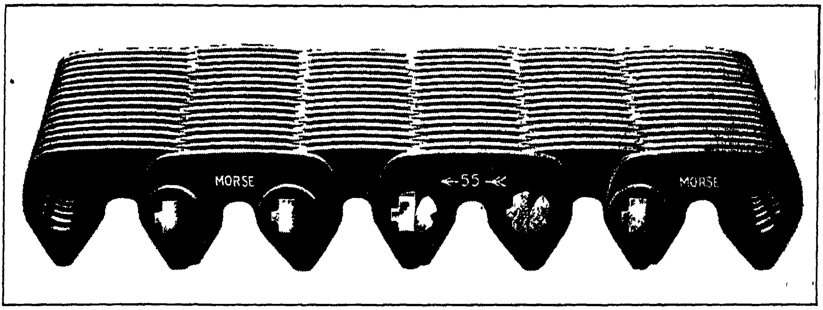


FIG. 12–30. Morse Chain Rocker Joint.

contact with the sprocket to open as the chain is bent. Note that the upper right links which are not flexed have their "scissors" closed, and contact is not made with the teeth until the chain is curved and the scissors open. The reverse action occurs when the chain leaves the sprocket, so that in both cases rubbing action is reduced to a minimum.

The second feature depends upon the flatness of the contact surfaces of tooth and link, so that as the pitch of the chain elongates in service, the links grip the teeth nearer their ends but at precisely the same pressure angle. So long as there is adequate contact area, there is little change noticeable in performance.

The types of silent chain manufactured by the various companies differ mainly in the bearing used at the joints. Pin-and-bushing bearings rub and require lubrication. In the Morse chain of Fig. 12–30 is seen a "rocking joint." The hardened-steel elements, indicated in outline, roll on each other as the chain flexes and straightens. The manner in which links are added to give chains of any desired width, may also be seen from this cut.

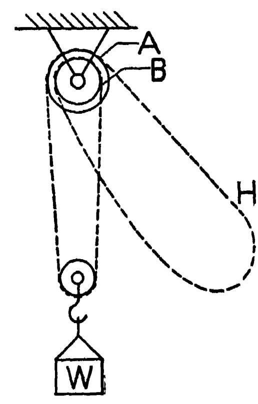


FIG. 12–31. Differential Hoist.

12–11. The Differential Hoist.—Oval-link chains, Fig. 12–22, are regularly used for chain hoists because maximum flexibility is required. The sprocket wheels are designed to hold the links of the particular chain used in recesses

which prevent slippage. In the differential hoist, Fig. 12–31, sprockets A and B are on the same kinematic link. The broken line represents a chain which cannot slip on A or B. When the chain is pulled at H so that A and B make one turn, the force at H has acted through a distance πD_A, where D_A is the pitch diameter of sprocket A. Meanwhile,

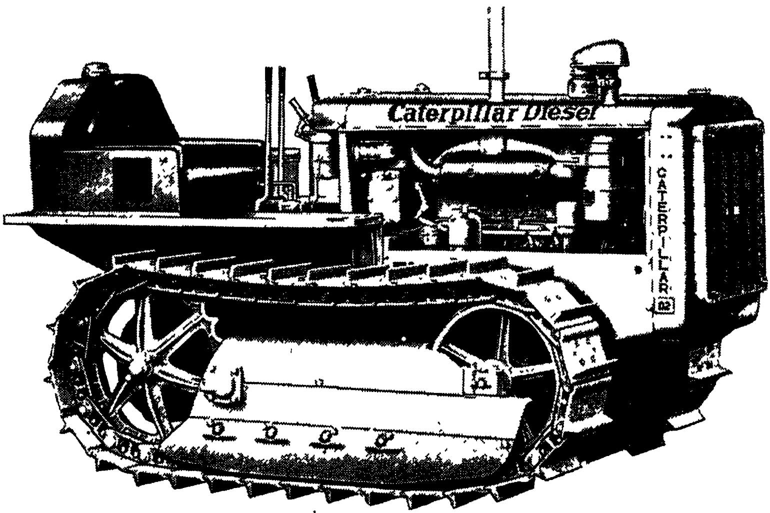


Fig. 12–32. Diesel Tractor.
Caterpillar Tractor Co., Peoria, Ill.

the weight chain is shortened πD_A on the left side and lengthened πD_B on the right side, a net shortening of $\pi(D_A - D_B)$. The weight W is raised half the distance that the lift portion of the chain is shortened; therefore

$$\frac{V\text{-}H}{V\text{-}W} = \frac{\pi D_A}{\frac{\pi}{2}(D_A - D_B)} = \frac{2D_A}{D_A - D_B} \qquad (8)$$

This, of course, is the mechanical advantage of the hoist. By making sprockets A and B nearly the same size, the mechanical advantage can be increased to any desired value and the device is still simple and compact.

12–12. **The Tractor.**—An important application of the chain is found in "track type" vehicles such as the tractor and the military tank. These vehicles lay their own track continuously, roll over it, and pick it up behind for reuse. This is made possible by the chain track, mounted over sprockets to which the power is applied. Increased contact with the road results in greater traction than is possible with wheels.

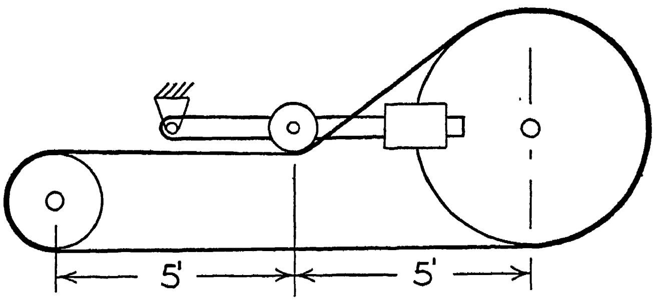
The tractor of Fig. 12–32 is driven by applying the engine power to the rear sprocket shafts through a differential-gear assembly similar to that used for an automobile. The lower half of each track chain is, therefore, in tension. Traction over soft earth is increased by attaching to each chain link a "grouser shoe" as shown. These chains must operate where protection against dirt is almost impossible, and reliance must be placed on the use of hard, wear-resistant metals at the link joints.

PROBLEMS

1. Two parallel shafts 8 ft apart are to be connected by open flat belt so that the speeds will be 250 rpm and 600 rpm. The smaller pulley is to be 10 in. in diameter.
   (*a*) What is the size of the other pulley?
   (*b*) What is the belt speed?
2. Two shafts 20 ft apart have pulleys of outside diameters 3 ft and 8 ft. Find the "tape line length" around pulleys for both open and crossed belts.
3. Find the sizes of all steps for a pair of cone pulleys operating on a span of 10 ft, and giving positive speed ratios of 0.4, 0.6, 0.9, and 1.35. Make the diameters for equal pulleys 2 ft. Use the trial and error method on a large Smith diagram. Check your results for the high and low ratios by the belt-length formula. What kind of progression is this?
4. A 1750-rpm motor is connected by open flat belt with a 250-rpm line shaft, the span being 8 ft and the motor being mounted on a Rockwood base. The motor pulley is 8 in. in diameter.
   (*a*) What is the angle of contact on the motor pulley?
   (*b*) What is the belt length?
5. Five speeds are required in geometrical progression, the lowest being 1000 rpm, and the highest 1874. Find the speeds and the common ratio. Ans. Speeds 1000, 1170, 1369, 1602, 1874; ratio 1.17.
6. The counter shaft for a drill press runs at 600 rpm. Cone pulleys are required to run with an open belt and give the machine five speeds in geometrical progression between 500 and 845 rpm. The smallest diameter on either cone is to be 8 in., and the span is 80 in. Find the size of all steps.
7. The normal planes of two nonintersecting shafts are at right angles. Make drawings showing how to connect these shafts by a reversing flat-belt drive using:
   (*a*) three idler pulleys,
   (*b*) two idler pulleys,
   (*c*) one idler pulley.

What limitation does (*c*) impose on the size and position of the idler pulley?

8. A compression-ignition engine having a speed of 650 rpm is to be connected by V-belts to an air compressor which has a rated speed of 106 rpm. The sheave pulleys stocked have pitch diameters in even numbered inches. Allowing for about 2% belt slip, choose sheave sizes, neither being smaller than 6 in. Belt speeds up to 4000 ft per min may be used.
9. What would be the mechanical advantage of a block and tackle similar to that of Fig. 12–16, if it had three pulleys in each block?
10. In this drive it was found that an open belt gave excessive slip, and tension control was difficult, so a one-foot weighted idler was placed to give 180° of contact on the small 2 ft pulley. The large pulley is 5 ft, all sizes being diameters. Find the belt length. Ans. 31.926 ft



11. The pitch of slow-speed conveyor chain is made less than the sprocket pitch to allow for wear. Sketch such a new roller chain on driver and driven sprockets, and demonstrate that the entering tooth of the driven sprocket and the leaving tooth of the driver take all the load. Discuss the velocity effects.
12. A drum-type elevator, where each cable is anchored at one end of a driven drum of 40 in. pitch diameter and wound on it in a helix, is required to serve 4 floors 16 feet apart. The cable is 3/4 in., and the laps lie $1\frac{1}{8}$ in. apart, center to center. Allowing 2 laps on the drum when the elevator is at the bottom and an overtravel of 4 ft at the top, find the width of each drum.
13. A block chain of 15/16 in. pitch initially was driven by a 20-tooth sprocket having a pitch of 1 in., and the radial distance from the under side of the chain to the top of the teeth was 3/8 in. How much increase in chain pitch due to wear would bring the chain to the point of jumping teeth? Assume 180° of contact.

Ans. 0.1803 in.

14. The elevator, Fig. 12–21, has a traction sheave of 4-ft pitch diameter.
    (*a*) At what rpm must the traction sheave turn to move the elevator at 300 ft per min?
    (*b*) If the counterweights weigh as much as the elevator plus half its maximum load, in which sense must torque be applied to raise the elevator when unloaded?
15. On the differential hoist of Fig. 12–31, the pitch diameter of A is 16 in., and of B 14 in.
    (*a*) What pull at H will balance 600 lb at W?
    (*b*) If the friction is 25% of the input, what pull at H would start the 600 lb upward?
16. Design a differential hoist having a mechanical advantage of 100, the larger sheave having a pitch diameter of 10 in.

CHAPTER XIII

MISCELLANEOUS MECHANISMS

13–1. The Universal Joint.—To connect two shafts end to end, whose center lines intersect but are not quite in line, so that torque can be transmitted continuously from one to the other, is a problem that frequently confronts the designer. If the misalignment angle is slight, there are many types of flexible connector available that have been developed to transmit quite large torques. However, if the misalignment angle is more than a few degrees, and particularly if it is variable, some form of universal joint is required.

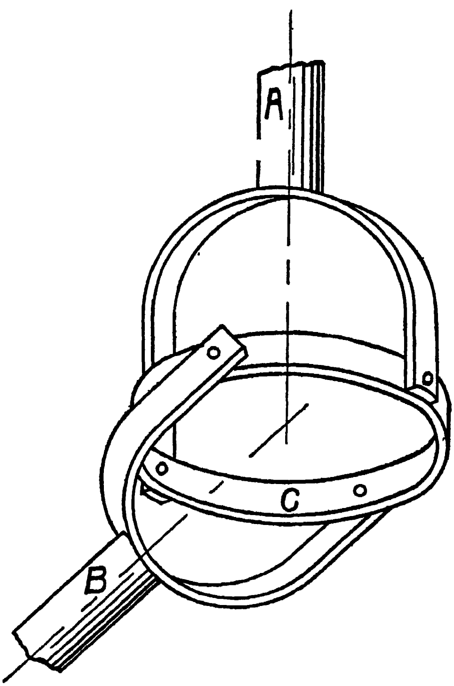


FIG. 13–1. The Hooke Joint.

The universal joint made of all metal parts is commonly called *Hooke's Joint* after its developer, and the essential parts are shown in Fig. 13–1. A characteristic of such joints is that the relative angular-velocity ratio of the connected shafts varies during the revolution, the variation depending on the misalignment angle. This may lead to serious trouble in application if the properties of the joint are not comprehended.

It will be seen from Fig. 13–1 that there are three main links, aside from the pin connections. Consider A, the driving shaft, to rotate at constant speed, ω_A radians per second. The intermediate link C is shown as a hoop. In some designs it is a cross, in others a block or sphere. It is required to find the speed of rotation of the third member, the driven shaft B, for any angular position.

In Fig. 13–2, the upper plan view is taken looking down on the plane of the two shafts A and B. The elevation view is taken looking along shaft A. The pin centers of the forks of A will travel in a circle of radius OK (lower elevation view), and the pin centers of B follow the circle of radius OL. The travel of the fork ends of A is, in this view, the circle KPR, and the corresponding locus of the fork ends of B appears as the ellipse LSP.

Assume that one fork end of A is initially at P. At 90° from P, measured along the intermediate hoop link C, will be found one of the fork ends of B at the point L. L is on the surface of a sphere with center at O and both the fork ends and the hoop C are at all times on this sphere.

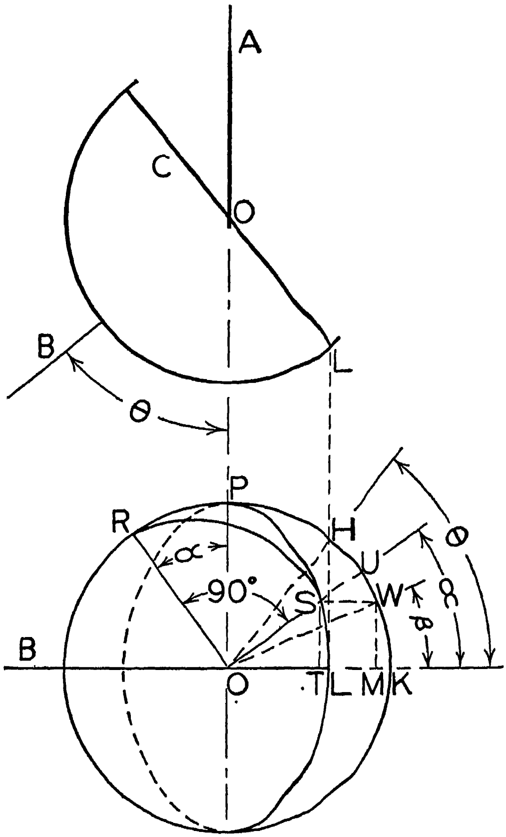


FIG. 13–2. Geometry of the Universal Joint.

Now let P move to R, the shaft A turning through the angle α from where the forks of A were vertical. L will move up its circle to some point S, such that angle $ROS = 90°$; for the quarter of the hoop C, initially at PL and now at RS, will always subtend 90°. The angle through which B has turned, β, is LOS, but does not appear in true size since it is not in the plane of the paper and is not 90°. Tipping the circle LP into the plane of the paper, it becomes the circle KP, while S moves horizontally to W, and β in true projection is seen to be WOK.

Similarly, to get the true size of the misalignment angle θ which is the angle between the circles KP and LP, the whole lower figure may be turned through 90° about OK as axis. Then L will move vertically to H, and θ in true size is seen to be HOK. Further, the angle $\alpha = 90° - POU =$ angle UOK, all in the plane of the paper.

Recalling that all the angles dealt with have been revolved into their true sizes in the plane of the paper,

$$\frac{\tan\beta}{\tan\alpha} = \frac{\frac{WM}{OM}}{\frac{ST}{OT}} = \frac{\frac{ST}{OM}}{\frac{ST}{OT}} = \frac{OT}{OM} = \frac{OL}{OK} = \frac{OL}{OH} = \cos\theta \tag{1}$$

The relation $OT/OM = OL/OK$ above is proved by observing that, if the circle PK is revolved about the axis PO through the angle θ, it becomes the ellipse PL, and OK is foreshortened to OL while OM is proportionately foreshortened to OT.

The relative angular velocity of shaft B to shaft A, expressed as $d\beta/d\alpha$, can be evaluated by differentiating (1), remembering that θ is constant.

$$\sec^2\beta\, d\beta = \sec^2\alpha\, d\alpha \cos\theta$$

$$\frac{\omega_B}{\omega_A} = \frac{d\beta}{d\alpha} = \frac{\sec^2\alpha\cos\theta}{\sec^2\beta} \tag{2}$$

Eliminating β by the use of (1),

$$\tan\beta = \tan\alpha\cos\theta$$

$$\sec^2\beta = 1 + \tan^2\beta = 1 + \tan^2\alpha\cos^2\theta \tag{3}$$

Hence

$$\begin{aligned}\frac{\omega_B}{\omega_A} &= \frac{\sec^2\alpha\cos\theta}{1+\tan^2\alpha\cos^2\theta}\\ &= \frac{\cos\theta}{\cos^2\alpha+\sin^2\alpha\cos^2\theta}\\ &= \frac{\cos\theta}{\cos^2\alpha+\sin^2\alpha(1-\sin^2\theta)}\end{aligned}$$

$$\frac{\omega_B}{\omega_A} = \frac{\cos\theta}{1-\sin^2\alpha\sin^2\theta} \tag{4}$$

Interpretation of this result depends on noting that the angle α is measured from where the forks of shaft A are normal to the plane of the two shafts. When α is zero, ω_B/ω_A equals $\cos\theta$, and when α is 90°, ω_B/ω_A equals $1/\cos\theta$, indicating that the curve of ω_B/ω_A plotted on degrees will pass through zero four times every revolution. If A is revolved at constant speed, B is compelled to accelerate and decelerate twice in each revolution, the severity of this action increasing directly as $1/\cos\theta$.

Consider now the effect of two universal joints in series. In Fig. 13–3, the forks of shafts A and D are vertical, and the forks on each end of B are in the same plane. It follows that ω_B/ω_A equals ω_B/ω_D for all phases, hence ω_D equals ω_A, and B only is subject to the accelerations. This is true only, provided the misalignment angles are equal and *the forks on*

the ends of B are in the same plane. If the forks of B are 90° out of plane, D will receive double the acceleration of B, the effect being cumulative. With D in position D', and the forks of B in the same plane, ω_D' equals ω_A for all phases.

In Fig. 13–4 is illustrated a combined universal joint and slip joint as used on the transmission shaft of an automobile between the gear box and the rear axle. The action of the rear springs causes almost constant action in this joint. Satisfactory performance over long mileage depends not a little on keeping the lubricant in and the dirt out. Provision for accomplishing this is well shown in the cut. Roller bearings of the small roller type (pin bearings) are used for the joints.

The Weiss universal joint uses steel balls as the intermediate link. These balls ride in axial grooves or races which are cut on the faces of lugs projecting from the two shaft members. It is claimed that the

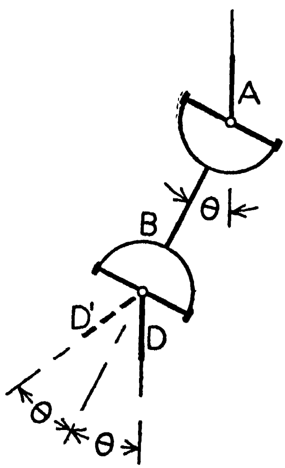


FIG. 13–3. Effect of Two Universal Joints in Series.

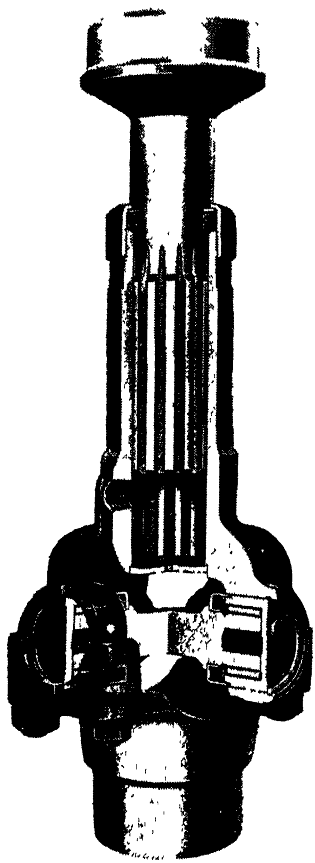


FIG. 13–4. Universal Joint for Automobile Service.

The Spicer Manufacturing Corp., Toledo, Ohio.

relative angular velocity of the connected members is uniform and unaffected by changes in the angle between the shafts.

There are many flexible couplings manufactured which will connect shafts satisfactorily if they are very nearly in line. These are, in general,

cheaper than universal joints. The latter are required when the shafts are considerably out of line.

13–2. Friction Transmissions.—Many ingenious mechanisms have been devised to give variable speed transmission using rolling friction contact. A measure of success has been achieved only where small power was transmitted.

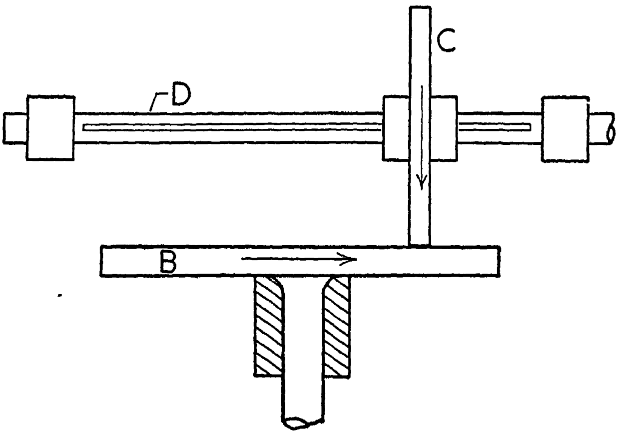


Fig. 13–5. The Brush Wheel.

The *brush wheel* is shown in Fig. 13–5. The driving shaft ends in a large disk B. The wheel C is given rotation by friction contact with the face of B, and transmits to its shaft D through a sliding key. A continuous pressure is required at the friction contact, and it is desirable that the thrust bearing behind B should be spring loaded. To change the speed ratio it is only necessary to slide C along D, but this requires the withdrawal of B. A lever arrangement is used to position C, and any speed can be selected over the range of the device. Reversal is obtained when C is moved to the left of center.

The capacity is increased by covering the rim of C with leather or fibrous material. B is regularly of metal. Overloading causes slip and the wearing of flats, which results in rough running. To approximate pure rolling, C should be narrow.

The *Transitorq*, known in England as the Hayes friction gear, is based on the same principle, namely, speed variation by selecting frictional contact at varying distances from the axis of rotating members. Here, both input and output shafts terminate in disks with the faces hollowed to accommodate intermediate friction disks. These are supported spherically, and transmit like idler gears. Fig. 13–6 shows only one, but there

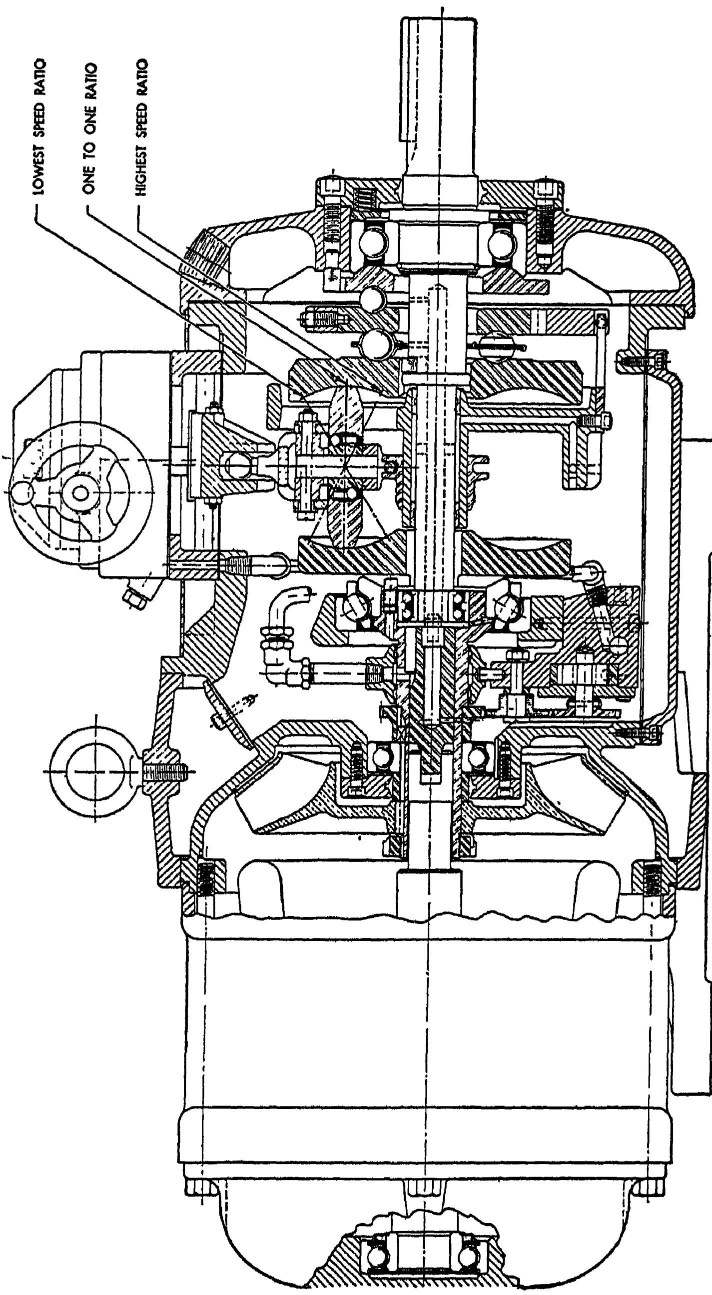
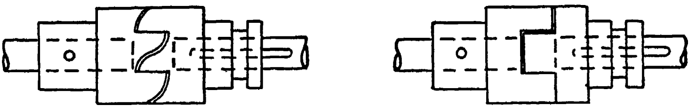


FIG. 13-6. The Transitorq.
The New Departure Mfg. Co., Bristol, Conn.

may be several. Engagement of the frames of all the idler disks with a central spool insures that they shift together and transmit at the same ratio.

13–3. **Clutches.**—Shafts in line can be permanently connected by flanged couplings, in which case they are solidly locked and become a single kinematic link. They can be connected by flexible couplings which allow a slight misalignment, or by universal joints which allow much greater freedom. From the standpoint of machine operation, these connections are permanent. When ready connection and disconnection is required, clutches are used.



(*a*) (*b*)

FIG. 13–7. Jaw Clutches.

There are two main classes, positive or jaw clutches, and friction clutches. Fig. 13–7 (*a*) shows a jaw clutch for transmission in one sense only. The left member is solidly keyed to its shaft, while the right member has a sliding-key connection to its shaft and is moved into and out of engagement by a lever-controlled fork (not shown) which rides in the channel.

At (*b*) is a square-jawed positive clutch suitable for transmitting torque in either sense. It should be used only where necessary, as it is difficult to engage, especially if well fitted to avoid backlash.

Friction clutches occupy the field where engagement must be made under load, or where the members may have different speeds of rotation at the instant of engagement. These conditions are common in vehicular and process machinery, also in the driving of machine tools. The friction clutch and its rival, the hydraulic coupling, perform under these conditions with maximum safety to the connected parts.

The Kinney clutch of Fig. 13–8 illustrates the type used where a pulley or sheave is to be controllably mounted on a through shaft. It can be substituted for a tight and loose pulley. First, consider that the shaft is to be driven by the pulley. The pulley is mounted on the sleeve A. The hub B is keyed to the shaft. The flange of B (but not the flange of A) is fastened to ring C by the cap screws shown. With shifter fork E moved to the right as shown, cam link F draws C and $C1$ together, and friction brings the flanges to the same speed. When E is moved to the left, cam F allows the springs to separate rings C and $C1$. Then A with its flange turns freely, and B with its shaft comes to rest.

Now suppose that the through shaft is the driver, and it is required to drive A and its pulley periodically. It is desirable to have the clutch mechanism (D, G, F, C, etc.) stationary when the clutch is disengaged.

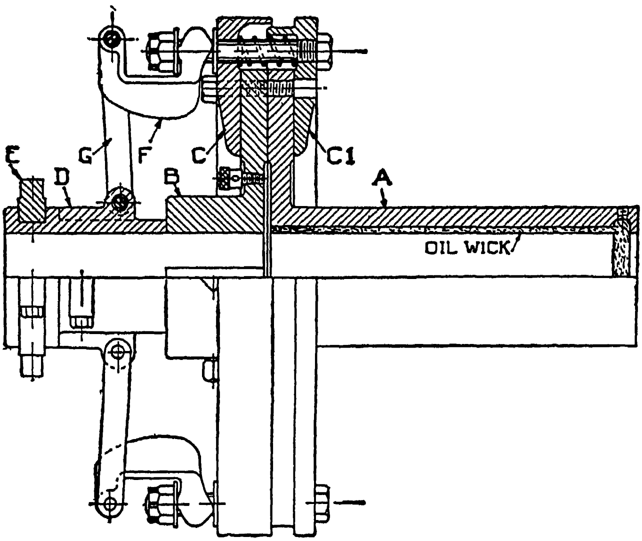


FIG. 13–8. Friction Disk Clutch.

Kinney Manufacturing Co., Boston, Mass.

To accomplish this, the cap screws are removed from C and the flange of B, and placed in the holes shown in $C1$ and the threads in the flange of A.

13–4. **Ratchets.**—The term ratchet is applied rather loosely to kinematic chains in which the drive of one link is in one direction or sense

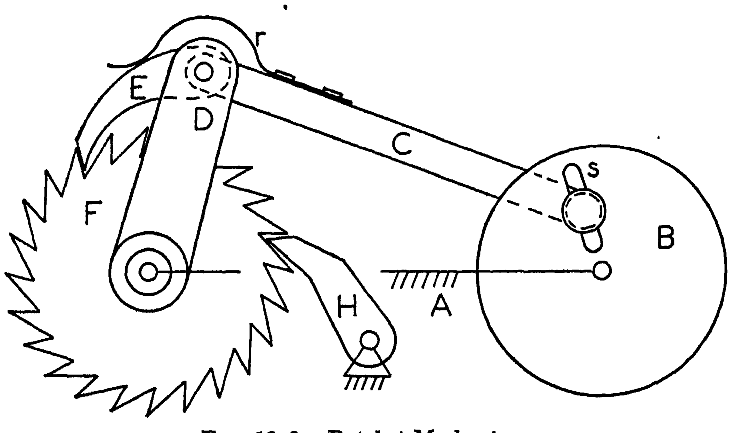


FIG. 13–9. Ratchet Mechanism.

and generally intermittent. They are really one-way clutches. Ratchets are used in feed mechanisms, lifting jacks, clocks, watches, and instruments.

Fig. 13–9 is an arrangement commonly used to operate feed screws on machine tools. The drive is taken from B which rotates at constant speed. The quadric chain A B C D gives reciprocating motion to the *pawl* E which may be held down by gravity, or better by a spring such as r. The ratchet wheel F will be driven intermittently, the stroke being adjustable by placement of the shoulder-bolt bearing s in the slot on B. If movements of F less than the pitch of its teeth are required, two or three pawls of slightly different lengths can be hinged at centro *ed*. Link H is called a *detent*. It is needed only if there is a tendency for F to drift back when the pawl is withdrawn. A notable application of the detent is on the main-spring ratchet of a watch.

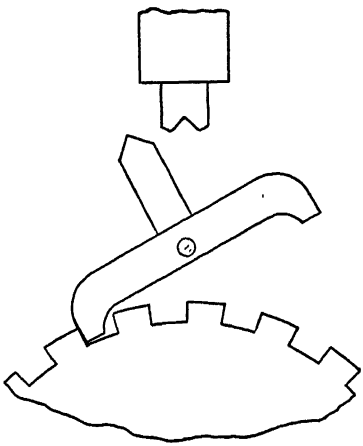


FIG. 13–10. Reversing Ratchet.

Feed mechanisms often require the reversing feature. Fig. 13–10 indicates a convenient solution. The frame of the double-ended pawl, which also carries the locking plunger, is given rotational reciprocation as in Fig. 13–9. In the position shown, the pawl drives the wheel counterclockwise. Reversal occurs when the pawl is snapped past the spring-loaded plunger, and when directly under the plunger it is safely in neutral position.

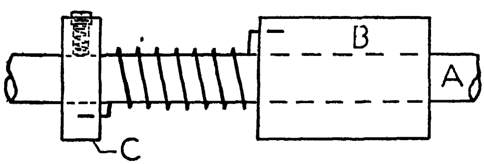


FIG. 13–11. Spring Ratchet.

When ratchet teeth are cut on a straight bar it becomes a ratchet rack. The pawl can be operated from the short end of a lever, and the result is the hoisting jack.

A helical spring can perform as a ratchet, Fig. 13–11. B and C ride freely on shaft A, except that C is given a slight frictional drag by its small spring. If B is turned clockwise, looking at the right end, A will be given only this small torque, since the large spring tends to open. Upon reversal of the drive on B, the large spring will grip the shaft and drive it. The friction between C and A must be adjusted so that the

large spring will be twisted sufficiently to cause it to grip initially. A flat-strip spring works well.

The three friction ratchets illustrated in Fig. 13–12 have been used as **free-wheeling clutches** on automobiles and other machines. In each case, when *A* is turned counterclockwise, *B* is driven by the wedging

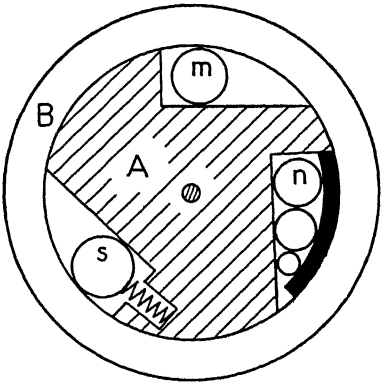


FIG. 13–12. Three Free-Wheeling Ratchets.

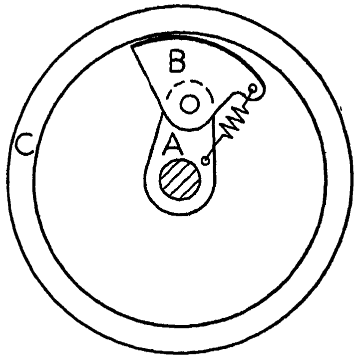


FIG. 13–13. Friction Ratchet.

action of the balls or rollers. When *A* is turned clockwise, they are rolled into the open ends of the spaces, and the pressure is released. Of course, the pressures are localized and severe, requiring hard materials. At *n*, a small plate of special material serves as a race for the three balls, and protects the driven ring *B*. The spring behind *s* is insurance that the clutching action will be prompt and reliable for any starting position. It also allows *B* to drive *A*.

The same principle is applied in the friction ratchet of Fig. 13–13. Here, *B* is a segment of a large roller. *C* will drive *A* clockwise and release it upon reversal. Of course, *B* could be used in similar manner on the outside of *C*, but, as with the annular gear, there is a better distribution of contact on the inside.

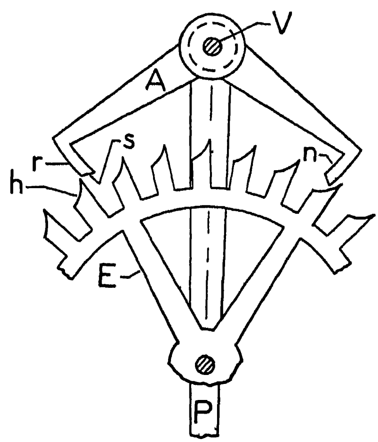


FIG. 13–14. Graham's Dead-Beat Escapement.

13–5. **Escapements.**—The mechanisms of clocks and watches convert the uniform vibrations of pendulums or loaded springs into dialed indications of time. The primary and controlling part of such mechanisms is the escapement.

Fig. 13–14 illustrates a design developed from many earlier clock escapements. The escape wheel E is actuated by springs or weights tending to turn it clockwise. The *pendulum* P is connected to the *anchor arms* A, all being supported by the shaft or *verge* V, which rides in fine bearings. The ends of the anchor arms which make contact with the teeth of the escape wheel are called *pallets*. In the position shown, the pendulum is swinging through central position toward the left, and tooth h is just escaping past the corner of pallet r. The end s of the pallet is tapered, so it is driven upward as h sweeps across it. This, and similar action at n when the pendulum returns, keep the pendulum link vibrating. When tooth h is freed by s, E turns until n contacts the face of the next tooth; hence E advances one tooth for each complete cycle of the pendulum. The surfaces r and n of the pallets are circular arcs about the axis

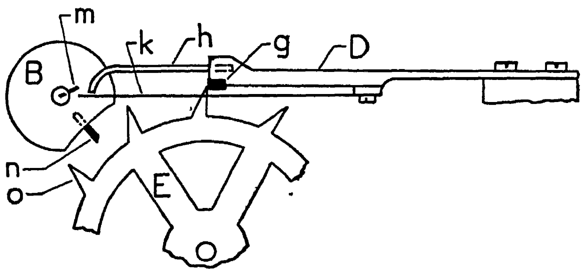


FIG. 13–15. The Chronometer Escapement.

of the verge. Therefore, however large the swing of the pendulum, the escape wheel will not be reversed. Hence this escapement is "dead beat."

The **chronometer escapement,** Fig. 13–15, uses a balance wheel B, controlled by a fine spiral spring called a *hair spring*. The characteristics of the spring and the balance wheel determine its rate of tortional vibration which, as in the case of the pendulum, is independent of the swing. A tooth of the escape wheel is shown in contact with the *locking stone* g, which is generally a piece of ruby. The locking stone is set in the detent link D, which also carries the flexible flat spring k. The balance wheel is about to turn clockwise, and pin m will deflect k and pass it. On the return stroke, m strikes k against the stiff member h. This raises the whole detent, and the tooth of E escapes past g. Now the energy to raise the detent must be restored to B. This is accomplished by o striking n, so that the restoration is immediate.

The simplest escapement, known as the cylinder or *Graham's cylinder*, is commonly used on Geneva watches. As illustrated in Fig. 13–16, B is a semicylinder supported axially. It carries the balance wheel con-

trolled by the hair spring. As B turns clockwise from the position shown, the left edge first releases the tooth of E which is driven clockwise by the main spring, then follows the inclined surface s, receiving the impulse necessary to overcome all resistance for a half cycle. When s has passed the left side of B, the right side is in position to receive the tooth on its inner surface. As B reverses, the surface s operates on the right edge of B, adding the energy necessary for the rest of the cycle. This is also a dead-beat mechanism.

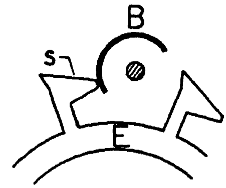


FIG. 13–16. Geneva Cylinder Escapement.

The extension of the use of electric clocks using small synchronous motors has, to some extent, transferred the work of time keeping to the governors of the turbines in the central generating stations. The usual procedure in the power station is to make adjustments every twenty-four hours to correct for the gain or loss in generator revolutions. In this way, cumulative errors in the time of electric clocks is avoided. Of course, the station operators must depend on chronometer time signals. Chronometer time is checked and adjusted, in turn, against celestial observations.

13–6. **Intermittent Gears.**—Fig. 13–17 illustrates a method of obtaining intermittent rotation from constant rotation by means of special gears. The driver B, considered to rotate at constant speed, will turn twice while C turns once, with two periods of dwell or rest. During the periods when C is at rest, constrainment is effected by radius r fitting into radius s. The sum of these radii must therefore equal the center distance.

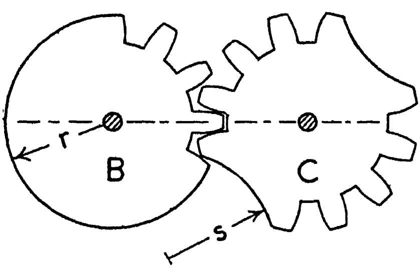


FIG. 13–17. Intermittent Gears.

When the first tooth makes contact after the dwell period, there is a peak of acceleration (theoretically infinite), hence such devices are limited to low speeds, unless special provision is made to distribute the change of velocity. Several methods are used, one being to attach B to its shaft by a stiff spring. The device as shown, however, if ruggedly built, will stand the speeds common to feed mechanisms and many applications on special machines.

13–7. **The Geneva Stop.**—Among the earliest of the intermittent mechanisms[1] is the Geneva Stop, one arrangement of which appears in

[1] For an unusually comprehensive collection of such devices see Ingenious Mechanism for Designers and Inventors, by Franklin D. Jones, The Industrial Press, New York

Fig. 13–18. The driver B carries a pin or roller with center at R. The follower C has four radial slots, and between these are four concave surfaces of such curvature as just to fit the shoulder H on the face of wheel B. Assume the driver B to revolve counterclockwise from the position shown. The pin with center at R is just entering a slot, and C will be started from rest clockwise. The shoulder H is cut back to the arc K to allow C to make the quarter turn, after which it again locks C stationary while B makes the remaining three-quarters of its revolution.

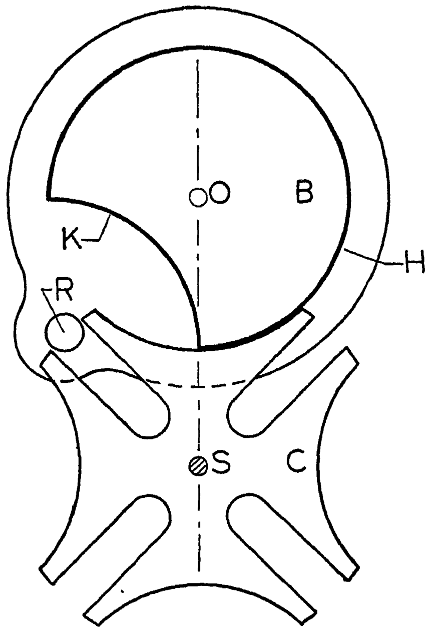


FIG. 13–18. The Geneva Stop Mechanism.

Driver B makes four revolutions to give C a complete turn, and C is moving only 25 per cent of the time. By increasing the size of C and the center distance OP, the number of slots and locking surfaces can be increased as desired. Further, by closing one of the slots, the number of turns that C can make continuously in one sense will be limited to the number of open slots in C. This mechanism has long been so used to prevent overwinding of the main springs of clocks and watches. B is attached to the winding end of the spring and as many open slots provided in C as the number of turns the spring should have when fully wound. From this, the oldest of its functions, the device received its name.

If the number of slots in C is increased to ten, the mechanism in multiple arrangement can be used for counting, and many devices using this principle are used on meters and computing machines.

When the early developers of the motion-picture projector were casting about for some means of giving the strip of film a quick advance between the periods of rest when it could be flashed on the screen, they tried the Geneva mechanism, and it served the purpose with some measure of success until better devices were developed. The requirements of the projector are briefly these. The film must be moved the length of one picture while the shutter is closed, and, to give the eye the sensation of continuity in the picture on the screen, this operation must be completed

in about 1/100 of a second. Using 4 slots in wheel C, it was easy enough to gear this wheel to the film so as to advance it one picture for 1/4 turn of C. The motion was completed in 1/4 turn of B, or B turned once in 0.04 sec, requiring a speed for B of 25 rps.

This is a high speed and would require excellent balance in the rotating parts. The speed of B might be reduced 50 per cent by using two pins and two cut-out portions but that would not reduce the speed of C, nor, what is more important, the acceleration of C. Since this type of mechanism appears frequently in the so-called "intermittors," we shall examine its acceleration characteristics.

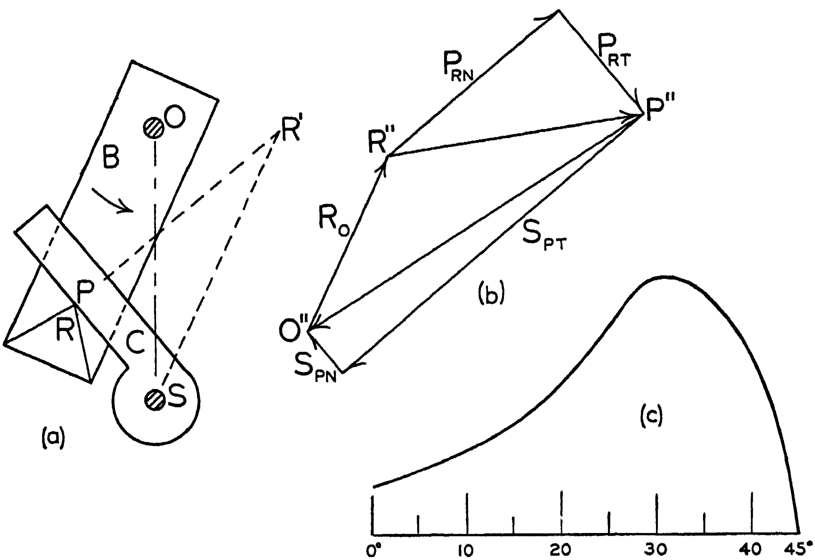


FIG. 13–19. Acceleration on the Geneva Stop.

(a) Slider-Plane Equivalent Mechanism. Scale, 1 in. = 1 in.
(b) Acceleration Diagram for 20° Phase. Scale, 1 in. = 1 in. per sec^2.
(c) Curve for Angular Acceleration of Link C. Scale, 1 in. = 4 radians per sec^2 ($\omega_B = 1$).

The roller driving a plane surface of a rotating link corresponds to the general case of Fig. 7–18, and indicates the use of the slider-plane equivalent mechanism. Consider B to rotate counterclockwise, calling its angle zero in the position shown. Then Fig. 13–19 (a) shows the equivalent mechanism for the 20° position. Point R on link B is in driving contact with P on C. Computations will be made on the basis of $\omega_B = 1$ rad per sec, assumed constant. R', the phorograph of R, is

located on C.

$$\omega_C = \frac{OR}{SR'}\omega_B = \frac{1}{1.54} \times 1 = 0.65$$

Using O, R, P, S as the order in which the points are taken, and using O'' as the initial point in the vector acceleration diagram,

$$R_{ON} = R_O = (OR)\omega_B^2 = 1 \times 1^2 = 1$$

As usually evaluated, the Coriolis component would be

$$R_{PN} = 2(R'R)\omega_C^2 = 2 \times 1.39 \times 0.4225 = 1.175$$

in direction R' to R. Here, however, the order requires P_{RN} which has the opposite sense and is so plotted. Passing to the terminal vector,

$$S_{PN} = (SP)\omega_C^2 = 0.66 \times 0.4225 = 0.279$$

Now P_{RT} and S_{PT} are of known directions and must close the diagram, locating P''.

Probably the most useful value is α_C, the angular acceleration of C.

$$\alpha_C = \frac{S_{PT}}{(SP)} = \frac{2.07}{0.66} = 3.14 \text{ rad per sec}^2$$

Several of these values for different angular positions of B yield the curve shown at (c), which covers the total acceleration period of 45°. The curve indicates that this type of motion is not well adapted to high speed. An ideal constant acceleration curve would have an ordinate much less than this maximum ordinate. In the succeeding 45° period of deceleration, curve (c) is reversed.

The value of α_C was found to be 3.14 for the 20° position with $\omega_B = 1$. What would it be if B turned at 25 rps as previously mentioned? Remembering that ω^2 enters into each value of acceleration,

$$\alpha_C = 3.14 \times (2\pi \times 25)^2 = 77{,}500 \text{ rad per sec}^2$$

and the maximum value of α_C would be about 135,000.

13–8. **Moving-Picture Projector Mechanism.**—A number of mechanisms for feeding the film have been developed which have better acceleration characteristics than the Geneva Stop. One of these appears in Fig. 13–20.

The driver B is made with a projection in the shape of an annular ring. The follower C turns in a fixed bearing and carries four pins or rollers which occupy the four corners of a square. If B is turned clockwise from the phase shown, the pin z will come out of the slot and z and w will be on the outside of the ring, while x will have come out of

its slot on the inside. In this position, the ring slides snugly between *z* and *w* on the outside, and *y* and *x* on the inside, so that *C* cannot turn. Now assume that *B* is turned counterclockwise. Pin *w* will ride up the cam on the outside of the ring on *B*, thus guiding *z* into the inclined slot leading from the outside, and *x* into the slot leading from the inside. Thus *C* is given a 90° rotation in any desired part of a turn of *B*.

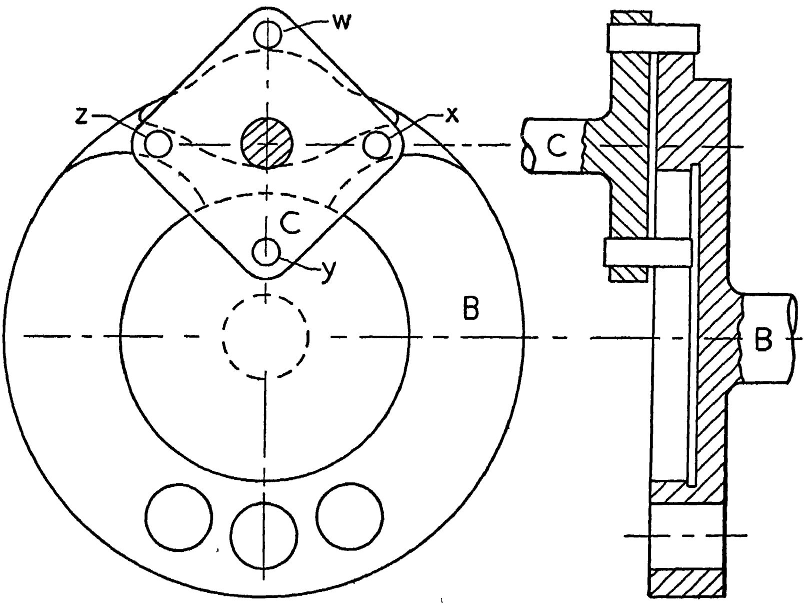


FIG. 13–20. Projector Feed Mechanism.

The film is geared to *C*, and looped above and below so that only a short piece is subject to the acceleration at each advance. Above and below these loops are constant speed feeders. The superiority of this machine lies in the control that the designer has over the contour of the cam surfaces. It is possible to lay them out for ideal acceleration characteristics (Chap. VIII), namely, for uniform acceleration of *C* during the first 45° of turn and uniform deceleration during the remaining 45°. The driver *B* is usually operated at 16 rps, and designed to give *C* the quarter turn in 1/6 revolution; hence the motion of *C* occupies 1/96 sec. The three holes in *B* opposite the cam slots are for balance.

13–9. **Latch Mechanisms.**—The designer is often confronted with the selection or design of a latch, trip, or trigger. Although generally minor parts of machines, their correct design may be of major importance. If an overspeed governor on a turbine or the trip of a circuit breaker oper-

ates when it should not, it causes inconvenience; if it fails to operate when it should, it may cause disaster. In general, the purpose of a latch is to control the operation of a large force by a very small force. Some of the fields of application are electric switches and circuit breakers, guns, governors, safety devices, and automatic machinery. Some of the principles involved will be treated.

Dead-center versus over-center operation. In the trigger latch, Fig. 13-21, if the sliding surfaces *s* are circular arcs described about *P* as center, the mechanism is classified as dead-center. As the trigger is pulled, there is no motion of the bolt *B*, and the resisting force is that due to friction only. If the radius of surface *s* passed below *P*, it would

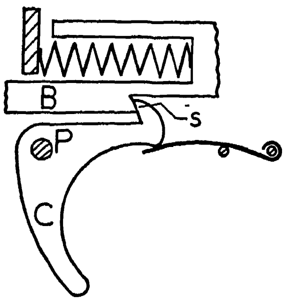


Fig. 13-21. Trigger Latch.

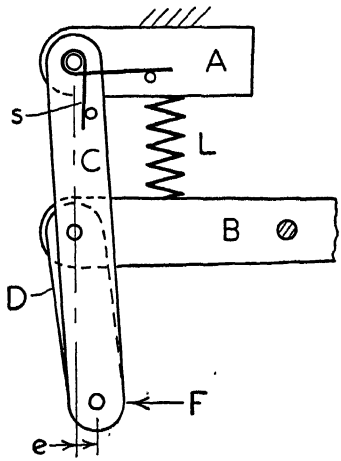


Fig. 13-22. Toggle Latch.

be an over-center latch. Then *B* would have to move back slightly against its spring as the trigger was operated, and the resistance would depend somewhat on the force exerted by the bolt spring, aside from friction.

In Fig. 13-22 we have an example of an over-center toggle latch. *B* might be the operating lever for an oil circuit breaker, with *L* the operating spring. The toggle is over center by the distance *e*, and the tripping force *F* must do the work of compressing *L* slightly as well as overcoming all friction. The resetting spring *s* has slight resistance, being only strong enough to insure that the toggle will swing to the right when *B* is reset.

The advantage of over-center operation is greater safety against accidental tripping due to vibration, shock, or wear. The disadvantage is the larger tripping force necessary.

Sliding versus rolling at the tripping surfaces. It is difficult to eliminate wear at rubbing surfaces which cannot be well lubricated. By

substituting rolling for sliding at the tripping surfaces, a closer approach to dead-center design becomes generally feasible. In Fig. 13–23, the surface of D in contact with the roller C is an arc about P as center, which is the condition for dead-center operation. All rubbing surfaces can be effectively lubricated, and this design may be considered to have the elements of the ideal sensitive latch.

There is great variety in the methods of applying the tripping force. In the control of electrical energy, the solenoid is generally used. As indicated in Fig. 13–23, it consists of an insulated coil of wire surrounding a movable soft-iron core. Current passed through the coil creates a magnetic field which moves the core. In over-speed governors, it is common to have a tripping member retained by a spring against centrifugal force as it revolves with the main shaft. When excessive speed throws this member out, it delivers a succession of blows on the stationary trip lever until the latch is released, closing the governor valve.

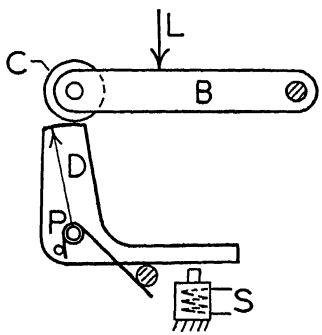


FIG. 13–23. Dead-Center Roller Latch.

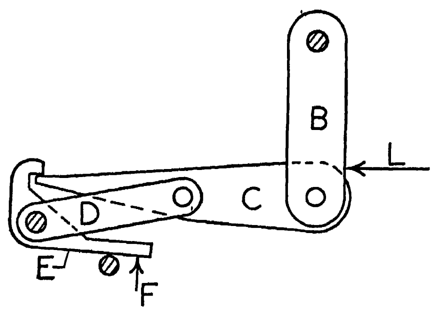


FIG. 13–24. Compound Latch.

Compound latches prove useful where operating forces are unusually large, or where the tripping force must be kept small. In Fig. 13–24 a compound latch is shown using a compression toggle. F is the tripping force, and L the operating force.

In many applications, but particularly in the electric transmission field, speed of action is a prime requirement. Parts must be as small and light as strength requirements will allow. Travels must be short, and the acceleration characteristics examined, for peaks of acceleration retard speed. Electro-magnetic latches [1] have proved satisfactory in certain high-speed applications.

A final example will be taken from the textile field. In the process of making cotton yarn, the cotton goes to a machine called the drawing

[1] Thumim, Carl, "Release Latches," Mechanical Engineering, Nov., 1939.

frame, as a loose fluffy strand called a *sliver* (rhymes with driver). It has little twist and therefore pulls apart easily. In the drawing frame, a number of slivers are fed together and drawn out to form one of much greater evenness. If any sliver breaks, the machine must be stopped instantly. Fig. 13–25 shows the latch. The spoon B is slightly over-

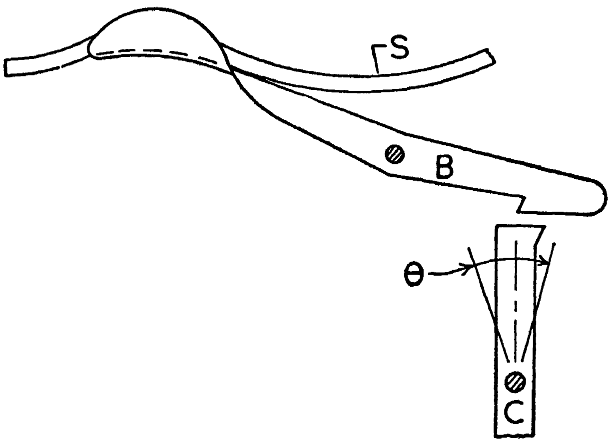


FIG. 13–25. Drawing-Frame Stop.

balanced on the right end. It is held in the position shown by the small force exerted by the sliver S passing over the left end. The lever C reciprocates through the angle θ, being driven from the main shaft. If the sliver breaks, B latches C, moving its fulcrum, and disengaging a clutch which stops the machine. There is a separate latch and spoon for each sliver.

13–10. **Locks.**—Since man has had articles of value to protect, he has felt the need for locks. In ancient and modern times alike, the ingenuity of the lock designer has been pitted against the dubious skill of the lock picker. Of the many interesting mechanisms evolved, two, which are most widely used at present, will be described.

Fig. 13–26 shows the lever lock. The frame is A, and a single lever C is shown hinged in A. The bolt B is in locked position. It carries the stop n, which projects through a cut-out in the lever so that the lever must be raised to bring n opposite the gate e, before the bolt can be moved to the right. Key k is in position to unlock when turned clockwise. Land m must be exactly high enough to raise the lever so that n will slide through e. Also h must have correct form to engage s and slide the bolt.

The lock can be given many levers in parallel, n projecting through them all, and each requiring a land of different height on the key. This gives the desired variety in keys.

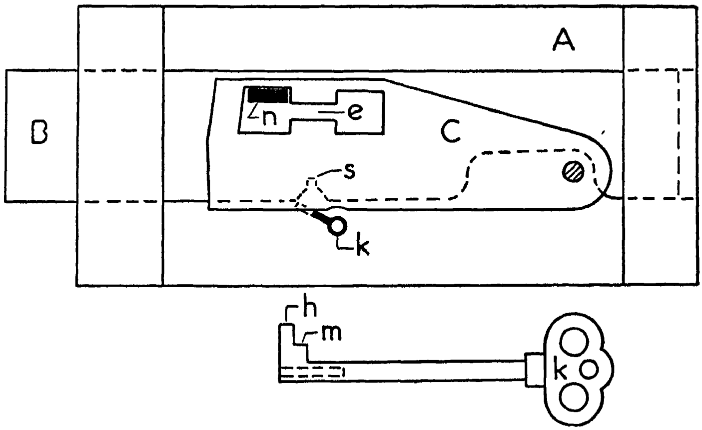


FIG. 13–26. The Lever Lock.

The Yale lock, Fig. 13–27, is most widely used in the United States. Inside the cylindrical frame is another cylinder which lies between the lines a and b in the sectional view. With the proper key in place, as shown, the five cylindrical plugs are each divided into two, along the surface of the cylinder ac. Torque on the key will therefore turn the cylinder, which moves the bolt by a tongue connection on the inner end (not shown). The plugs prevent the withdrawal of the key until the cylinder is returned to the phase where the plug segments are in line. Then the plugs can rise, compressing their springs as much as necessary to allow the removal of the key. With the key out, the springs drive the plugs down, and the upper segments all key the cylinder, preventing rotation. Any key which does not raise every plug to exactly the designed level cannot turn the lock. The bevelled end of the key enables it to raise all plugs as it is inserted.

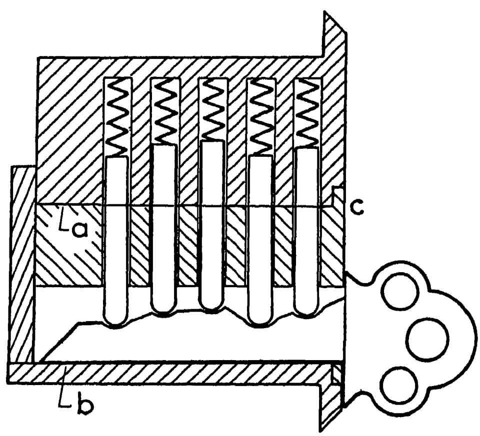


FIG. 13–27. The Yale Lock.

PROBLEMS

1. Shaft B rotates at the constant speed of 60 radians per second and drives shaft C through a universal joint. Shaft C makes an angle of 20° with the projection of B. Draw a curve showing the speed variation of C for a cycle. Plot every 15°.
2. A manufacturer desired to know if it would be feasible to replace both clutch and gear box in a small automobile by using a brush-wheel friction gear. For an engine speed of 3000 rpm, he desired rear-axle drive speeds ranging from 1500 rpm forward, to 200 rpm in reverse. Report on the kinematic phases of the problem.
3. On an escapement for a very large clock, it was desired to use rollers on the escape wheel so that the contact with the anchor arms would be rolling contact. Make a lay-out of the anchor arms and a sector of the escape wheel to give dead-beat operation.
4. Design and draw a pair of intermittent gears such that the driver, in each revolution, will turn the driven gear through 90° while the driver is turning 120°.
5. On the Geneva Stop of Fig. 13–18, determine the angular acceleration of link C, when link B has moved 60° counterclockwise from the position shown, and compare your result with the 30° ordinate of the curve shown in Fig. 13–19 (c).
6. A projector mechanism, Fig. 13–20, has the circular part of the outside of B, 6 in. in diameter. The pins, w, x, y, and z, are 0.4 in. in diameter, and have their centers on a square of sides 2 in. C turns 90° while B turns 60°. Determine the outside cam curve on B for ideal parabolic acceleration of C.
7. Design a compound trip mechanism such that the force at the tripping surfaces (contact between C and E in Fig. 13–24) will be one-tenth of the controlled force, neglecting friction. (Note that forces are inversely proportional to the components of velocities in the directions of the forces.)
8. Four doors are to be equipped with Yale locks arranged so that four individual keys will unlock only one door each, but one master key will unlock all four. There are several ways to do this. Present one workable plan.
9. It has been proposed that two shafts at right angles and in the same plane be connected by one or more universal joints so that one shaft would drive the other at 1000 rpm. Make a report covering the kinematic phases of the problem.

10. A box manufacturer requires a mechanism that will *translate* a part in a rectangular path, and cause it to come to rest at each corner. Devise the mechanism, to be driven from a shaft rotating at constant speed.

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APPENDIX I

GRAPHICAL SOLUTIONS AND PROBLEMS FOR THE DRAFTING ROOM

The effectiveness of the graphical method in solving many types of problems in kinematics has been abundantly demonstrated. Some remarks on procedure may be helpful in assuring satisfactory results and controllable accuracy.

The tools should first be given attention. The left edge of the drawing board or table should be straight and smooth. T-squares and set squares should be tested by the turn-over method to insure that they are square and straight. If drafting machines are used, they should be tested for looseness or lack of firmness. Of course, many of the simpler graphical solutions can be made working on any plane table with set squares only.

Quite thin, sharp, light lines should be used. Pencils of 3H or 4H hardness are best. They remain sharp, they do not fog the drawing, and the lines, if light, are easily erased. After all results are obtained, the drawing can be quickly lined in with softer pencil for record or for reproduction.

The scales selected for mechanism and vectors should be such as to give figures of moderate size. Figures larger than the normal range of the instruments used are unlikely to give increased accuracy. Location of points by the intersection of lines at very small angles is a frequent source of error. Often such values can be checked by an alternative method.

A convenient size of drawing for many problems is 18 × 24 in. A suggested arrangement is shown in Fig. *A*. The left margin will serve for binding. The title block is placed in the lower right corner for convenience in filing.

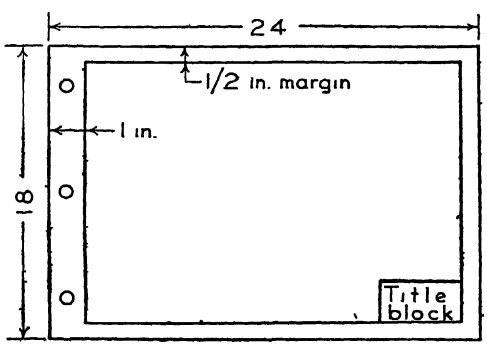

Fig. A.

The title block may contain:

Title

Scale

Date started

Date completed

Desk No. Name

If various scales are used, specify each under the part to which it relates.

The following problems will be found adaptable for solution in the drafting room:

Chap. I—8, 9, 10, 11, 12, 13, 14, 15, 16.
Chap. II—7, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23.
Chap. III—4.
Chap. IV—8, 9, 10, 11, 12, 13, 14.
Chap. V—4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17.
Chap. VI—6, 7, 8, 9, 10, 11, 12, 13, 14.
Chap. VII—7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17.
Chap. VIII—6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22.
Chap. IX—28, 29.
Chap. X—12, 14, 20, 21.
Chap. XII—3, 6, 7, 11.
Chap. XIII—1, 3, 4, 5, 6, 7, 10.

APPENDIX II

RITTERHAUS CONSTRUCTION FOR ACCELERATION ON THE SLIDER-CRANK MECHANISM

The Ritterhaus method is a time saving graphical construction applicable to the slider-crank chain only. The conventional acceleration diagram is shown in broken line in Fig. B (see also Fig. 7–5) except that the scale of acceleration is chosen such that P_{ON} which is $B\omega_B^2$ is represented by the length of the crank B. Assuming first that the mechanism is drawn to full size, the acceleration scale becomes

$$1 \text{ in.} = \omega_B^2 \text{ in. per sec}^2$$

If the conventional diagram is revolved about O through 180° it will have the position $OPKR''$ and all vectors will have reversed sign.

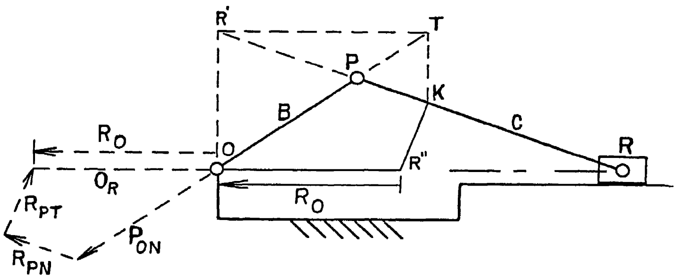

FIG. B.

R_{PN} will now be represented by PK. To locate K, first draw through the phorograph of R on B the line $R'T$ parallel to the path of R giving point T at the intersection with the projection of B, then through T draw TK parallel to $R'O$ giving K at the intersection with C. The proof follows.

By construction, triangles $OR'P$ and TKP are similar, also ORP and $TR'P$.

$$R_{PN} = C\omega_C^2 = \frac{C^2\omega_C^2}{C} = \frac{(R'P)^2\omega_B^2}{C}$$

To obtain the length of R_{PN} on the drawing divide by ω_B^2.

$$R_{PN} = \frac{(R'P)^2}{C} \doteq \frac{PT}{B}\,(R'P) = (PT)\,\frac{PK}{PT} = PK$$

If the machine is not drawn full size but to a scale 1 in. = m in., the acceleration scale becomes 1 in. = $m\omega_B^2$ in. per sec^2. The method fails to give results for the dead-center phases.

APPENDIX III

KLEIN'S CONSTRUCTION FOR ACCELERATION ON THE SLIDER-CRANK MECHANISM

Klein's construction, like that of Ritterhaus, applies only to the slider-crank chain. Assuming that the mechanism, Fig. C, is drawn to full size, and that the acceleration scale is 1 in. = $\omega_B{}^2$ in. per sec^2, the acceleration diagram can be quickly constructed in the following manner.

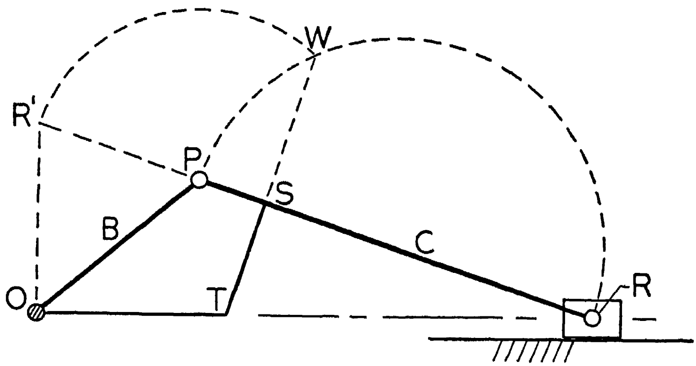

Fig. C.

Draw a semicircle PWR on the connecting rod as diameter. Through R', the phorograph of R on B, draw the arc $R'W$ with P as center. Through W draw WS normal to C. Then $OPST$ is the required acceleration diagram for the phase shown, and TO represents the acceleration of R. The diagram is similar to that of Fig. 7–5 except that all vectors have reversed sign, also that the scale, as given, must be such that the length of the crank represents the normal acceleration of the crank pin.

The proof depends on the evaluation of PS as representing the acceleration R_{PN}. Since the triangles PWR and PSW are similar by construction,

$$\frac{PS}{PW} = \frac{PW}{C} \qquad \text{or} \qquad PS = \frac{(PR')^2}{C}$$

Also,

$$\frac{\omega_C}{\omega_B} = \frac{PR'}{C} \qquad \text{or} \qquad (PR')^2 = \frac{C^2\omega_C{}^2}{\omega_B{}^2}$$

Therefore,

$$PS = \frac{C\omega_C{}^2}{\omega_B{}^2}$$

Multiplying by the scale gives the acceleration represented by PS.

$$PS = C\omega_C{}^2 = R_{PN}$$

If the mechanism is drawn to a scale of 1 in. = m in., the acceleration scale becomes 1 in. = $m\omega_B{}^2$ in. per sec^2.

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